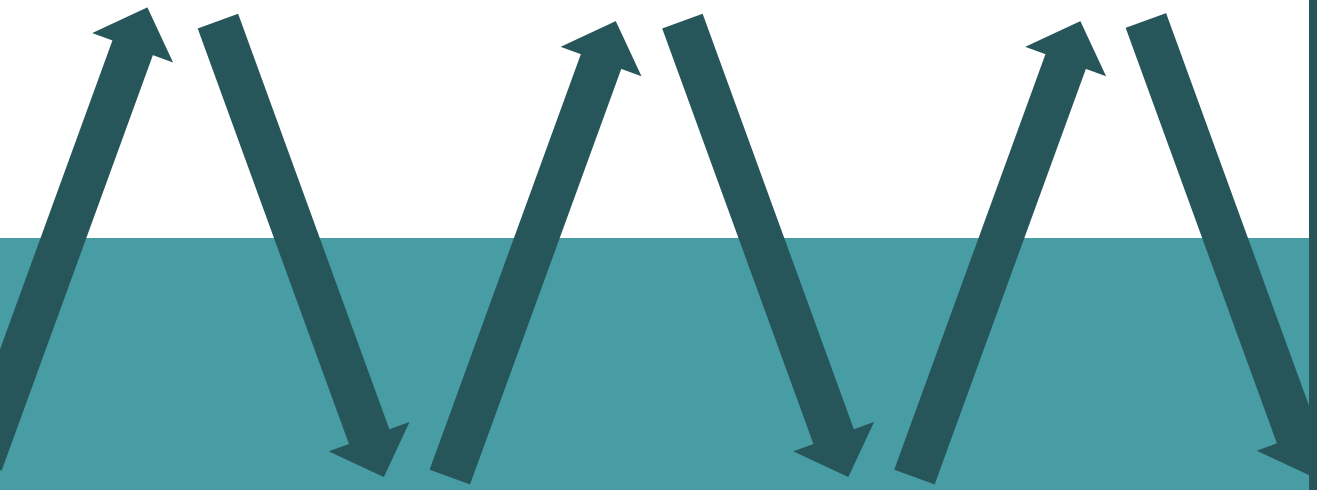
Bombs don't explode, the forgers get rich:



an adaptive attack on Wiesner's quantum $.

QCRYPT 2014

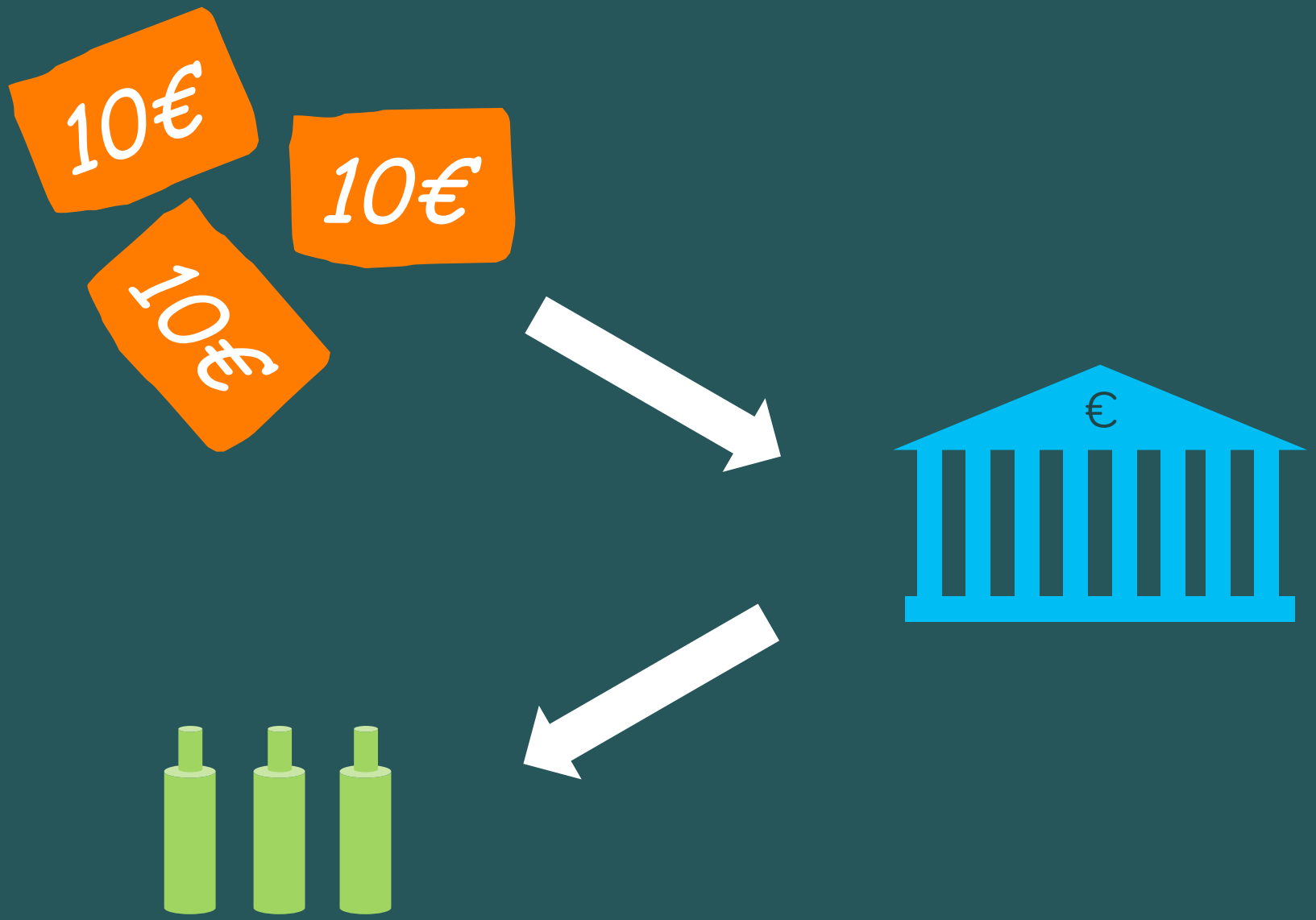
Or Sattath
Aharon Brodutch
Daniel Nagaj

1404.1507

SAV

Simons Institute for the Theory of Computing

10€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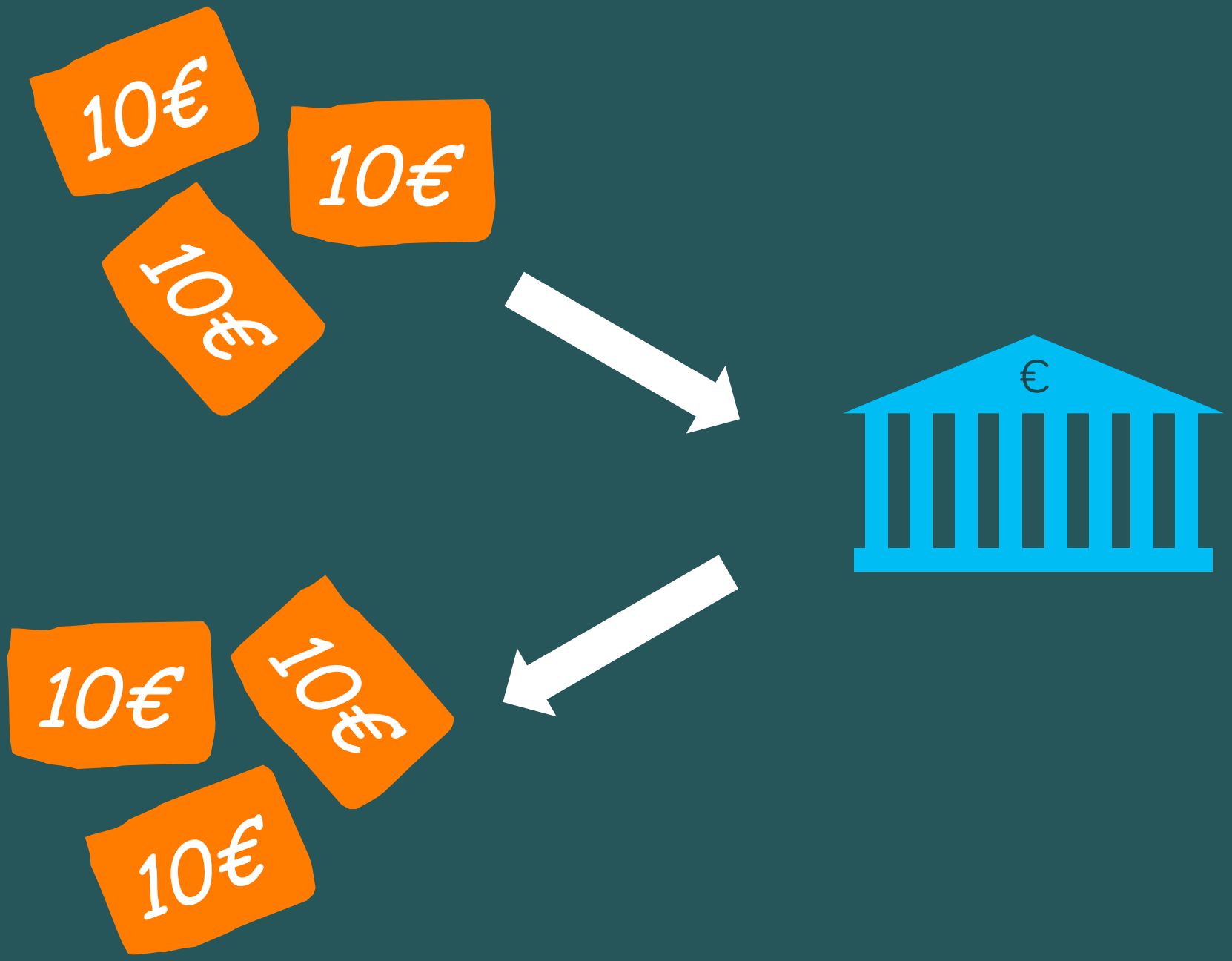
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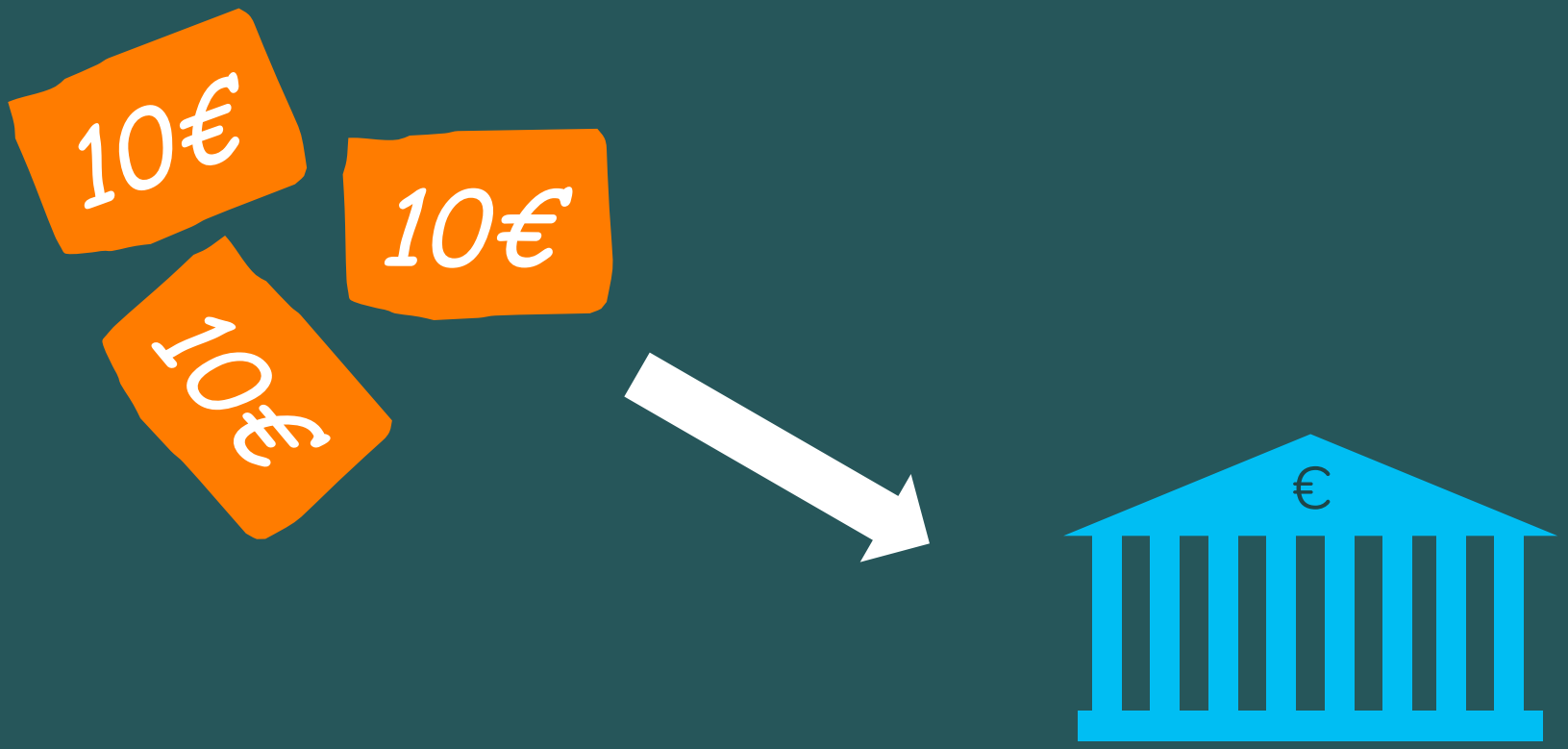
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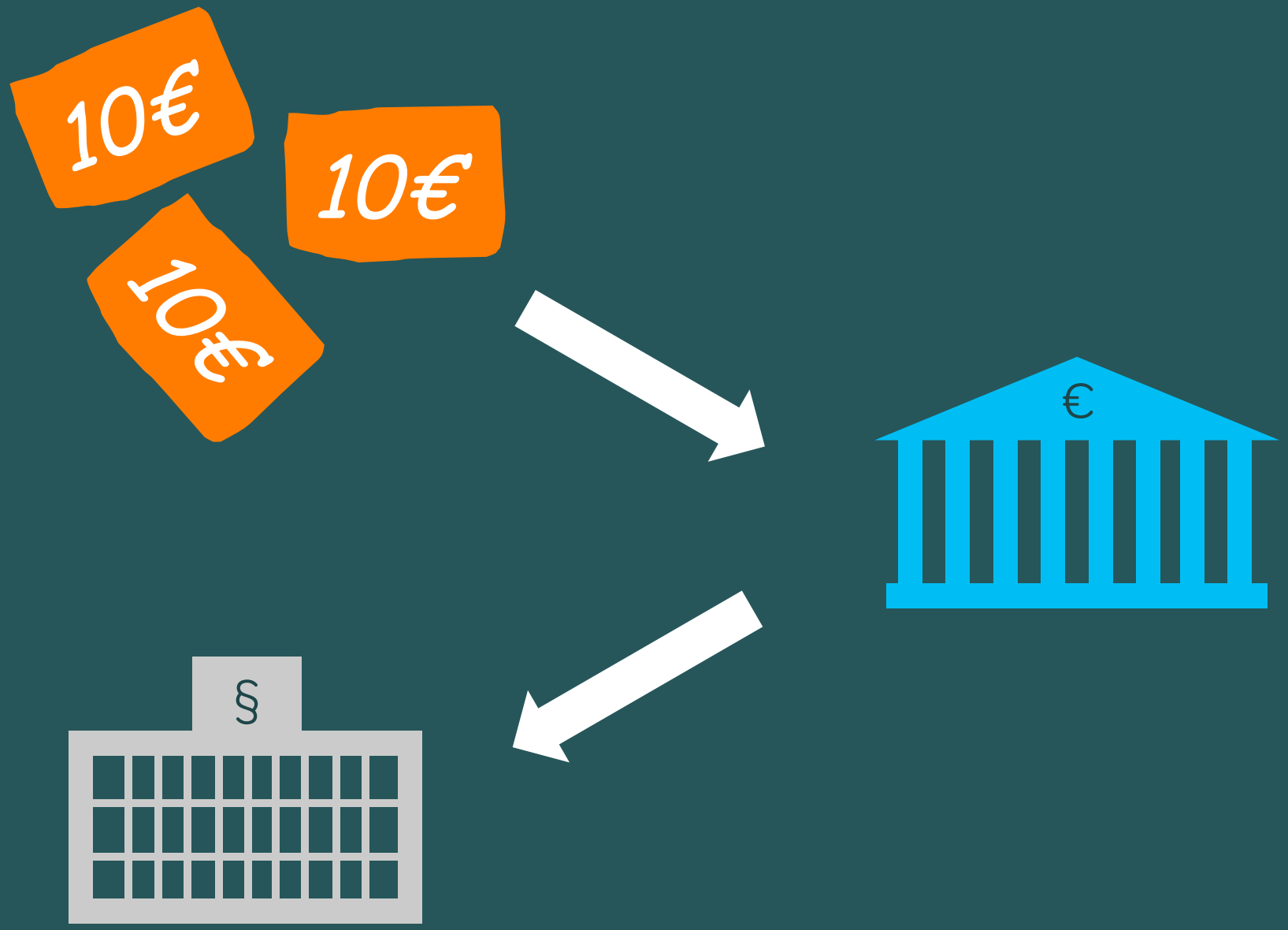
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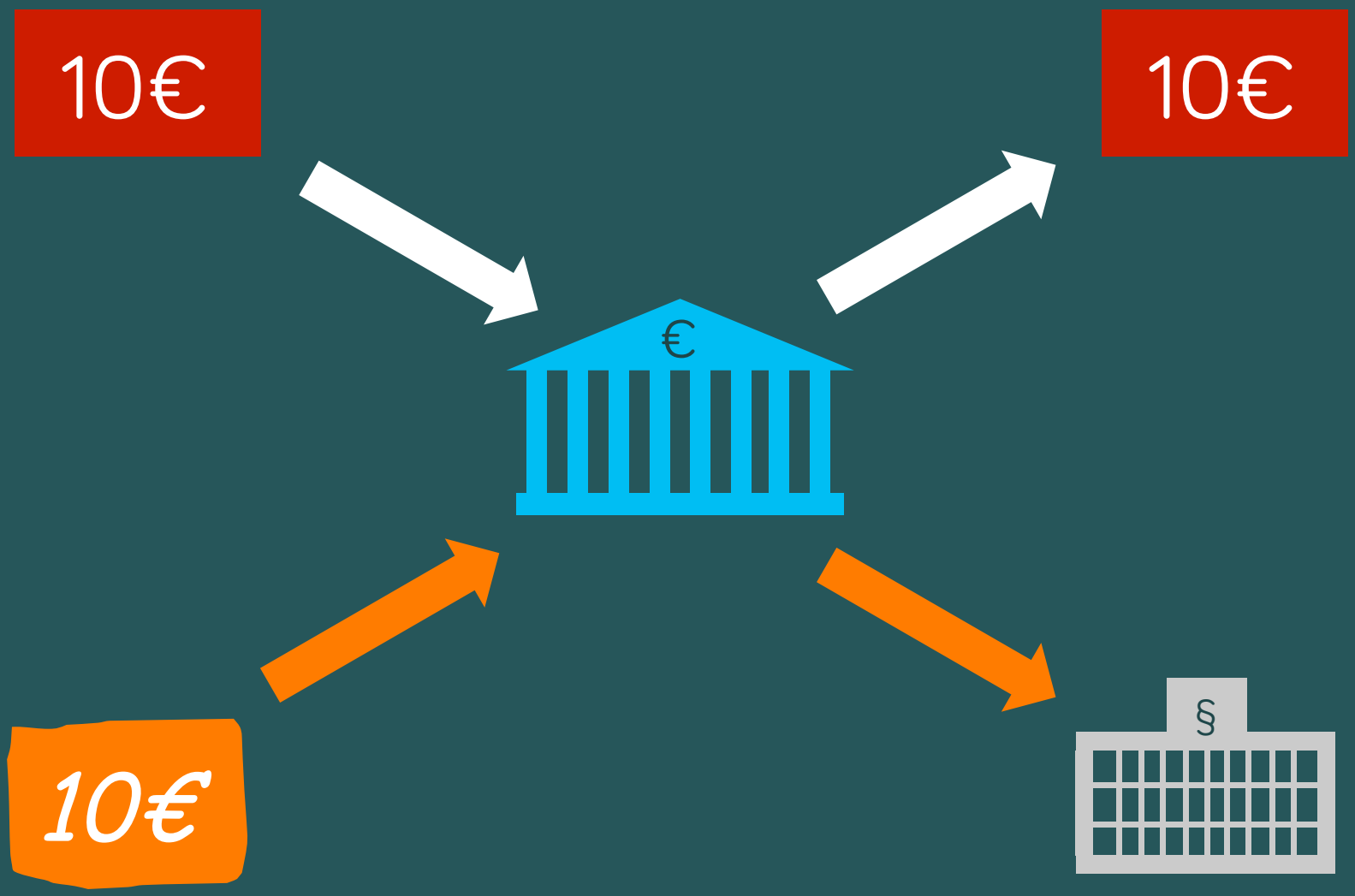
strict testing



1 expensive states

validity, (re)usability & strict testing

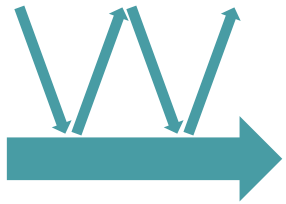
100€

2 money and bombs

quantum Zeno effect & successful forgery

3 measuring weakly

strict testing & single-copy tomography



strict testing &
Wiesner's money

1 Wiesner's quantum money ['69, published '83]

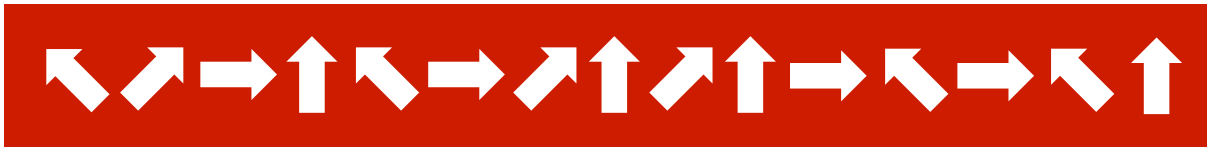
serial number **s**

secret state $|\psi_s\rangle = |c_1^s\rangle|c_2^s\rangle \cdots |c_n^s\rangle$

$c_i^s \in \{0, 1, +, -\}$

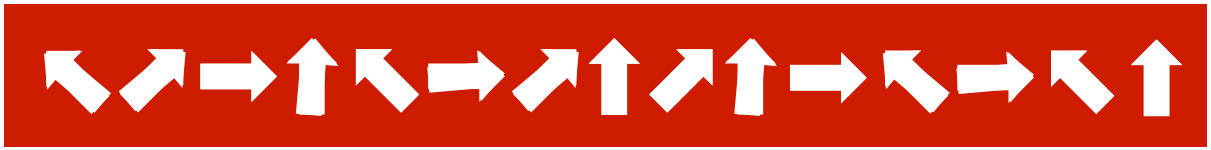
1 Is it secret? Is it safe?

- verify-only memory, unforgeable tokens [BBBW '83]



- guaranteed safe for a single use [Molina et al. '12]

$\left(\frac{3}{4}\right)^n$ safest: 6 states $\left(\frac{2}{3}\right)^n$

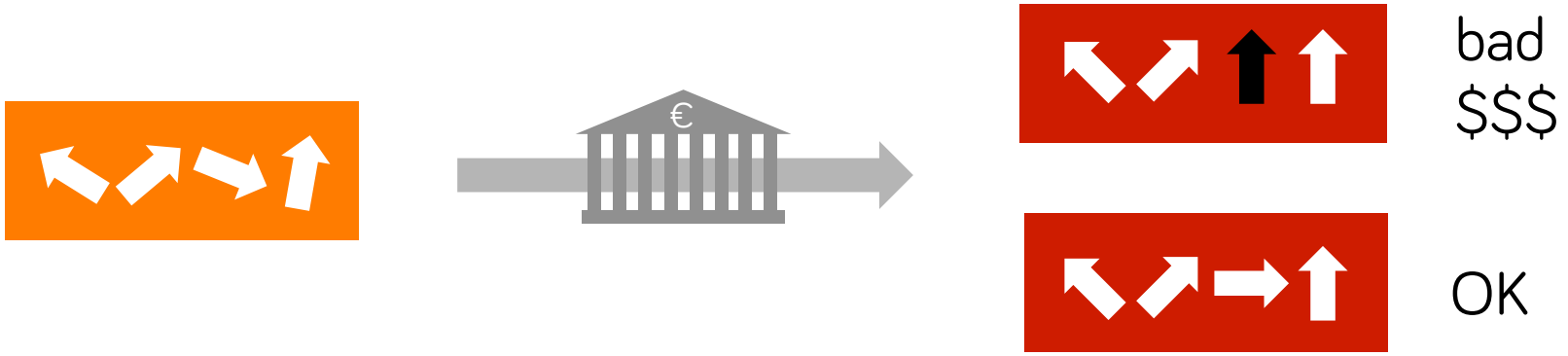


- OK with some noise [Pastawski et al. '11]
  classical communication is enough [also Gavinsky '11]

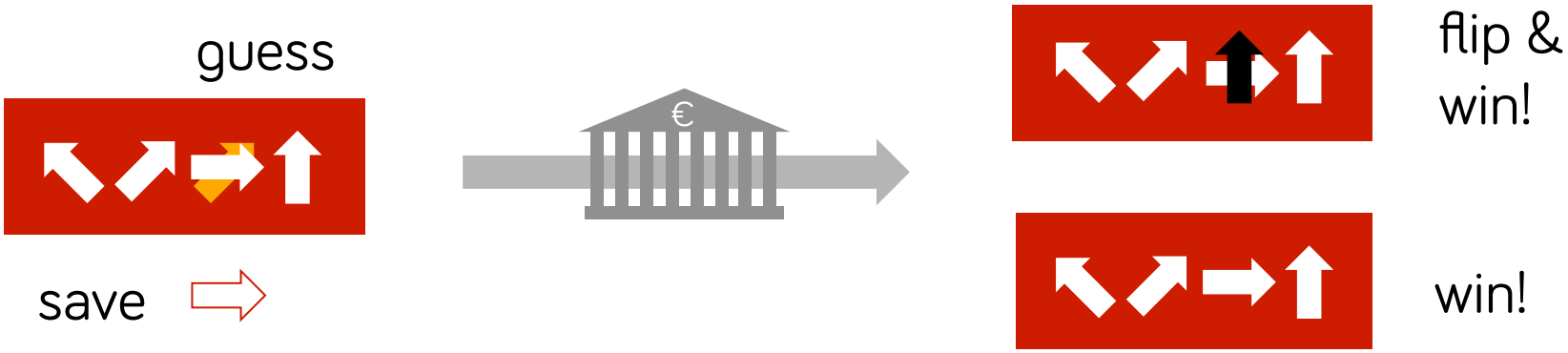
+ × + × × × + × + + × + + × +

1 Asking for “repairs” (and returns of bad states)

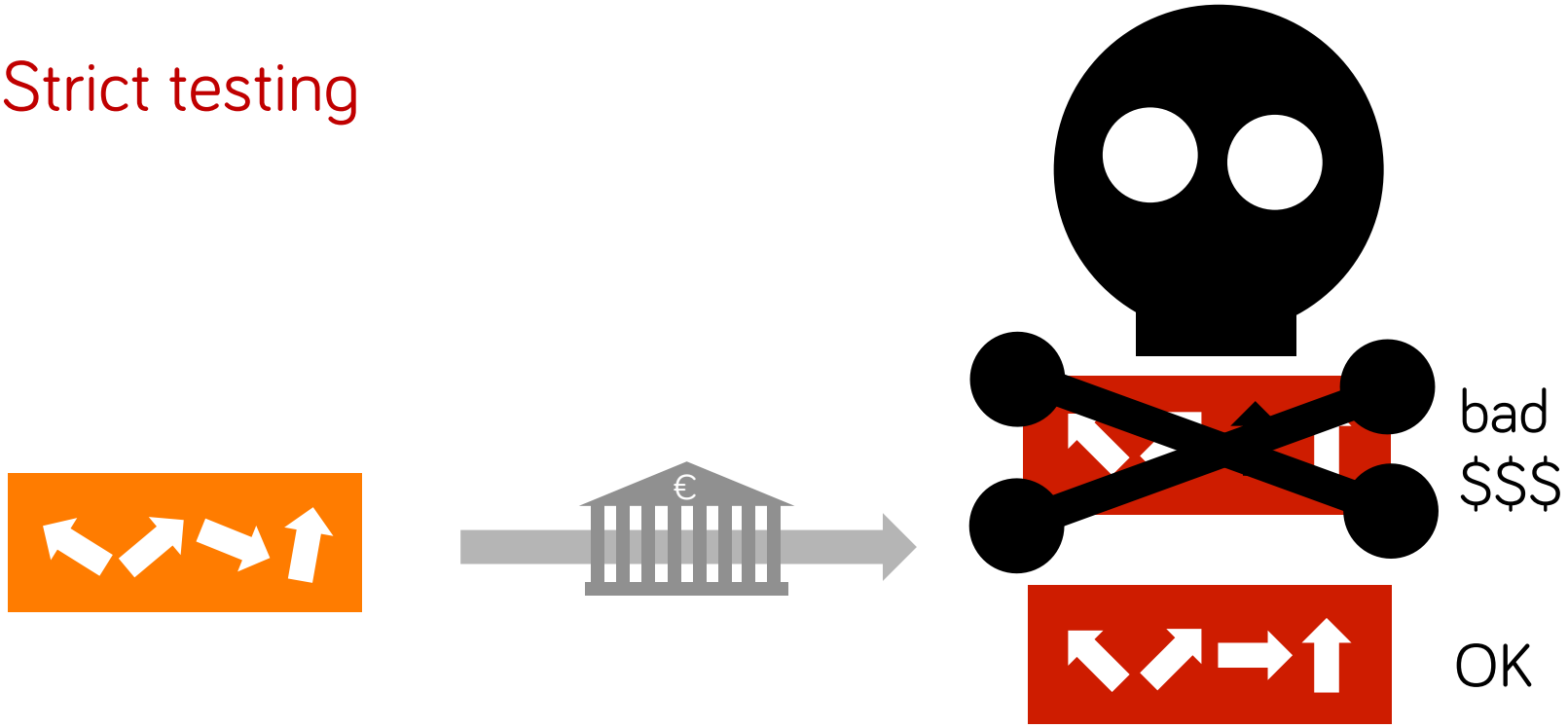
- validating “old” bills



- Lutomirski’s attack



1 Strict testing



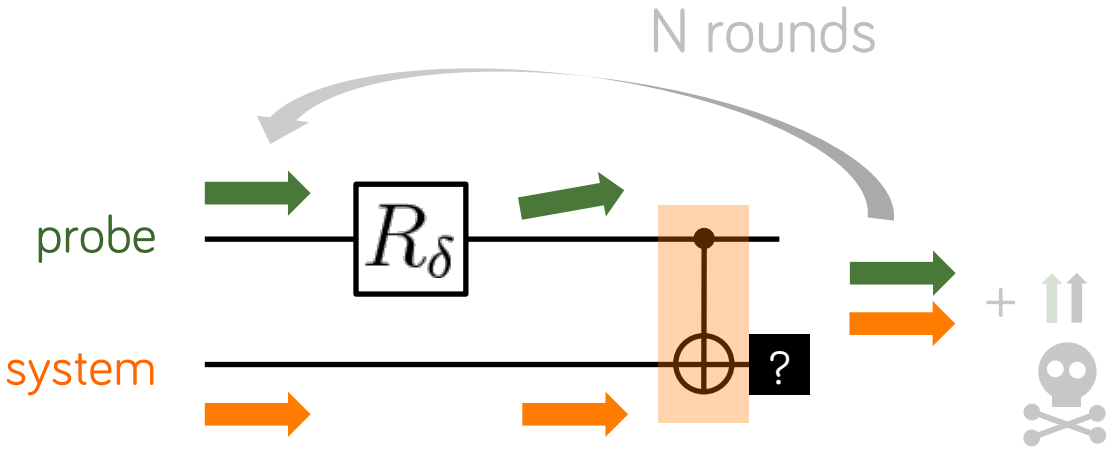
- is forgery still worth it?

We show an efficient adaptive attack on Wiesner's quantum money scheme (and its variant by Bennett et al.), when valid money is accepted and passed on, while invalid money is destroyed. Our approach is based on the quantum Zeno effect, also known as Elitzur-Vaidman's bomb tester. [1404.1507]

testing
quantum
bombs
carefully

2 The Elitzur-Vaidman bomb tester

- “bomb”



final state

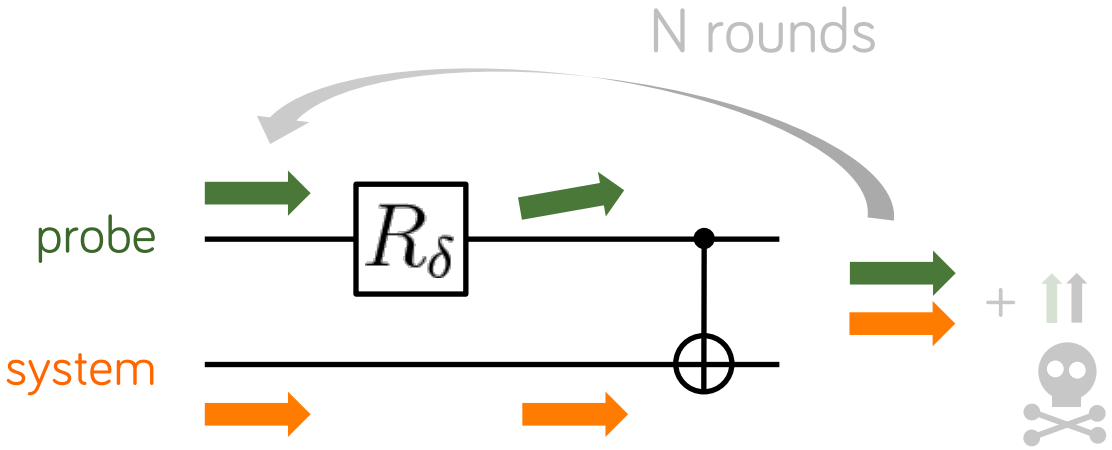
be careful!

$$\delta = \frac{\pi}{2N}$$

$$p_{\text{☠}} \propto N\delta^2 \propto \frac{1}{N}$$

2 The Elitzur-Vaidman bomb tester

- "bomb"



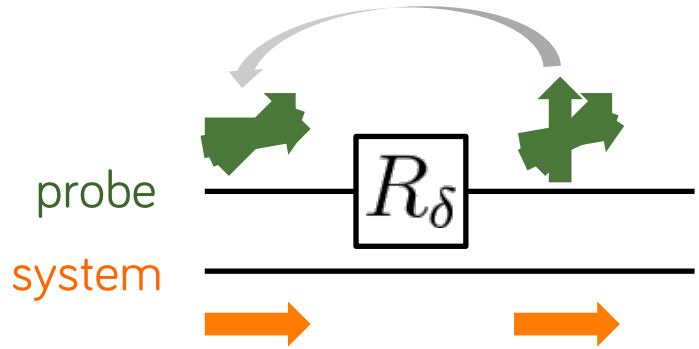
final state

be careful!

$$\delta = \frac{\pi}{2N}$$

$$p_{\text{☠}} \propto N\delta^2 \propto \frac{1}{N}$$

- "no bomb"

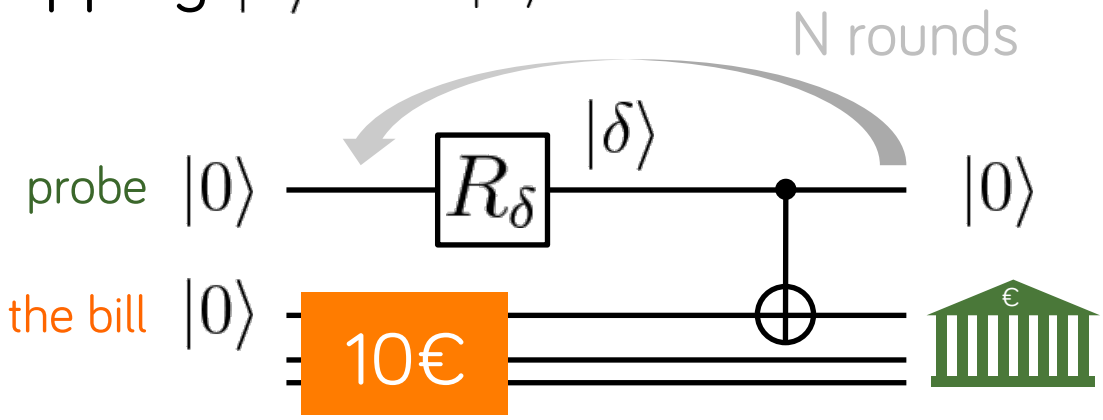


final state

all clear

2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$



final state: $|0\rangle$

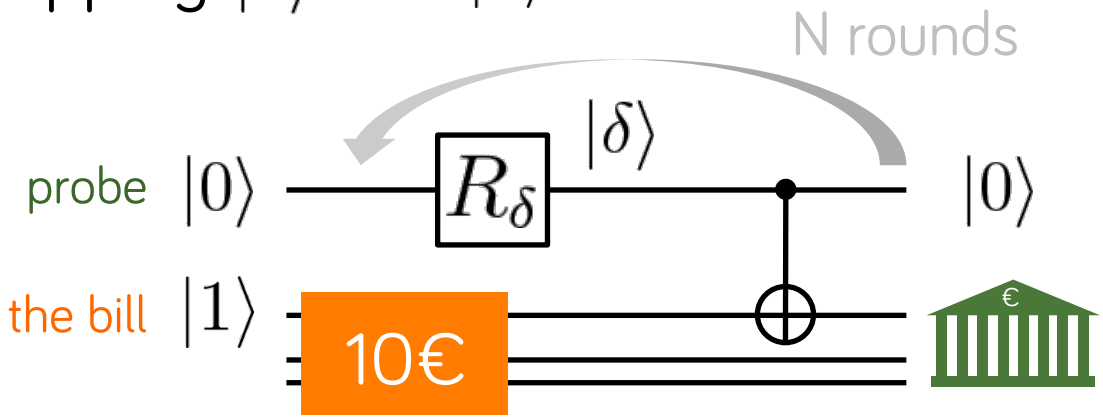
$$p_{\text{☠}} \propto \frac{1}{N}$$

$\cos\delta\,|0\rangle|0\rangle$ €

$+\sin\delta\,|1\rangle|1\rangle$ ☠

2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$



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$\cos\delta\,|0\rangle|1\rangle$ €

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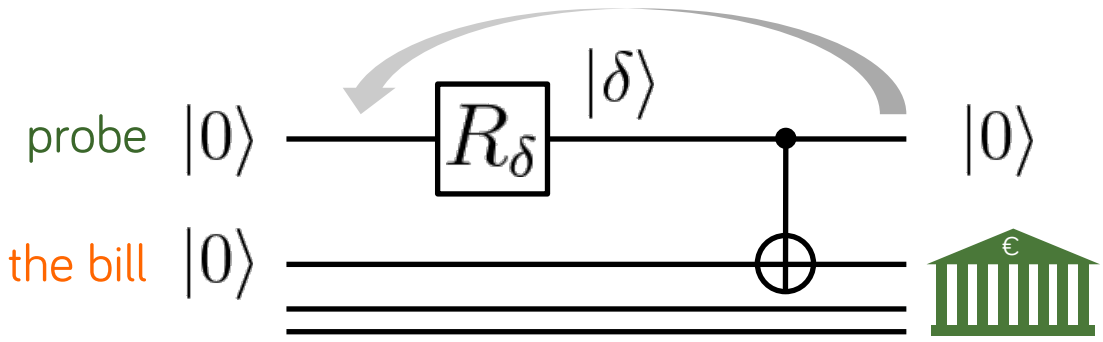
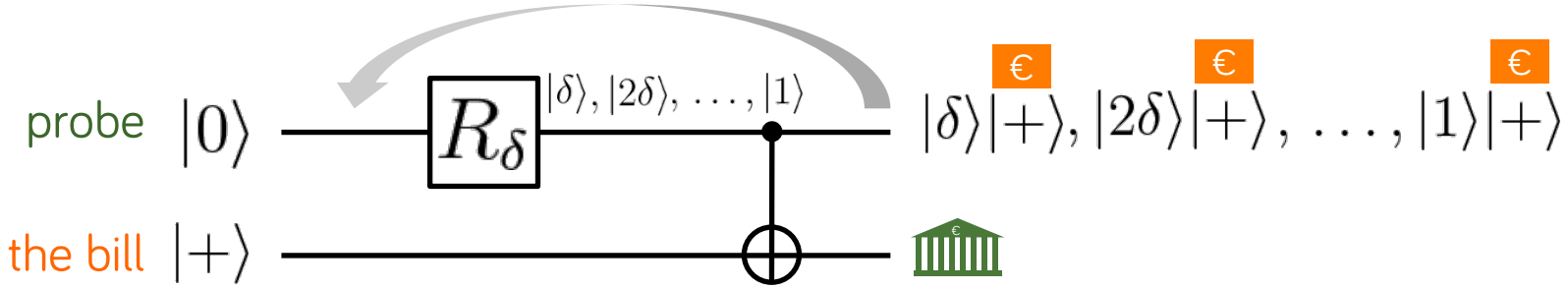


- keeping $|+\rangle$

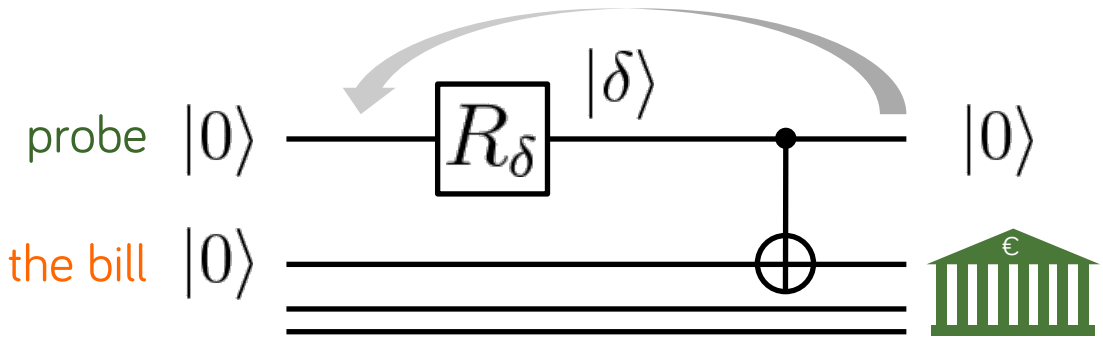
final state: $|1\rangle$



2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$

final state: $|0\rangle$



$$p_{☠} \propto \frac{1}{N}$$

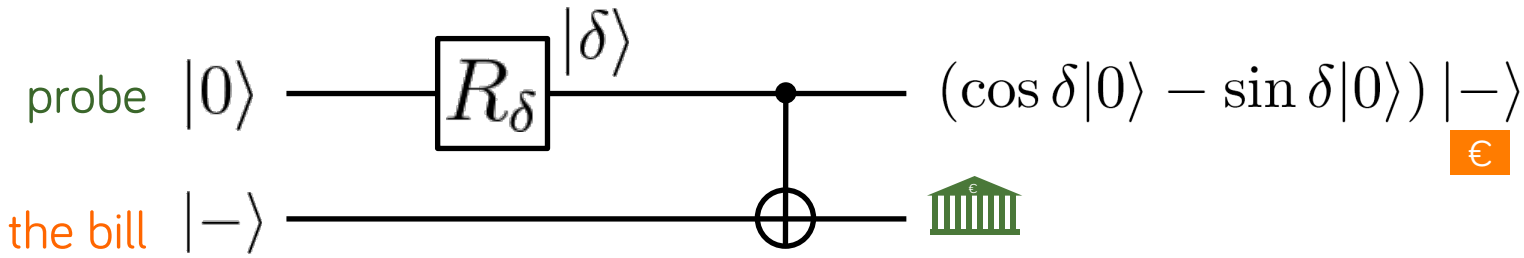
$$\cos\delta\,|0\rangle|0\rangle$$ €

$$+\sin\delta\,|1\rangle|1\rangle$$ ☠

- keeping $|+\rangle$

final state: $|1\rangle$

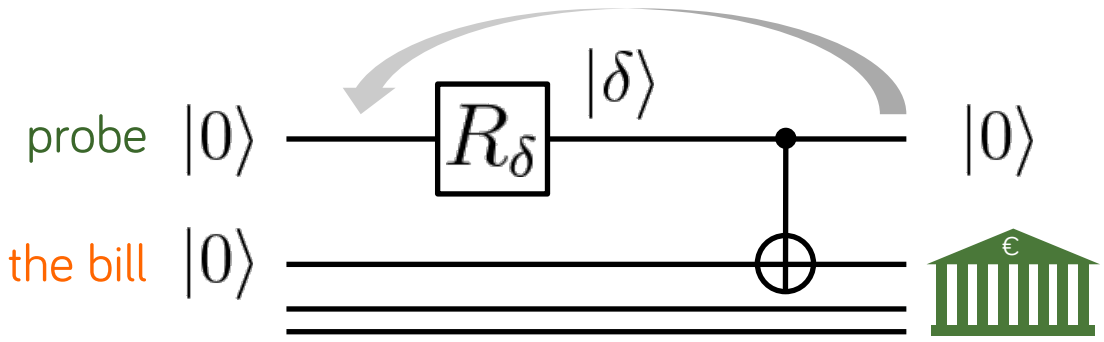
- fun with phases on $|-\rangle$



2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$

final state: $|0\rangle$



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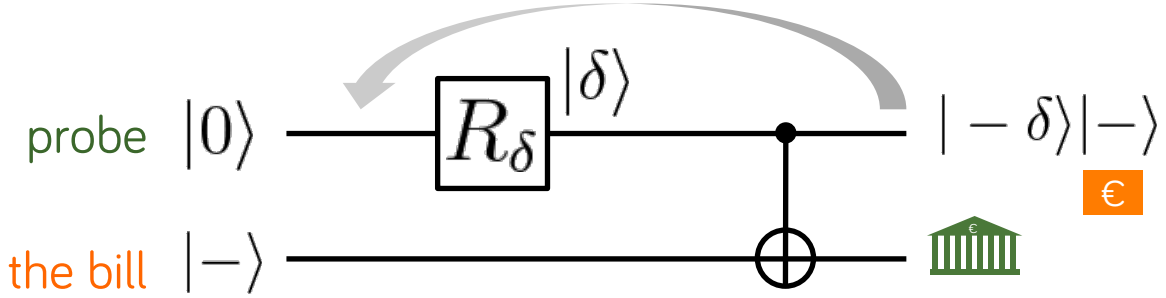

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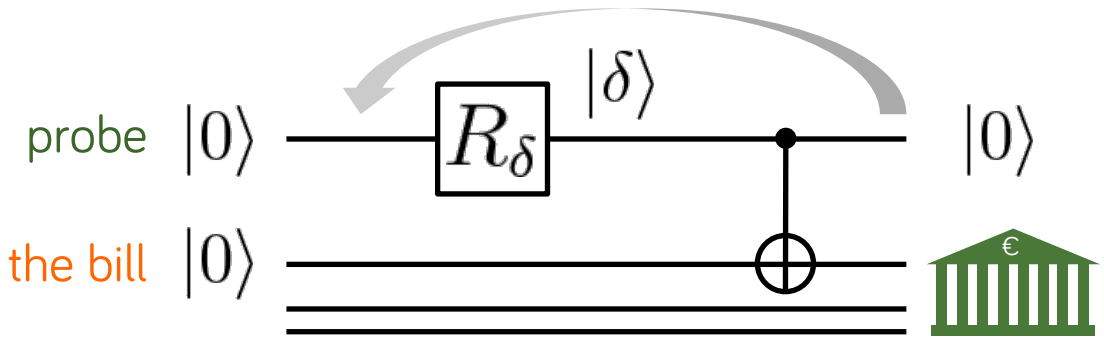
- flipping $|0\rangle$ and $|1\rangle$

final state: $|0\rangle$



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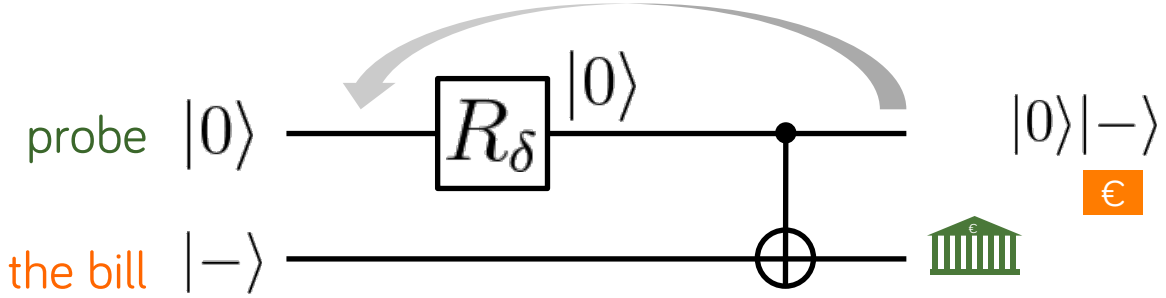

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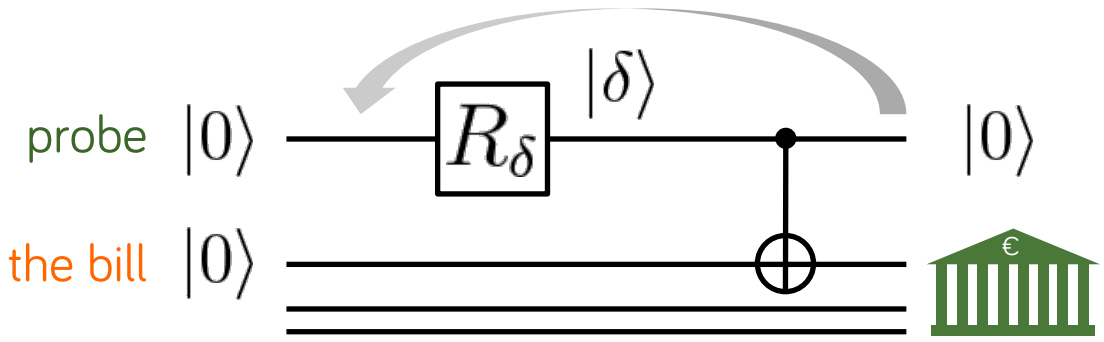

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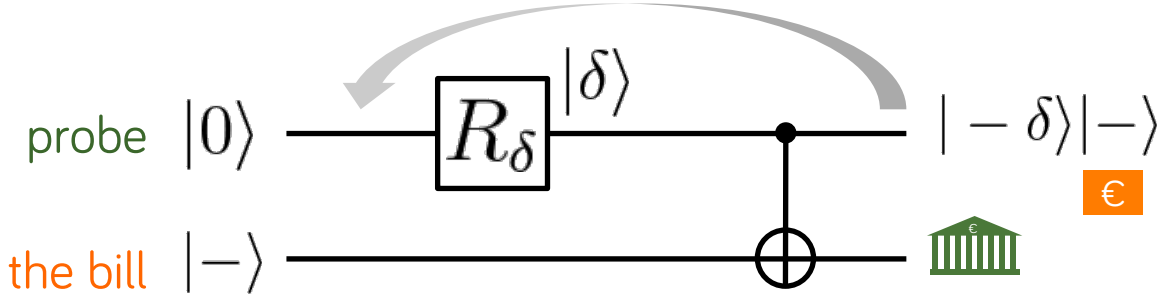

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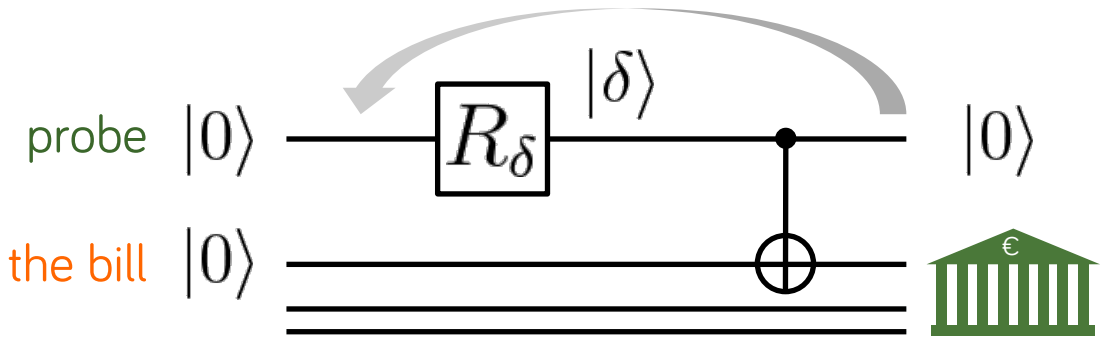

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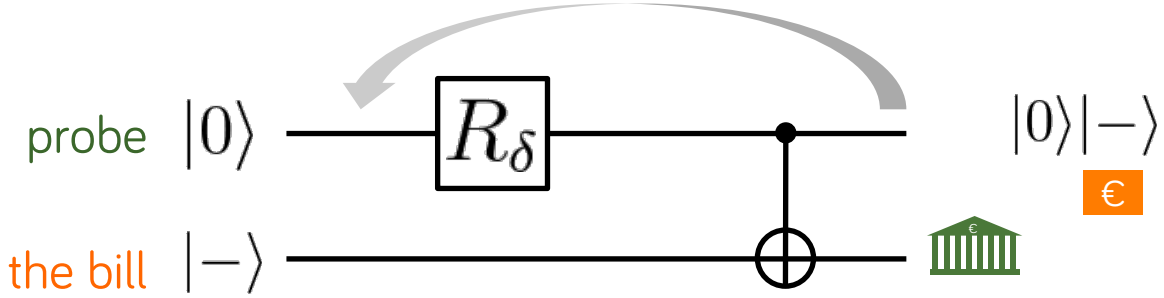
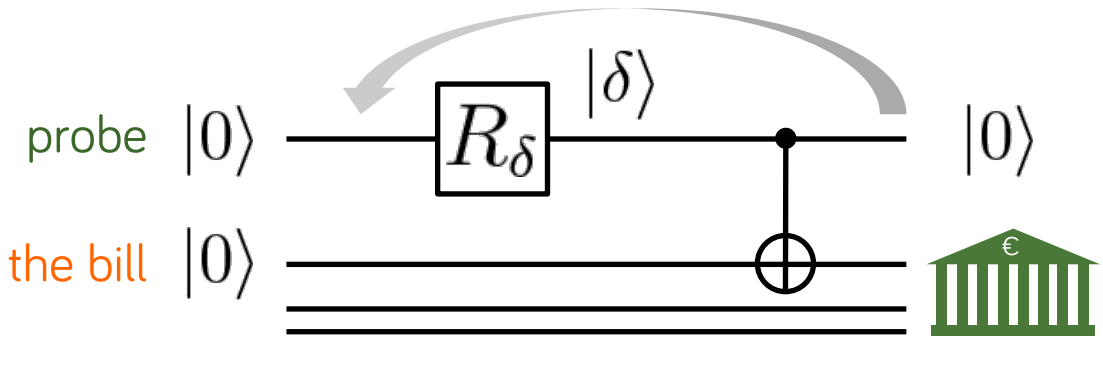


2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$

final state: $|0\rangle$



- keeping $|+\rangle$

final state: $|1\rangle$

- fun with phases on $|-\rangle$

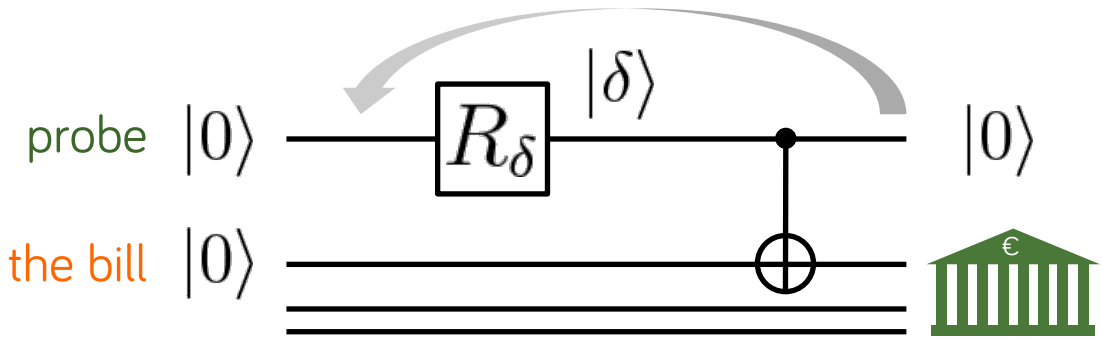
final state: $|0\rangle$

- identifying a state besides $|+\rangle$?

2 Validating slightly modified quantum money

- flipping $|0\rangle$ and $|1\rangle$

final state: $|0\rangle$



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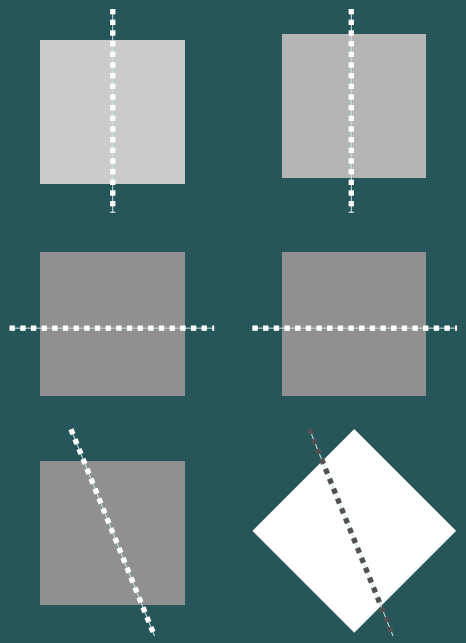
- keeping $|+\rangle$

final state: $|1\rangle$

- fun with phases on $|-\rangle$

final state: $|0\rangle$

adaptive verification + bomb-testing = $$$



single-copy tomography

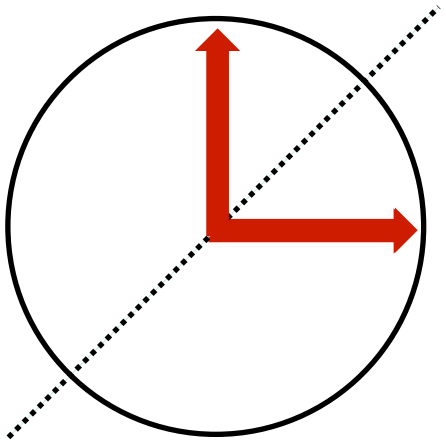
from strict testing

3 Generalizing Wiesner's money

- a different list of states? $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle, |y+\rangle, |y-\rangle, \dots\}$
  not a problem

- completely unknown states?
  guess an axis to flip about … imperfect bombs

a "bomb"



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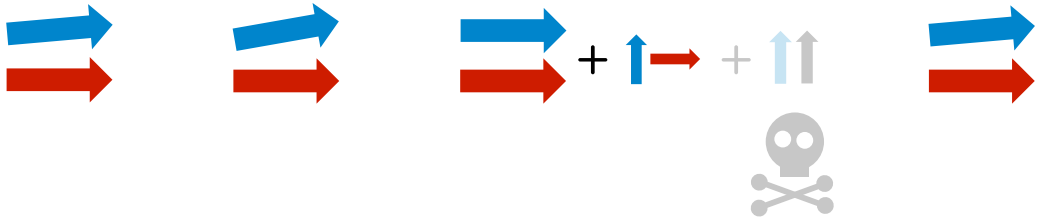
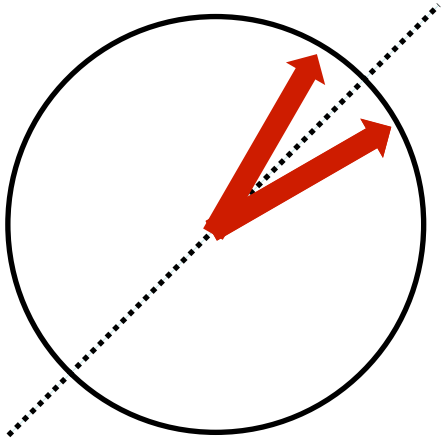
almost a "bomb"





the probe converges to a small fixed angle

3 Generalizing Wiesner's money

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  not a problem

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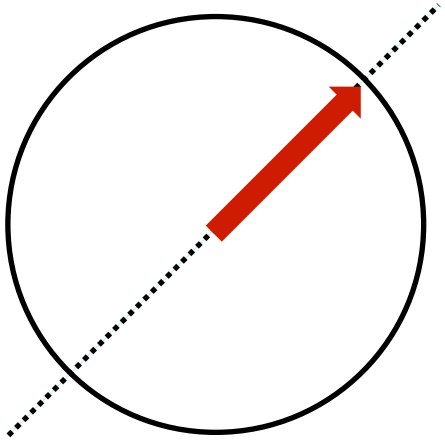
almost a "bomb"

almost "no-bomb"



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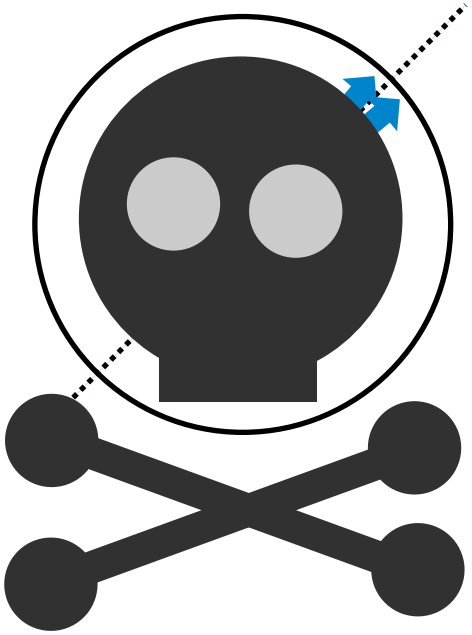
  guess an axis to flip about … imperfect bombs

almost a "bomb"

a messy case

almost "no-bomb"



seems safe … probe more & more …

3 Generalizing Wiesner's money

- a different list of states?
  not a problem

$$\{|0\rangle, |1\rangle, |+\rangle, |-\rangle, |y+\rangle, |y-\rangle, \dots\}$$

- completely unknown states?
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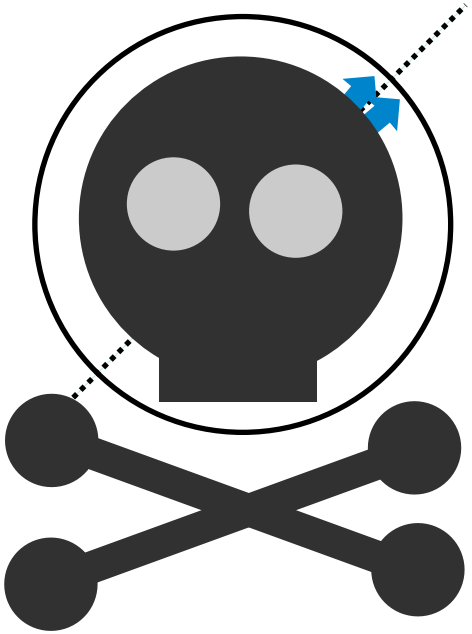
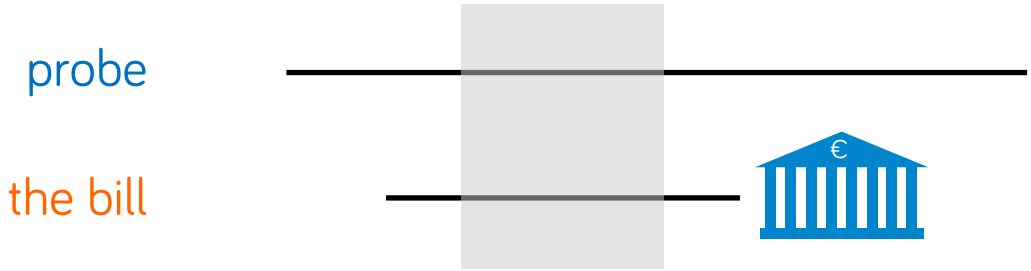


- tomography with strict testing?

3 Modular weak measurement

- an interaction that is always weak



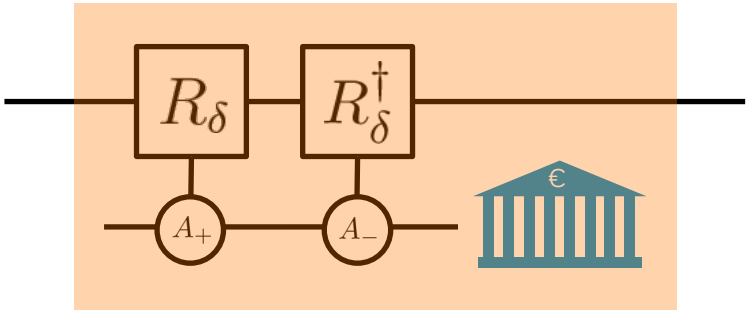
$$p_{☠} \propto \delta^2$$

3 Estimating $\langle A \rangle$ for a Pauli operator

- how much does A mess up the state?

probe

the bill



$p_{☠} \propto \delta^2$

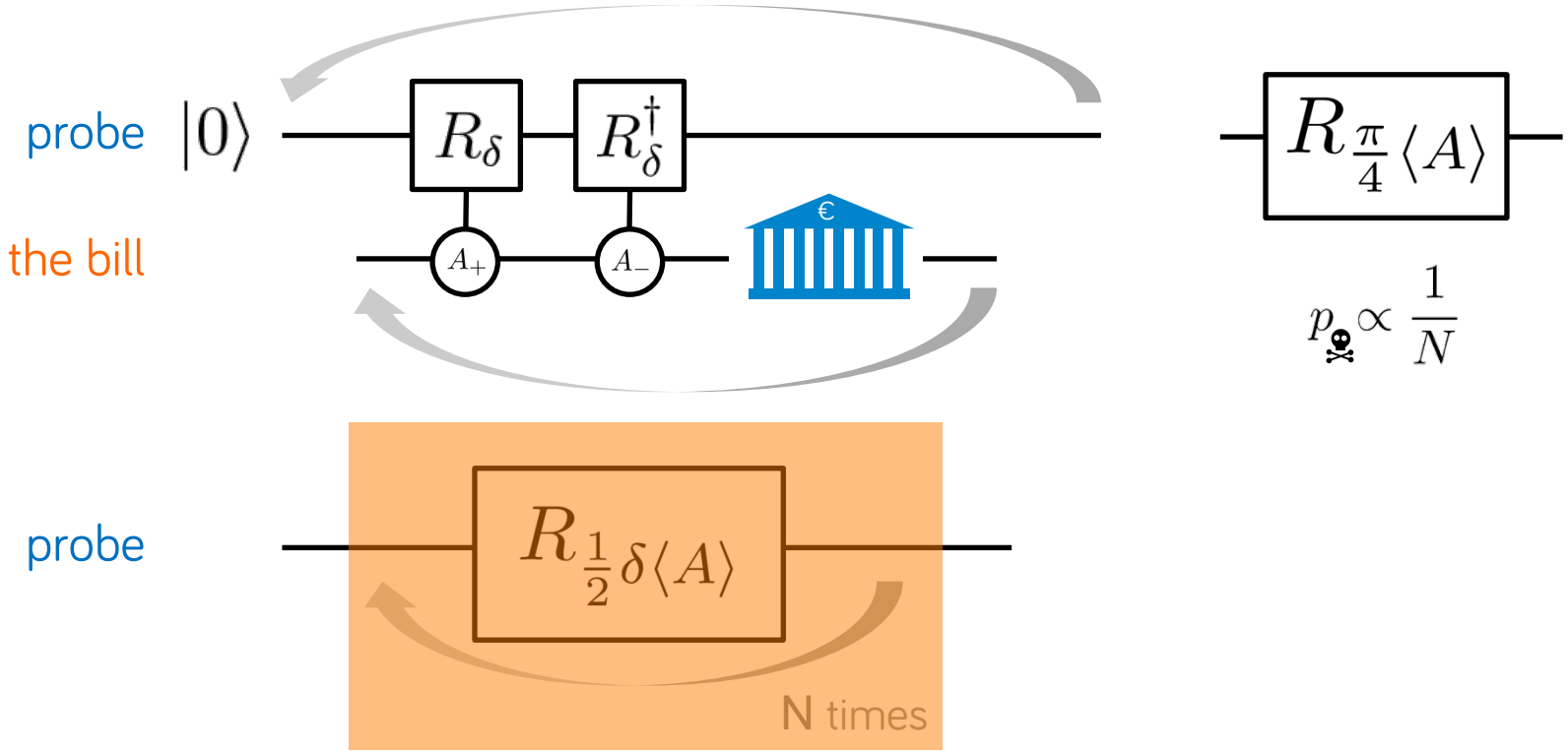
probe

$$R_{\frac{1}{2}\delta\langle A\rangle}$$

if we don't die
(approximately)

3 Estimating $\langle A \rangle$ for a Pauli operator

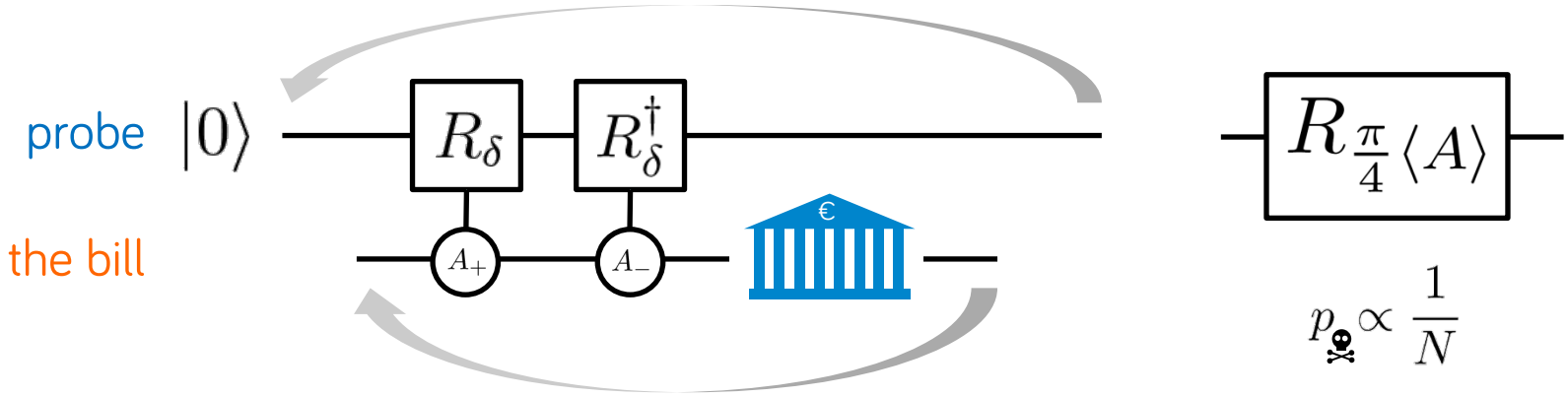
- how much does A mess up the state?



- phase estimation to required precision
- use operators A = { X, Y, Z } (or do it adaptively)

3 Estimating $\langle A \rangle$ for a Pauli operator

- how much does A mess up the state?



single-copy tomography from strict testing

- phase estimation to required precision
- use operators A = { X, Y, Z } (or do it adaptively)

1 destroy bad bills

or print/prepare new ones!

100€

2 quantum Zeno

how to copy {0, 1, +, −} without dying

3 tomography

with a single copy and strict-testing