Three-observer Bell inequality violation on a two-qubit entangled state

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Introduction - In a bipartite system, the most known Bell inequality is the so called Clauser-Horse-Shimony-Holt (CHSH) inequality [?]. Two observers, usually called Alice and Bob, perform independent measurements on their subsystem. They choose randomly between two different measurements $\mathcal{A}$ or $\mathcal{A}'$ for Alice and $\mathcal{B}$ or $\mathcal{B}'$ for Bob. The CHSH inequality is written as $I_{\text{CHSH}} \equiv \langle \mathcal{A} \otimes \mathcal{B} \rangle + \langle \mathcal{A}' \otimes \mathcal{B} \rangle + \langle \mathcal{A} \otimes \mathcal{B}' \rangle - \langle \mathcal{A}' \otimes \mathcal{B}' \rangle \leq 2$. Bipartite entangled states may violate such inequality: in particular, for a two-qubit maximally entangled state, the Bell parameter I_{CHSH} may reach the Tsirelson's bound $2\sqrt{2}$. The violation of the CHSH inequality certifies the presence of entanglement and rules out the possibility of describing quantum physics with a local hidden variable model, as recently demonstrated experimentally.

An intriguing property of quantum entanglement is its *monogamy*: given a tripartite state $\rho_{AB_1B_2}$, the larger is the entanglement between two observers, the lower is the entanglement of the third observer with any of the other two. A similar monogamy argument holds for "non-local-realistic" correlations, whose presence is associated with the violation of Bell inequalities. Indeed, given three observers (Alice, Bob1 and Bob2) and assuming non signaling, it is impossible to have a simultaneous violation between Alice-Bob1 and Alice-Bob2.

However, as realized in [?], this restriction no longer holds if the non-signaling hypothesis is dropped. Therefore, it is possible to violate the CHSH inequality between two different pairs of observers by using a single two-qubit entangled state and allowing the state received by Bob2 to be first measured by Bob1. In this case, the state received by Bob2 is dependent on Bob1's basis choice and therefore there is signaling between Bob1 and Bob2. However, the two Bobs do not have to agree on a common measurement strategy and can in principle be unaware of each other's presence, so they may be considered as independent.

Here [?] we introduce a model to test the limit of the monogamy in the case of a photonic bipartite entangled state, by varying the strength of the weak measurement performed by Bob1 in the presence of an independent strong measurement realized by Bob2.

Description of the experiment - The scheme allowing the double violation of the CHSH inequality is presented in Fig. **??**. Three observers, Alice (A), Bob1 ($B1$) and Bob2 ($B2$) perform some measurements on a two-particle entangled state $|\Psi^-\rangle$. In particular, each of the observer choose independently between two different dichotomic observables by using a single random bit

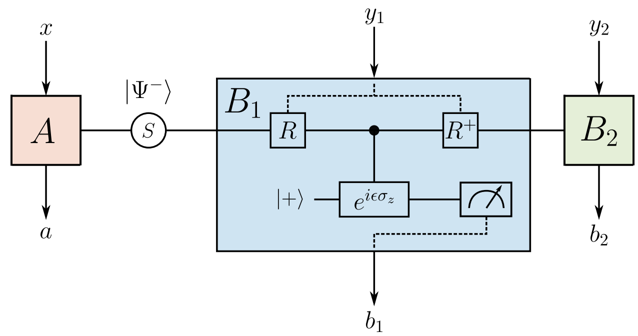

Figure 1: Circuit model of the scheme used for the double violation of the CHSH inequality.

(respectively denoted by x, y_1 and y_2 for the three observers) taking values in 0 or 1. The binary outcomes ($\pm$) of the dichotomic measurements are respectively given by a, b_1 and b_2. By measuring the probability of the outcomes, two CHSH parameters I_{CHSH} can be evaluated as:

$$
\begin{aligned}
I_{\text{CHSH}}^{(1)} &= \sum_{x,y_1} (-1)^{x \cdot y_1} [p(a = b_1 | x, y_1) - p(a \neq b_1 | x, y_1)], \\
I_{\text{CHSH}}^{(2)} &= \sum_{x,y_2} (-1)^{x \cdot y_2} [p(a = b_2 | x, y_2) - p(a \neq b_2 | x, y_2)].
\end{aligned} \tag{1}
$$

In order to violate both CHSH inequalities $I_{\text{CHSH}}^{(j)} \leq 2$ it is necessary that Bob1 performs a weak measurement on his subsystem. More precisely, a weak measurement scheme consists of entangling the system under measurement with an ancillary system and then strongly measuring the ancilla. The measurement scheme of Bob1 exploits a controlled phase gate $\text{CP}_\epsilon = |H\rangle\langle H| \otimes \mathbb{1} + |V\rangle\langle V| \otimes e^{i\epsilon\sigma_z}$ to entangle an ancillary qubit, prepared in the state $|+\rangle$, with the system coming from the source. The ancilla qubit is then measured in the $\{|+\rangle, |-\rangle\}$ basis. Depending on the amount of rotation ϵ in the controlled phase gate it is possible to vary the strength of the measurement. The above scheme implements a controllable-strength measurement of the system in the $\{|H\rangle, |V\rangle\}$ basis. In order to generalize the measurement to an arbitrary basis $\left\{|\omega_{y_1}\rangle, |\omega_{y_1}^{\perp}\rangle\right\}$ it is necessary to rotate the state with a rotation matrix R_{y_1} such that $R_{y_1}|\omega_{y_1}\rangle = |H\rangle$ and $R_{y_1}|\omega_{y_1}^{\perp}\rangle = |V\rangle$.

If Alice chooses to measure in the directions $-(Z+X)/\sqrt{2}$ or $(-Z+X)/\sqrt{2}$ and the Bobs choose to measure in the Z or X directions, the above probabilities predict the following values of the CHSH parameters:

$$
\begin{aligned}
I_{\text{CHSH}}^{(1)} &= 2\sqrt{2}\sin^2\epsilon, \\
I_{\text{CHSH}}^{(2)} &= \sqrt{2}(1+\cos\epsilon),
\end{aligned} \tag{2}
$$

being $I_{\text{CHSH}}^{(1)}$ ($I_{\text{CHSH}}^{(2)}$) the CHSH parameter of Alice and Bob1 (Bob2).

Results and conclusions - We observe that the correlations that both Bobs have with Alice experimentally violate the corresponding CHSH inequalities, for $\sin^2\epsilon = 3/4$ (see Fig. **??**). The crucial ingredient to achieve such double violation is the weak measurement performed by Bob1. The underlying concept that we exploited here is that by performing the weak measurement, a lower amount of information is obtained about the system with respect to a projective, or strong, measurement. However, such less information is compensated by a lower degree of disturbance on the measured state.

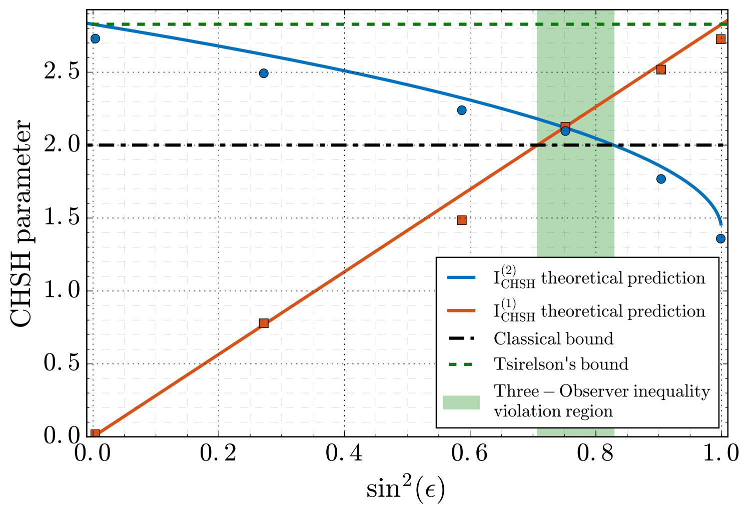

Figure 2: Measurements of $I^{(1)}_{\mathrm{CHSH}}$ (squares) and $I^{(2)}_{\mathrm{CHSH}}$ (circles) for several values of ϵ. The red and green solid lines show the expected values of $I^{(1)}_{\mathrm{CHSH}}$ and $I^{(2)}_{\mathrm{CHSH}}$ (Eq. **??**), while the dash-dotted and dashed lines indicate classical and Tsirelson's bounds respectively. The green region highlights the values of ϵ in which double violation is expected. In particular, the measurement for $\sin^2\epsilon = 3/4$ experimentally show the double violation. Poissonian errors are within the dimension of the points.

The achievement of double violation and the realization of a simple weak measurement scheme have important applications for Quantum Key Distribution exploiting weak measurements [**?**]. Our setup can implement a QKD protocol in which Alice and the two Bobs want to share a secret key. Assuming that Bob1 and Bob2 are in the same laboratory and that Eve cannot control the channel from Bob1 to Bob2, Alice and Bob1's measurement are used to estimate the channel while Alice and Bob2's measurements are used for the key generation (for this case Bob2 should use the measurements $-(Z+X)/\sqrt{2}$ or $(-Z+X)/\sqrt{2})$. As stated in [**?**], such a protocol is immune to a very powerful class of measurement-side device attacks, while achieving a secret key rate essentially equal to that of BB84, with decoy states, and significantly higher than MDI-QKD for realistic system parameters.

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