Constructing optimal quantum error correcting codes from absolute maximally entangled states

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The maximally entangled states of two qubits, the so-called EPR states, are pure states of 2-qubits having reduced density matrices on each half of the system that are maximally mixed. A very intriguing question is whether systems made out of more than two parties can exhibit this property that all reduced states of at most half of the system size are maximally mixed. Such states are called Absolutely Maximally Entangled (AME) states and are pure multi-partite generalizations of the bipartite maximally entangled states.

AME states are known to play an important role in quantum information processing when dealing with many parties. They are useful for multipartite teleportation and in quantum secret sharing [1]. AME states have also deep connections with apparently unrelated areas of mathematics such as combinatorial designs and structures [2], classical error correcting codes [3], and quantum error correcting codes (QECC) [4]. Recently, they have gained new relevance as building blocks for holographic theories and in high energy physics. There they allow for the construction of tensor network states that realize discrete instances of the AdS/CFT correspondence and holography [5, 6, 7, 8].

At the same time it is still largely unknown for which n and q AME states exist and how they can be constructed. In the case of qubits for instance, it has been proven analytically that there are no AME states for $n = 4$ and $n \geq 7$. The non-existence in the cases $n = 4$ and $n \geq 8$ was proven by finding a contradiction in a linear program [9, 4]. Qubit AME states for $n = 2,3$ were long known, a state for $n = 5$ was found in [10] and more recently such for $n = 5,6$ were found numerically in [11, 12, 13]. Only very recently it was shown that there can not be a qubit AME state for the case $n = 7$ [14].

We work out in detail the connection between AME states of minimal support and classical maximum distance separable (MDS) error correcting codes and, in particular, provide explicit closed form expressions for AME(n,q) states of n parties with local dimension q a power of a prime for all $n \leq q+1$. Further, from a single AME state, we show how to produce an orthonormal basis of AME states. Based on our construction of minimal support AME states, we derive generators for stabilizer operators and conjecture the existence of a family of QECC whose code spaces are spanned by AME states. For every $q \geq n-1$ prime power we construct QECCs that encode a logical qudit into a subspace spanned by AME states up to $n = 8$, by finding several suitable incompressible operators. Under a conjecture for which we provide numerical evidence, this construction produces a family of quantum error correcting codes $[\![n,1,n/2]\!]_q$ for n even, saturating the quantum Singleton bound (called quantum MDS codes). Our method provides quantum MDS codes, for smaller local dimension q than all previously known QECC with otherwise identical parameters and we explicitly construct them for $n = 4$, 6, 8. Also, our proposal has a very clear physical motivation and nicely complements other constructions of non-binary QECC (see for example [15, 16] for an overview and [17] for tables of known codes with $q = 2$).

The abstract is based on [18].

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