Exploiting no-Signalling Extremal Distributions to find Bell Inequalities

Thomas Cope[1] and Roger Colbeck[1]

[1] *University of York*

(Dated: July 10, 2017)

I. EXTENDED ABSTRACT

Bell inequalities[1] form an important part of our understanding of quantum physics, bounding the region between probability distributions with a local hidden variable scheme, and those that are fundamentally quantum in nature. Despite being so useful for our purposes, our knowledge of Bell inequalities is quite limited; this is mainly due to the speed at which the number of extremal inequalities grows as the dimension increases.

We begin with an overview of two main approaches to obtaining Bell inequalities; the first finding the faces of the local polytope, the set of probabilities describable with a local hidden variable, which are defined by these inequalities. This method is known as *facet enumeration*[2, 3], and provides a deterministic method of obtaining all Bell inequalities. The second is using a *linear programming algorithm*[4, 5] to minimize the inner product with a known non-local probability distribution, whilst constraining the value at the extremal local distributions above a certain value. This method is probabilistic, but practically may be more feasible in certain scenarios.

We then introduce a result of Jones and Masanes[6], who give a complete description of the no-signalling distributions in the scenario where two non-communicating parties are limited to binary outcomes. Specifically, they present the extremal distributions of the no-signalling polytope, from which all no-signalling distributions may be formed.

Our result is to explain how, by applying this knowledge, it is possible to simplify the task of enumerating the facets - taking advantage of the dual description, where Bell inequalities are instead represented as points, we use the no-signalling distributions to define a series of smaller, easier to solve problems which overall give us the full solution. It is also shown how the linear programming approach may also benefit from this knowledge. We also discuss some practical details of the implementation.

To illustrate our approach, we provide a full list of extremal Bell inequalities for the scenario with two parties, each of whom have four input choices and binary outcomes. The number of these was already known from [2]. Also included is an overview of the algorithm we used for this task, and partial results for scenarios with higher dimension.

Finally, a discussion on the problems still faced by this approach and for the overall effort, and suggestions on how these problems may be tackled. For example, even when we do not have the luxury of knowledge of the extremal no-signalling points,we may still use a probabilistic approach utilizing entangled quantum states to obtain new inequalities.

---

[1] J. Bell, Physics **1**, 195 (1964).
[2] M. Deza and M. Sikiriȉ, International Transactions in Operational Research **23**, 853860 (2015).
[3] D. Avis and K. Fukuda, Discrete and Computational Geometry **8**, 295 (1992).
[4] G. Walsh, *An Introduction to Linear Programming* (Holt, Rinehart and Winston Ltd., 1971).
[5] S. Pironio, *Aspects of quantum non-locality*, `quic.ulb.ac.be/_media/publications/2004-thesis-stefano.pdf` (2004).
[6] N. Jones and L. Masanes, Physical Review A **72**, 052312 (2005).