Violation of the CGLMP inequality with entangled time-bin ququarts

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Abstract: Recently, many experiments have investigated violation of the CGLMP inequality, a Bell-type inequality for high-dimensional quantum entanglement, by using photonic entanglement based on orbital angular momentum or time-energy uncertainty. Here we report the generation and observation of four-dimensional time-bin entanglement. We performed projection measurements for the CGLMP inequality test with cascaded delay Mach-Zehnder interferometers and successfully observed a clear violation of the CGLMP inequality by 18 standard deviations.

1. Introduction

Quantum entanglement is one of the key elements in quantum information technology (QIT), such as quantum key distribution (QKD) and quantum computation. An entangled photon pair with a higher dimension has more noise tolerance than that with two dimensions, and thus more powerful applications in QIT are expected to be achieved, such as QKD with higher resistance against eavesdropping [1]. The Collins-Gisin-Linden-Masser-Popescu (CGLMP) inequality test is a measure that shows the entanglement between two particles with dimensions higher than two [2]. A family of Bell-parameters for d dimension S_d introduced in [2] satisfies the following inequality in the local variable theory

$$S_d \leq 2, for\ all\ d \geq 2\,. \tag{1}$$

However, this inequality is violated using quantum entanglement; thus, a violation of this inequality is good evidence of the entanglement in high dimensions. Violation of the CGLMP inequality has been observed by using high-dimensional entanglement based on orbital angular momentum (OAM) of photons [3,4] or time-energy uncertainty [5,6]. Here, we report the generation and observation of four-dimensional time entanglement, i.e., time-bin entangled ququarts. Using cascaded delay Mach-Zehnder interferometers (MZIs) made with planar lightwave circuit technology (PLC), we performed precise and stable projection measurements of four-dimensional quantum states, and successfully observed a clear violation of the CGLMP inequality.

2. Cascaded delay MZIs

In the CGLMP inequality test, we performed projection measurements to the Fourier transform measurement bases, which is given by

$$|\theta\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\exp(ik\theta)|k\rangle\,, \tag{2}$$

where d is a dimension of one photon and $|k\rangle$ is a state vector encoded on an OAM mode or a time bin. Figure 1 shows concept of projection measurement for time-bin ququarts based on cascaded delay MZIs [6]. The first and second MZIs have T and $2T$ delays, respectively, where T denotes the temporal interval between time slots. Phase shifts between the short and long arm in one interferometer are synchronized so that the phase shift in the second one is twice of that in the first MZI θ. When we input a time-bin ququart into this cascaded delay MZI, we can possibly detect a photon in one of seven time slots at the MZI output. A photon detection at the fourth time slot (shown by purple pulses in Fig. 1) corresponds to a projection to a state shown by Eq. (2) with d=4. Therefore, we can implement the projection measurements for the CGLMP inequality test by extracting photon detection events in the central time slot at the cascaded MZI outputs. Note that we can similarly implement 2^n-dimensional projection measurements by cascading n MZIs. Therefore, this scheme can be easily extended to the measurement of entangled pairs with even higher dimensions.

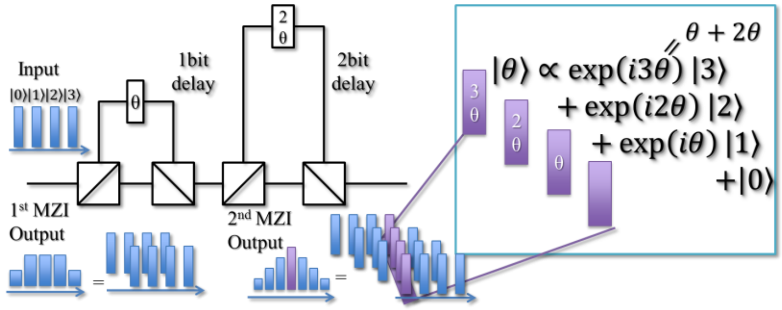

Figure 1 : Concept of projection measurement with cascaded delay MZIs.

3. Experimental setup

Figure 2 shows the experimental setup. A continuous-wave light from a 1551.1-nm wavelength laser is modulated into four sequential pulses by a lithium niobate intensity modulator, whose pulse duration, temporal interval between pulses, and repetition frequency are 100 ps, 1 ns and 125 MHz, respectively. This pulse train is then launched into the first periodically poled lithium niobate (PPLN) waveguide so that we can generate a 780-nm pump pulse train via second harmonic generation (SHG). The 780-nm, four-sequential pump pulses are input into the second PPLN waveguide, where a time-bin ququart denoted by the following formula is generated via spontaneous parametric down conversion (SPDC) :

$$|\Psi\rangle = \frac{1}{2}\sum_{k=0}^{3}|k\rangle_A|k\rangle_B \tag{3}$$

The two generated photons, denoted as photon A and B and whose wavelengths are 1555 and 1547 nm, respectively, are launched into a wavelength demultiplexing filter to separate them. Photons A and B are sent to Alice and Bob, respectively. Alice and Bob perform the projection measurements denoted by Eq. (2) using cascaded MZIs followed by superconducting nanowire single photon detectors (SNSPD). The detection signals from the SNSPD are input into a time interval analyzer (TIA) for coincidence analysis. A polarization controller is placed in front of each MZI to obtain a >20 dB extinction ratio by optimizing the polarization state. The detection efficiencies and dark count rates of the SNSPDs at Alice and Bob are 19 and 17 %, and 7 and 2 Hz, respectively.

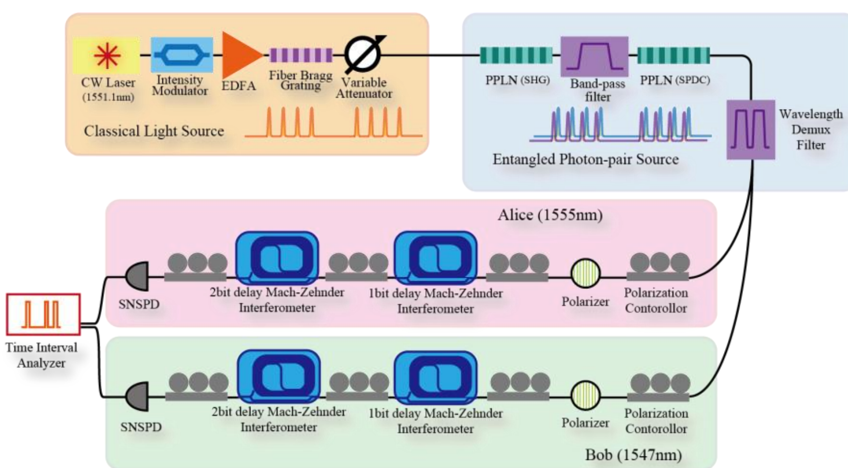

Figure 2 : Experimental setup.

4. Results

We performed coincidence fringe measurements when Alice swept her phase of cascaded MZI θ_A from 0 to 2π while Bob set his phase θ_B at 0 or $\frac{\pi}{4}$. Figure 3 shows the result. Red and blue dots denote the coincidence counts for $\theta_B = 0$ and $\frac{\pi}{4}$, respectively, and solid and dotted lines show the fitted curves obtained with the Levenberg-Marquardt algorithm. The average photon number per ququart was about 0.01. We observed four coincidence peaks when θ_A was changed by 2π, which is a clear signature of entangled ququarts. In addition, we observed clear fringes for two non-orthogonal measurement bases at Bob, confirming the existence of a non-classical correlation between ququarts. From the fitted curve, we obtained the visibility of 99.65 ± 0.24 % for $\theta_B = 0$ (the error was also estimated from the standard error of the fitted curve). The S_4 value estimated from the visibility was 2.877 ± 0.012, which exceeded the maximum S_2 value associated with entangled qubit, $2\sqrt{2}$. We also performed a direct measurement of S_4 by undertaking coincidence measurements for 64 combinations of θ_A and θ_B. We measured coincidence counts while θ_A and θ_B were set at 41 and 8 points dividing phase range $[0, 2\pi]$ and $\left[\frac{\pi}{8}, \frac{15\pi}{8}\right]$, respectively. Figure 4 shows the coincidence count result for 41×8 data points. From 64 data points out of these counts, we obtained S_4 of 2.741 ± 0.040, which violated the CGLMP-Bell inequality by 18 standard deviations. Here, we did not subtract any noise counts from our coincidence measurement data, including accidental coincidences due to detector dark counts or multi-photon emissions. Thus, we confirmed a clear violation of the CGLMP-Bell inequality with four-dimensional time-bin entanglement using cascaded MZIs.

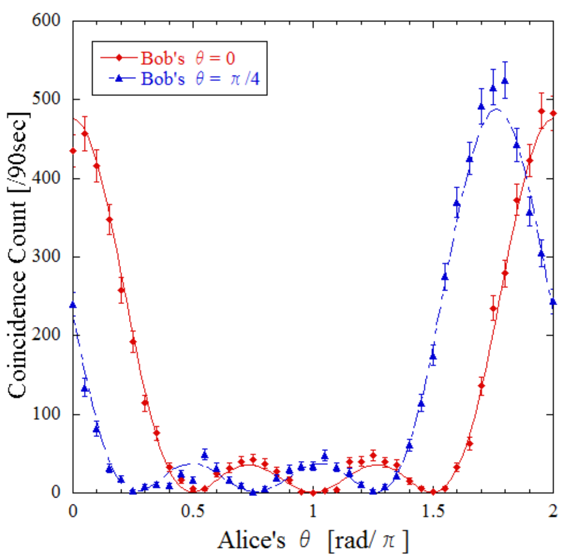

Figure 3 : Coincidence fringe as function of Alice's phase.

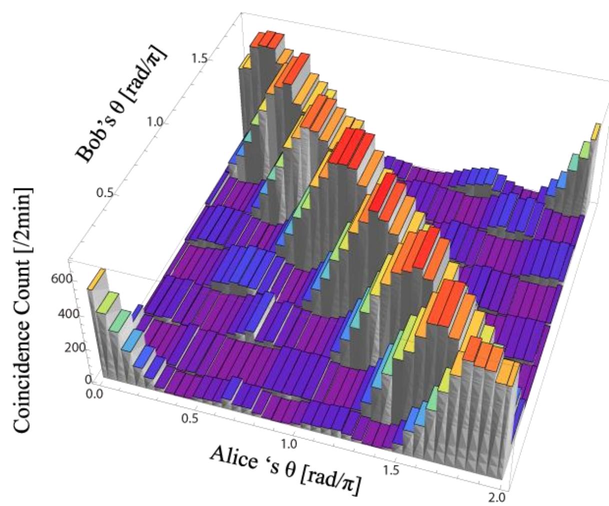

Figure 4 : Coincidence fringe as function of Alice's and Bob's phase.

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