

On Security Notions for Encryption in a Quantum World

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Motivations

Classical Security

Classical adversaries, classical communication

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Post-quantum Security

Quantum adversaries, classical communication

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Quantum adversaries, classical communication

Fully-Quantum Security [BZ'13, DFNS'14, GHS'16]

Quantum adversaries, **quantum** communication.

- ▶ Running classically obfuscated programs in quantum computers
- ▶ Exotic quantum attacks: frozen smart-card attack

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This paper: fully-quantum security for classical **encryption**.

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Classical adversaries, classical communication

Post-quantum Security

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Prior Results

- ① **Superposition** access to encryption oracle

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$$\sum_x \alpha_x |x, y\rangle \mapsto \sum_x \alpha_x |x, y \oplus \text{Enc}(x)\rangle$$

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- ② Superposition access to decryption oracle in **chosen-ciphertext** security.

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Open Quantum challenges, in the standard oracle model, for both PKE and SKE with CCA security.

- ▶ No-cloning Theorem
- ▶ Measurement destructiveness
- ▶ Impossibility for any Left-or-Right indistinguishability notion [BZ'13]

Main Results

An achievable, meaningful **quantum** notion of **chosen-ciphertext** security for both **secret-** and **public-**key encryption.

Our Work

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Our Techniques

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Our Techniques

- ▶ Switching from a **Left-or-Right** indistinguishability notion to a **Real-or-Random** indistinguishability notion.

Our Work

Main Results

An achievable, meaningful quantum notion of chosen-ciphertext security for both secret- and public-key encryption.

Our Techniques

- ▶ Switching from a Left-or-Right indistinguishability notion to a Real-or-Random indistinguishability notion.
- ▶ Adapting **Zhandry's compressed oracle** technique to randomized functions.

[Zhandry'19]

Zhandry's Recording Technique [Zhandry'19]¹

- ▶ Goal: on-the-fly simulation of random oracles in the quantum setting.

¹ © Zhandry, CRYPTO'19

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Four steps

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$$\text{Measuring } \sum_h |h\rangle = h \stackrel{\$}{\leftarrow} \mathcal{U}$$

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- ① Quantum-ify
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$$\boxed{|x, y\rangle_{\mathcal{A}} |h\rangle_{\mathcal{O}} \mapsto |x, y \oplus h(x)\rangle_{\mathcal{A}} |h\rangle_{\mathcal{O}}} \xrightleftharpoons[\text{Transform}]{\text{Fourier}} \boxed{|x, y\rangle_{\mathcal{A}} |h\rangle_{\mathcal{O}} \mapsto |x, y\rangle_{\mathcal{A}} |h \oplus P_{x,y}\rangle_{\mathcal{O}}}$$

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Initial Oracle State $D = \{\}$. Query(x, y, D):

- 1 If $\nexists(x, y') \in D$, $D = D \cup \{(x, 0)\}$
- 2 $D = D \setminus \{(x, y')\} \cup \{(x, y \oplus y')\}$
- 3 $D = D \setminus \{(x, 0)\}$ if $\exists(x, 0) \in D$

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The oracle now has information about the adversary's queries.

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Perfect Simulability

This is a perfect simulation for quantum random oracles.

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Consider a randomized function $f(x; r)$

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$$\boxed{|x, y\rangle_{\mathcal{A}} |r, f_r\rangle_{\mathcal{O}} \mapsto |x, y \oplus f(x; r)\rangle_{\mathcal{A}} |r, f_r\rangle_{\mathcal{O}}} \xrightarrow{\text{QFT}} \boxed{|x, y\rangle_{\mathcal{A}} |r, f_r\rangle_{\mathcal{O}} \mapsto |x, y\rangle_{\mathcal{A}} |r, f_r \oplus P_{x,y}\rangle_{\mathcal{O}}}$$

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The above simulation needs 3 applications of U_f . Thus it is not useful for:

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Simulation with 1 call to U_f

In the randomized setting, the database is always initialized to “zero”, thus we can simulate with only **one** call to U_f .

IND-CCA (Real-or-Random)²

Expt_{S \mathcal{E}} ^b(λ , \mathcal{A}):

- 1 $\mathbf{k} \xleftarrow{\$} \mathcal{K}()$
- 2 $(x, \text{state}) \leftarrow \mathcal{A}_1^{\text{Enc}_k, \text{Dec}_k}(\lambda)$
- 3 $x_0 \leftarrow x, x_1 \xleftarrow{\$} \mathcal{X}$
- 4 $y^* \leftarrow \text{Enc}_k(x_b)$
- 5 $b' \leftarrow \mathcal{A}_2^{\text{Enc}_k, \text{Dec}_k^*}(y^*, \text{state})$
- 6 **return** b'


$$\blacktriangleright \text{Dec}_k^*(y) = \begin{cases} \perp & \text{if } y = y^* \\ \text{Dec}_k(y) & \end{cases}$$

$$|\Pr[\text{Expt}_{S\mathcal{E}}^1(\lambda, \mathcal{A}) = 1] - \Pr[\text{Expt}_{S\mathcal{E}}^0(\lambda, \mathcal{A}) = 1]| \leq \epsilon$$

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²Single-Challenge

IND-CCA (Real-or-Random)²

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- 1 $\mathbf{k} \xleftarrow{\$} \mathcal{K}()$
- 2 $(x, \text{state}) \leftarrow \mathcal{A}_1^{\text{Enc}_k, \text{Dec}_k}(\lambda)$
- 3 $\pi \xleftarrow{\$} \Pi$
- 4 $y^* \leftarrow \text{Enc}_k(\pi^b(x))$
- 5 $b' \leftarrow \mathcal{A}_2^{\text{Enc}_k, \text{Dec}_k^*}(y^*, \text{state})$
- 6 **return** b'

$$\blacktriangleright \text{Dec}_k^*(y) = \begin{cases} x & \text{if } y = y^* \\ \text{Dec}_k(y) & \end{cases}$$

$$\blacktriangleright \pi^b = \begin{cases} \pi & \text{if } b = 1 \\ \mathbb{1} & \text{if } b = 0 \end{cases}$$

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qIND-qCCA (Real-or-Random)

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③ $\pi \xleftarrow{\$} \Pi$

④ $\sum_{x,y} \alpha_{x,y} \underbrace{|x, y \oplus \text{Enc}_k(\pi^b(x)), \phi_{x,y}\rangle}_{|\Psi\rangle} \otimes D_{x,y} \leftarrow \text{Enc}_k \circ \pi^b |\Phi\rangle$

Use compressed oracle here



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qIND-qCCA (Real-or-Random)

Expt_{SE}^b(λ, A):

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⑥ **return** b'

$$\text{Dec}_k^* |y, z\rangle \otimes D = \begin{cases} |y, z \oplus \text{Dec}_k(y)\rangle & \text{if } \nexists (w, y) \in D \\ |y, z \oplus w\rangle & \text{if } \exists (w, y) \in D \end{cases}$$

Properties & Achievability

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- ▶ qIND security \Rightarrow IND security
- ▶ Composability
- ▶ IND-qCCA \Leftrightarrow qIND-qCPA

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 - ▶ One-time pad encryption style (stream cipher, GCM, CFB, OFB, CTR ...) is **insecure**.

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- ▶ qIND security \Rightarrow IND security
- ▶ Composability
- ▶ IND-qCCA $\not\Leftrightarrow$ qIND-qCPA
 - ▶ One-time pad encryption style (stream cipher, GCM, CFB, OFB, CTR ...) is insecure.

Achievability

- ▶ Encrypt-then-MAC is qIND-qCCA
- ▶ IND-qCCA PKE + OWF \Rightarrow qIND-qCCA PKE

Thank you!