General bounds for sender-receiver capacities in multipoint quantum communications

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Today a huge effort is devoted to the development of robust quantum technologies, inspired by quantum information theory. The most typical communication tasks are quantum key distribution (QKD), reliable transmission of quantum information and distribution of entanglement. The latter allows two parties to implement powerful protocols such as quantum teleportation. Unfortunately, any practical realization of such quantum tasks is affected by decoherence. This is the very reason why the performance of any point-to-point protocol of quantum and private communication, i.e., in the absence of quantum repeaters, suffers from fundamental limitations, which become more severe when the distance is increased.

In this context, an open problem was to find the optimal rates for quantum and private communication that are achievable by two remote parties, say Alice and Bob, assuming the most general strategies allowed by quantum mechanics, i.e., assuming arbitrary local operations (LOs) assisted by unlimited two-way classical communication (CCs), briefly called adaptive LOCCs. These optimal rates are known as two-way (assisted) capacities and their determination has been notoriously difficult. Only recently, after about 20 years [1], Ref. [2] finally addressed this problem and established the two-way capacities at which two remote parties can distribute entanglement (D_2), transmit quantum information (Q_2), and generate secret keys (K) over a number of fundamental quantum channels at both finite and infinite dimension, including erasure channels, dephasing channels, bosonic lossy channels and quantum-limited amplifiers.

For the specific case of a bosonic lossy channel with transmissivity η, Ref. [2] proved that $D_2 = Q_2 = K = -\log_2(1-\eta)$ corresponding to $\simeq 1.44\eta$ bits per channel use at high loss. The latter result completely characterizes the fundamental rate-loss scaling that affects any point-to-point protocol of QKD through a lossy communication line, such as an optical fiber or free-space link. The novel and general methodology that led to these results is based on a suitable combination of quantum teleportation with a LOCC-monotonic functional, such as the relative entropy of entanglement (REE). Thanks to this combination, Ref. [2] was able to upper-bound the two-way capacity $\mathcal{C} = D_2, Q_2, K$ of an arbitrary quantum channel $\mathcal{E}$ with computable single-letter quantities.

The goal of the present paper [3] is to extend such "REE+teleportation" methodology to a more complex communication scenario, in particular that of a single-hop quantum network, where multiple senders and/or receivers are involved. The basic configurations are represented by the quantum broadcast channel where information is broadcast from a single sender to multiple receivers, and the quantum multiple-access channel, where multiple senders communicate with a single receiver. More generally, we also consider the combination of these two cases, where many senders communicate with many receivers in a sort of all-in-all quantum communication or quantum interference channel. In practical implementations, this may represent a quantum bus where quantum information is transmitted among an arbitrary number of qubit registers.

In all these multipoint scenarios, we characterize the most general protocols for entanglement distillation, quantum communication and key generation, assisted by adaptive LOCCs. This leads to the definition of the two-way capacities $\mathcal{C} = D_2, Q_2, K$ between any pair of sender and receiver. We then consider those quantum channels (for broadcasting, multiple-accessing, and all-in-all communication) which are teleportation-covariant. For these channels, we can completely reduce an adaptive protocol into a block form involving a tensor product of Choi matrices. Combining this reduction with the REE, we then bound their two-way capacities by means of the REE of their Choi matrix, therefore extending previous methods [2] to multipoint quantum/private communication.

Our upper bounds applies to both discrete-variable (DV) and continuous-variable (CV) channels. As an example, we consider the specific case of a 1-to-M thermal-loss broadcast channel through a sequence of beamsplitters subject to thermal noise. In particular, we discuss how that the two-way capacities Q_2, D_2 and K between the sender and each receiver are all bounded by the first point-to-point channel in the "multisplitter". This bottleneck result can be extended to other Gaussian broadcast channels. In the specific case of a lossy broadcast channel (without thermal noise), we find a straighforward extension of the fundamental rate-loss scaling, so that any sender-receiver capacity is bounded by $-\log_2(1-\eta)$ with η being the transmissivity of the first beamsplitter.

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