Non-interactive classical verification of quantum computation

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Verifiable quantum advantage

nature

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

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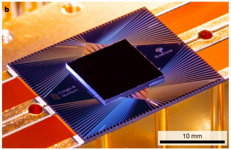

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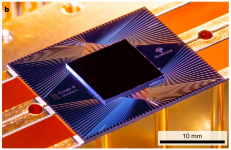

When a quantum cloud is available for remote access...

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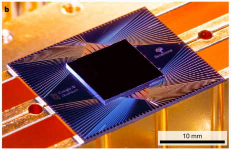


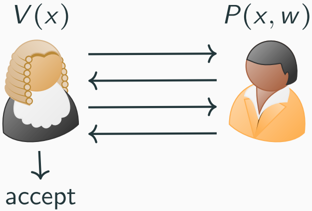
When a quantum cloud is available for remote access...

How do you know if you can trust it via classical communication (e.g., email messages)?

Interactive proofs/arguments

An interactive proof (or argument) system for language L is a protocol which is both complete and sound.

Completeness: for $x \in L_{yes}$,



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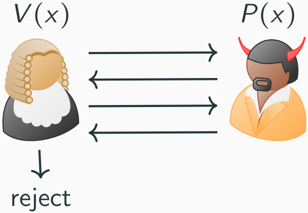
Soundness: for $x \in L_{no}$,



Interactive proofs/arguments

An interactive proof (or argument) system for language L is a protocol which is both complete and sound.

It is sometimes desirable that the interaction conveys no information about the witness.

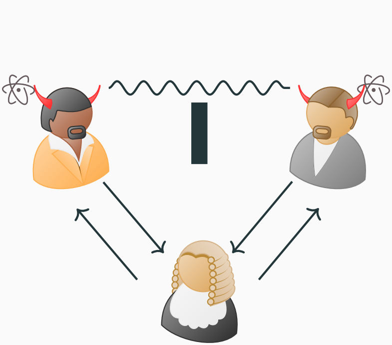
Zero knowledge: there exists a simulator $\mathcal{S}$ who outputs an indistinguishable view.

$V(x)$ $\qquad\qquad$ $P(x, w)$

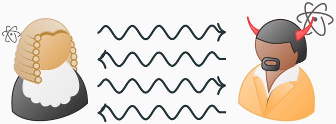
$$\approx \qquad \mathcal{S}(V, x)$$

Testing quantum computers

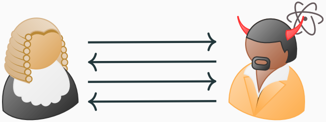
How do we classically verify quantum computers when classical simulation is impossible?



Multiprover interactive proofs with pre-shared entanglements. [RUV13, M16, GKW15, HPDF15, FH15, NV17, CGJV19, G19]



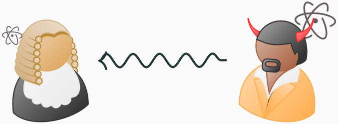
Interactive proof systems with a limited quantum verifier. [B18, ABEM17, MHF18]



$\leq$ LWE

Interactive arguments with a bounded quantum prover. [M18]

An XZ verification protocol for BQP/QMA



Verifier(H):

- measures ρ in X or Z bases, and checks the parity of 2 qubits.

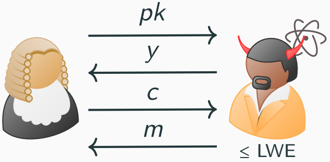
Prover(H):

- prepares the ground state ρ and sends it.

For this approach to work [MHF18],

- the ground state energy of Hamiltonian $H = \sum_i p_i \Pi_i$ is either $\leq a$ or $\geq b$ with $(b - a) > n^{-c}$;
- for every problem L in BQP there is a corresponding Hamiltonian for every instance;
- for QMA, the prover is given access to a quantum witness.

The Mahadev protocol



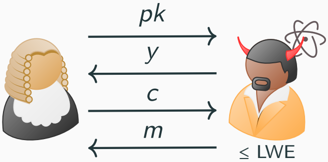
Assuming LWE is hard against quantum adversaries, there is a 4-message protocol for BQP. [M18]

- Verifier publicizes the key pk, and keeps sk secret;
- tosses a random coin c;
- checks $m = (b, x)$,
  - if $c = 0$, $f_{pk}(b, x) = y$;
  - if $c = 1$, the decryption of b or y is accepted to the XZ verification protocol.

- Prover prepares state $|\Psi\rangle = \sum_b \alpha_b |b\rangle|x\rangle|f_{pk}(b, x)\rangle$ and performs partial measurement;
- measures $|\psi_y\rangle$
  - if $c = 0$, in Z basis;
  - if $c = 1$, in X basis;

to get m.

The Mahadev protocol



Assuming LWE is hard against quantum adversaries, there is a 4-message protocol for BQP. [M18]

For this protocol to work,

- The key pairs (pk, sk) encode the bases.
- The function f_{pk} is either 2-to-1 or 1-to-1.
- Hard to prepare the preimage superposition for a fixed y without sk.

There exists an instantiation based on plain LWE. [M18]

The soundness error is constant.

Overview of our protocols

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Question

Can quantum computation be certified with a single message, up to instance-independent preprocessing?

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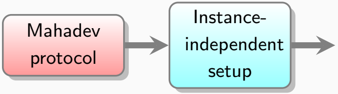

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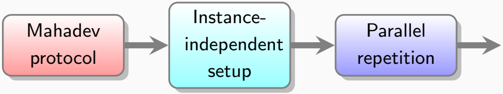

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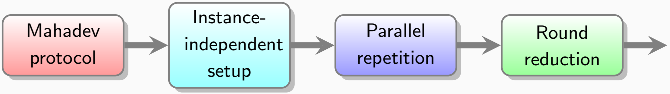

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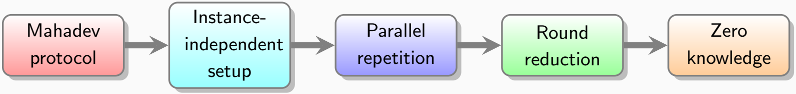

Instance independent setup

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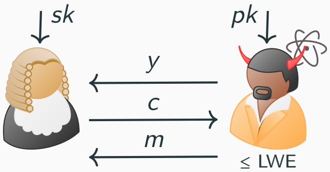
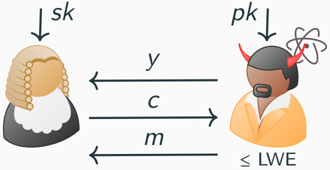


Theorem

The key sampling can be preprocessed prior to verification.

Proof.

Instance independent setup



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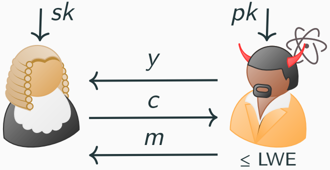

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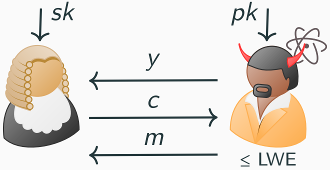

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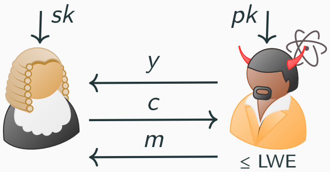

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- Sample bases S randomly and the keys according to the bases.
- V samples the real bases S' according to the Hamiltonian.
- If $S \neq S'$, the verifier accepts; otherwise run the same verification protocol as before.
- Since the Hamiltonian is 2-local, with probability $1/4$ they match $\Rightarrow$ the gap decreases by a factor of $1/4$.

A parallel repetition theorem

Hardness amplification

Given a protocol Π with small completeness-soundness gap,
two possibilities to amplify the gap:

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- Parallel repetition (PR)
  Run Π in parallel, accept if many copies are accepted.
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  ☹ Not always reduce the soundness error.

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- Parallel repetition (PR)
  Run Π in parallel, accept if many copies are accepted.
  ☺ Additional interaction is not required.
  ☹ Not always reduce the soundness error.
  - There exists a protocol for which the soundness error stays the same using two-fold PR.

A parallel repetition theorem

Theorem

The soundness error of a k-fold protocol is $2^{-k} + \epsilon$ *for negligible* ϵ.

Proof.

[1] In the sense that $\mathcal{P}$ is quantum efficient and only knows the public keys.

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$$\forall a \neq b, \underset{pk,sk,s}{\mathbb{E}}[\mathrm{tr}(\rho_{pk}\{M_{sk,s,a}, M_{sk,s,b}\})] \leq \mathrm{negl}(n).$$

Otherwise, there exists an adversary who wins the single-copy protocol w.p. close to 1.

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  Otherwise, there exists an adversary who wins the single-copy protocol w.p. close to 1.
- Thus any prover can win at most a single challenge (out of 2^k possibilities).

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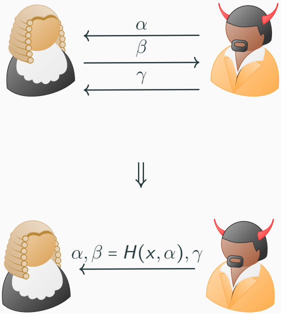
Round reduction

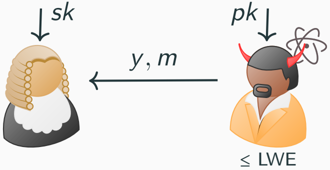
The Fiat-Shamir paradigm

The Fiat-Shamir transform turns a Σ-protocol (3-message, public-coin), into a non-interactive protocol.

In the QROM, FS is secure with an $O(q^2)$ loss against a q-query adversary to the random oracle.



Round reduction for BQP verification



Theorem

The FS-transformed BQP verification has negligible soundness error.

Proof.

- Assuming the existence of an FS-breaking adversary $\mathcal{A}$, there must be a noticeable fraction of bad keys (pk^*, sk^*).

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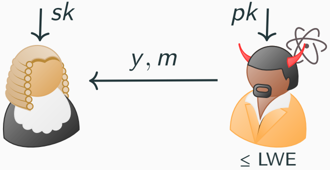

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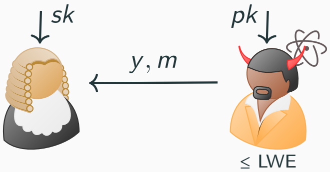

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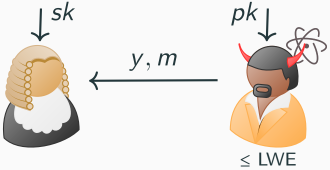

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- There exists an adversary $\mathcal{B}(pk^*)$ who wins the Σ-protocol w.p. arbitrarily close to 1, using the same reduction as [DFMS19].
- The adversary $\mathcal{B}$ breaks the original protocol.

Classical NIZK for BQP/QMA

Making the protocol zero-knowledge

Theorem

There exists a classical NIZK for QMA in the QROM, assuming the existence of a circularly secure FHE and a NIZK for NP.

Sketch of construction.

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- ☹ The first message is obtained by querying f_{pk} on the witness.
  $\Rightarrow$ Prover encrypts the witness state with quantum one-time pad and commits to the keys.

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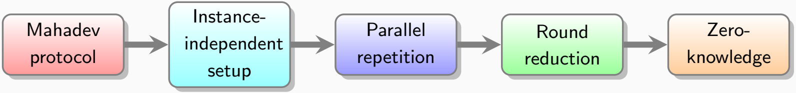
- ☹ The first message is obtained by querying f_{pk} on the witness.
  $\Rightarrow$ Prover encrypts the witness state with quantum one-time pad and commits to the keys.
- ☹ The prover gets accepted by sending the openings and the measurement outcomes.
  $\Rightarrow$ Viewing these as the witness to the NP relation.
  $\Rightarrow$ Sending a homomorphically evaluated NIZK proof.

Summary

We showed classical verification of quantum computation can be performed non-interactively and in zero-knowledge.



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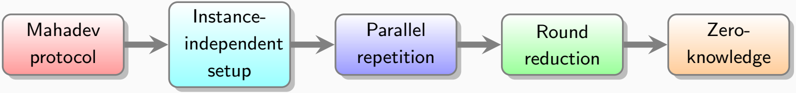

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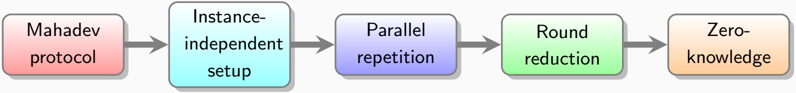

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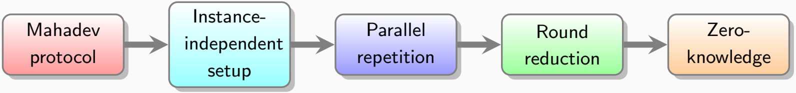

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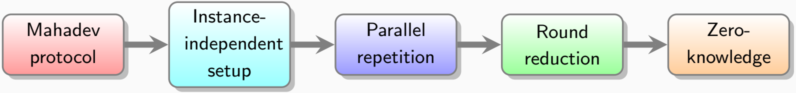

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- Simpler NIZK arguments for BQP/QMA?