QCRYPT 2023

Single-qubit loss-tolerant quantum position verification protocol secure against entangled attackers

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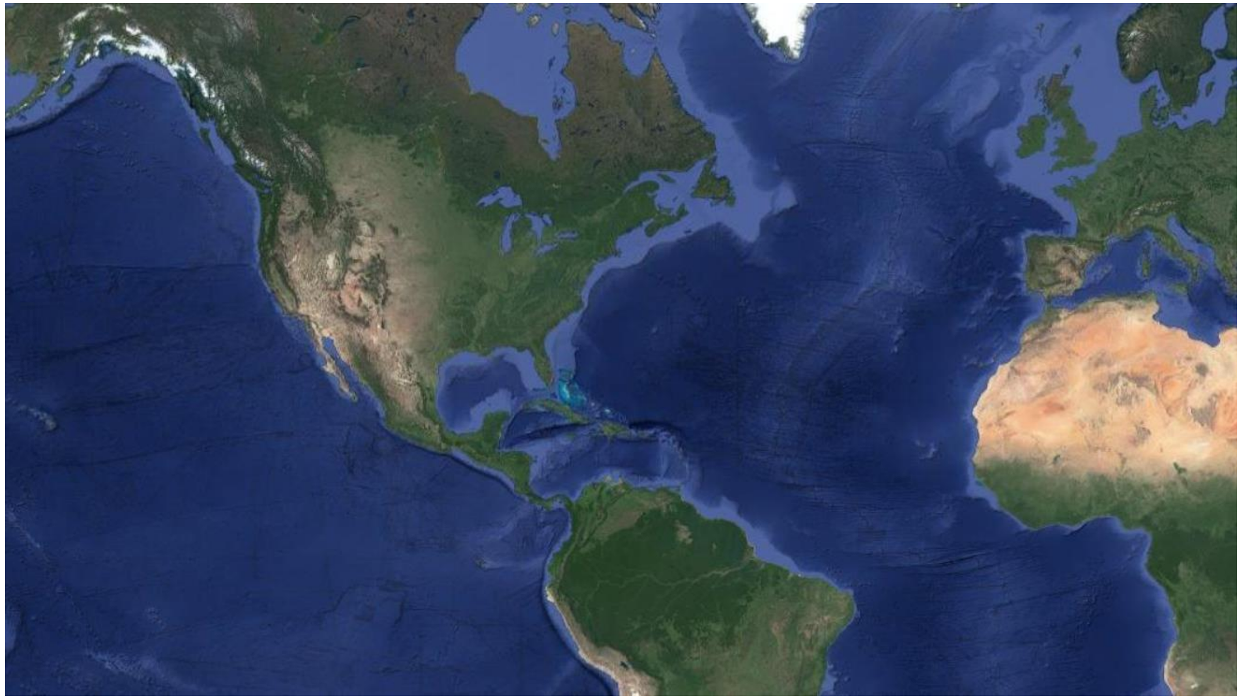
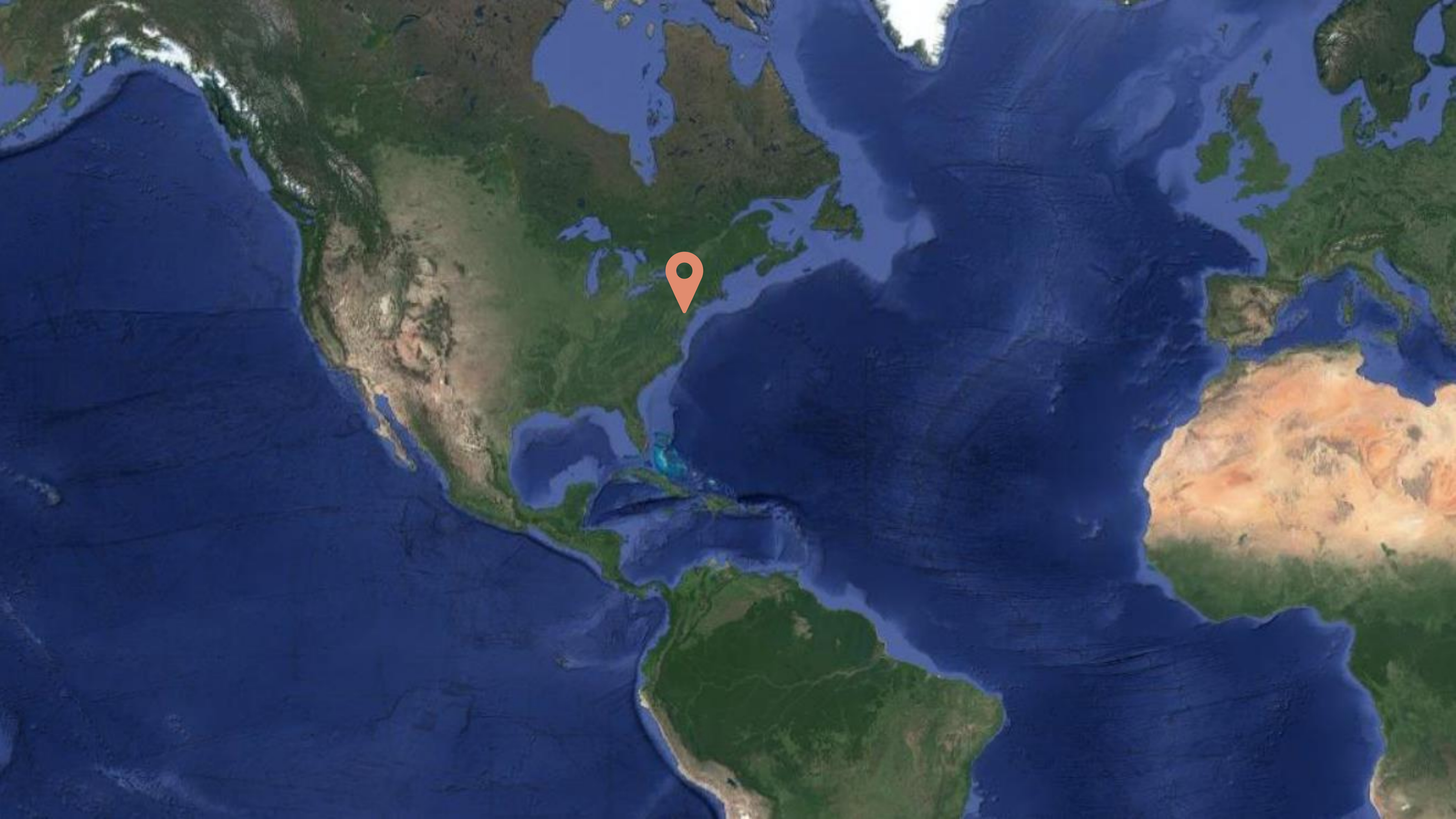
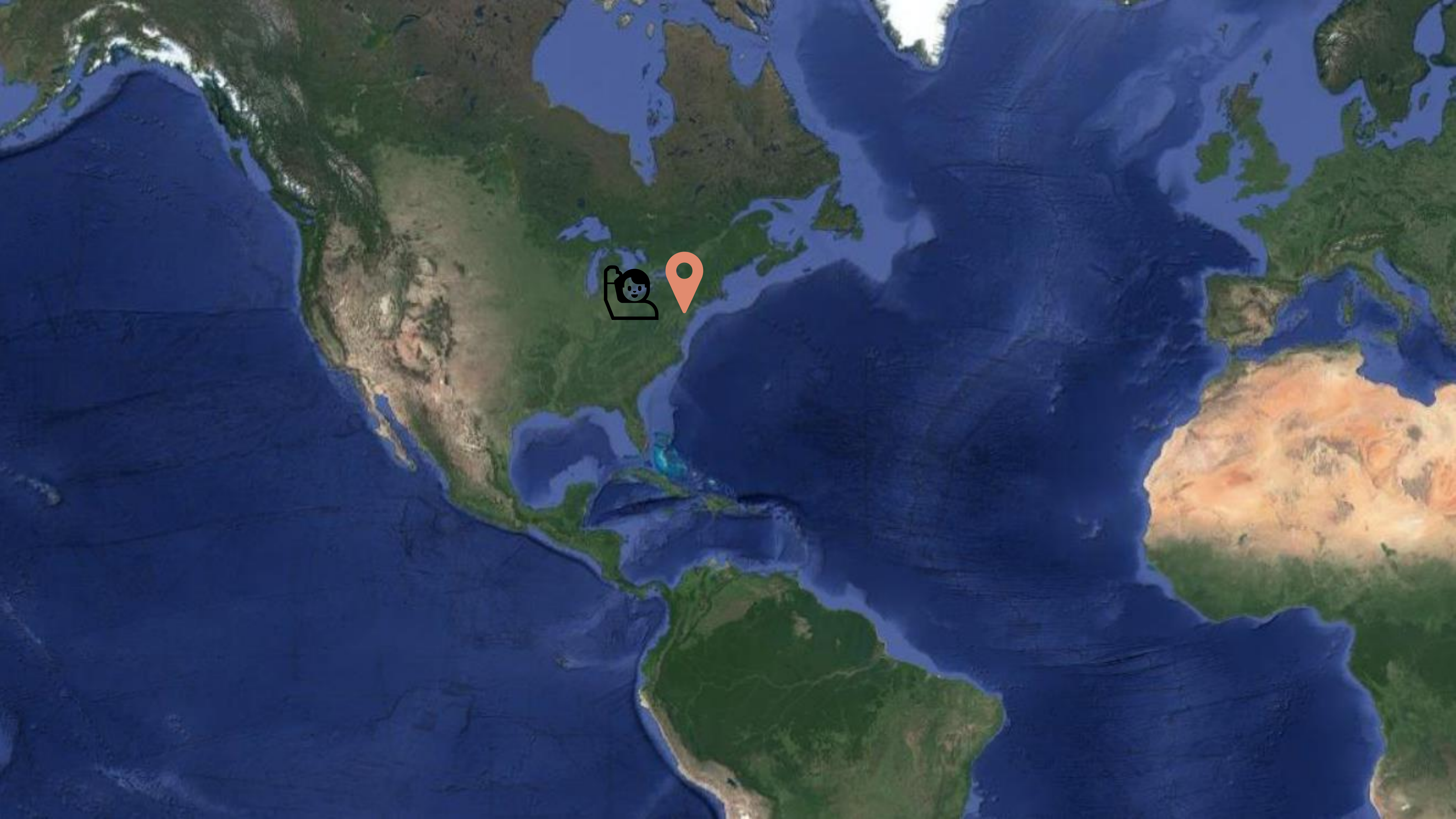
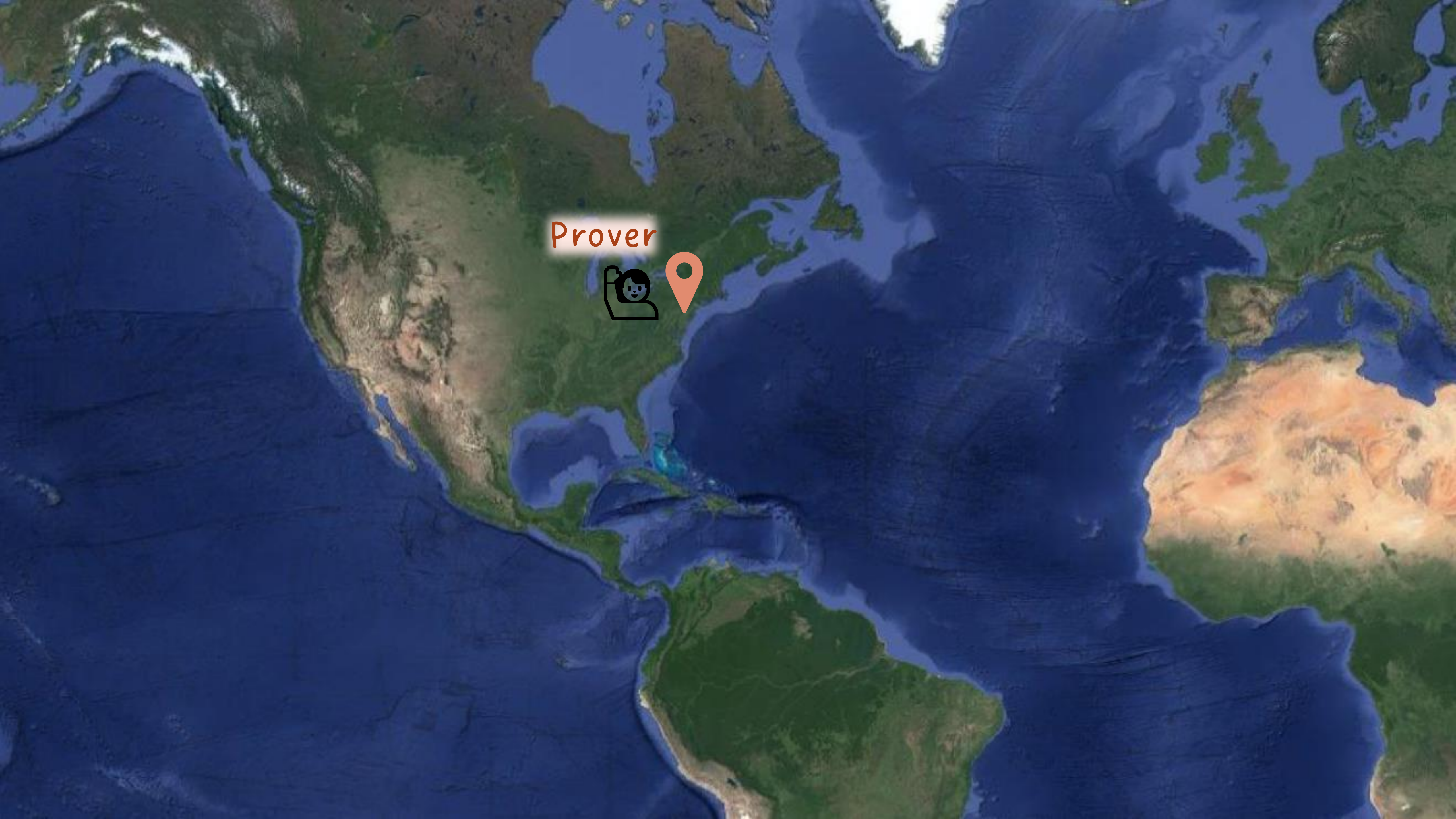
What is Position Verification?





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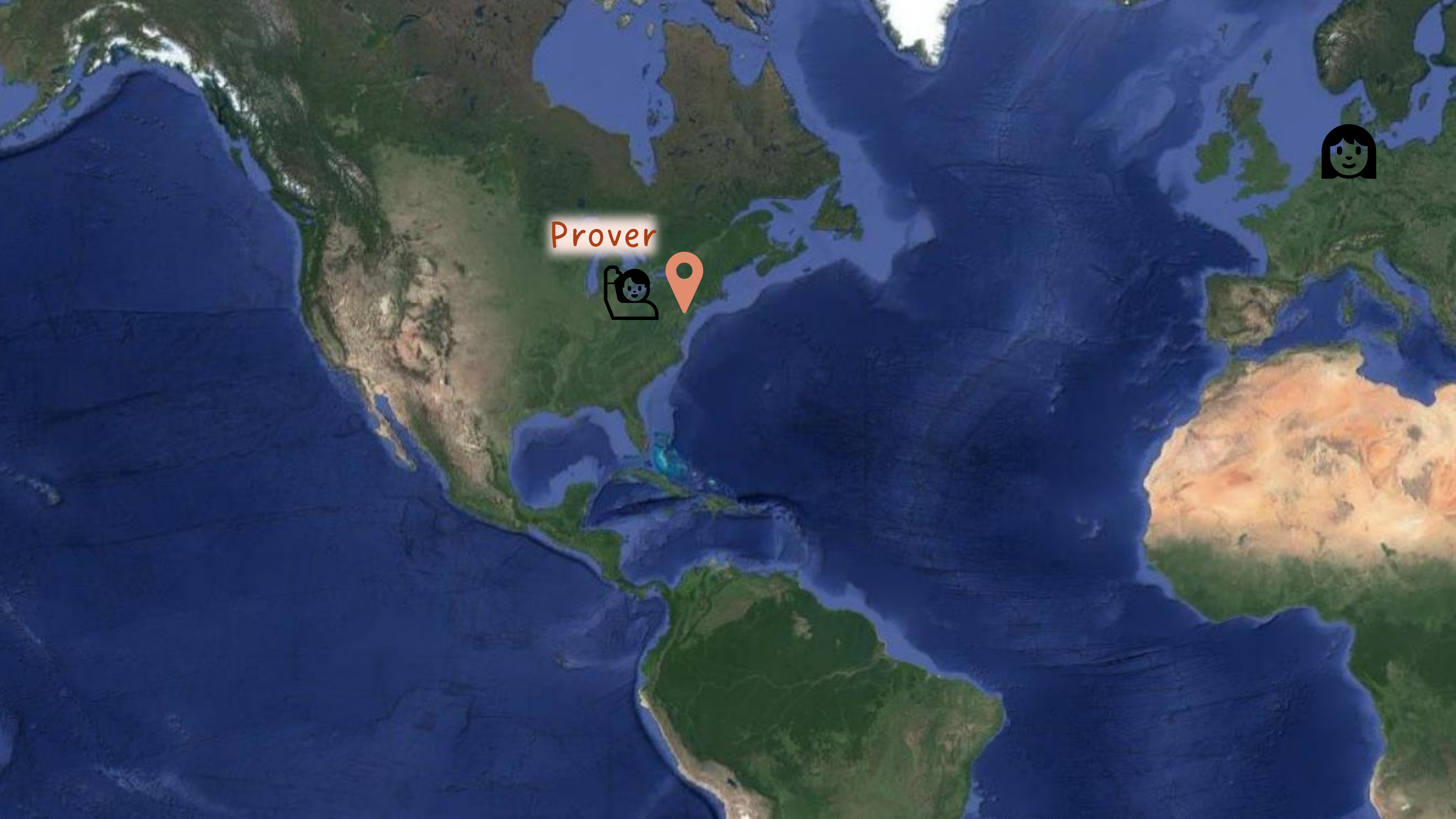
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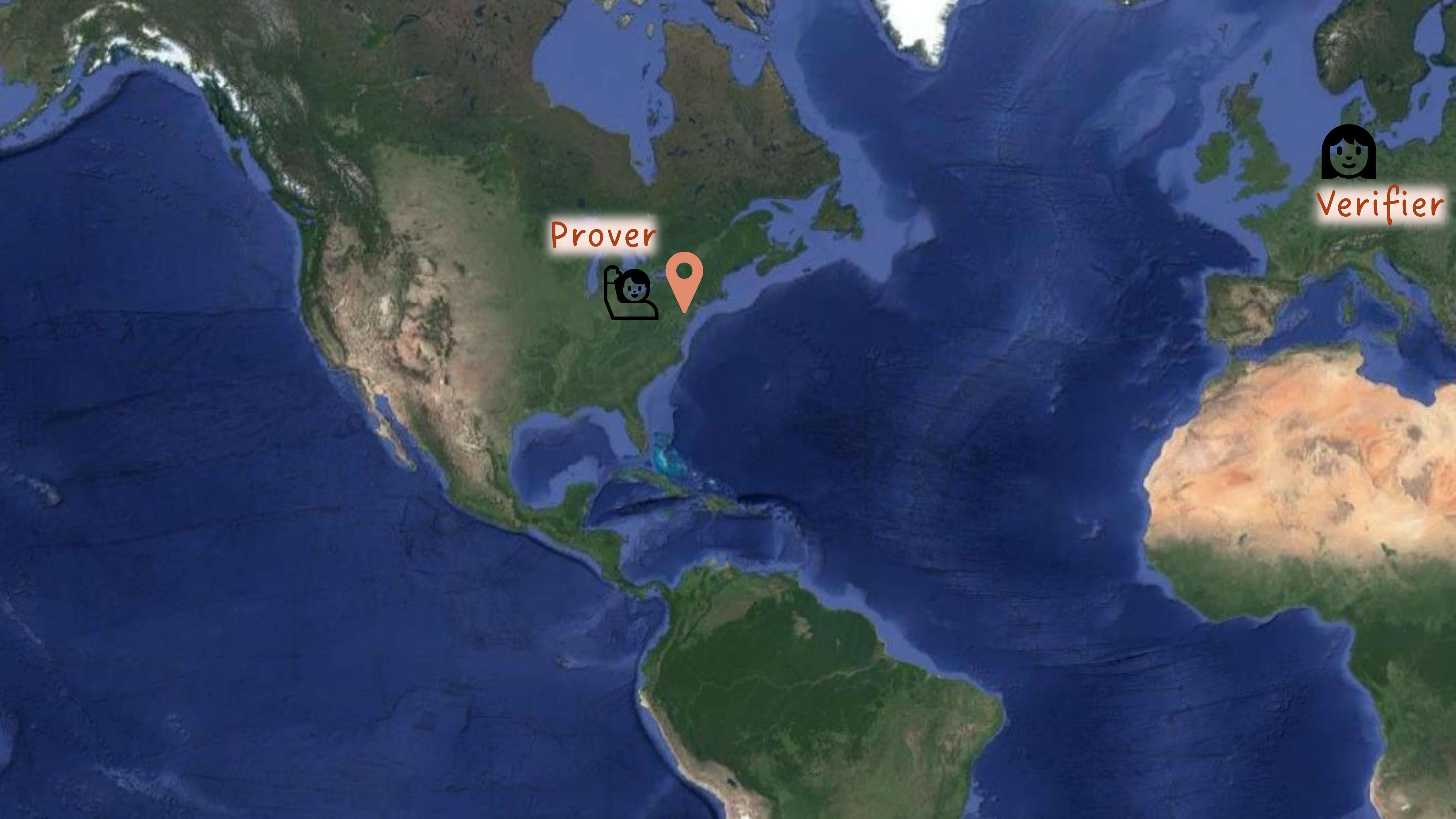
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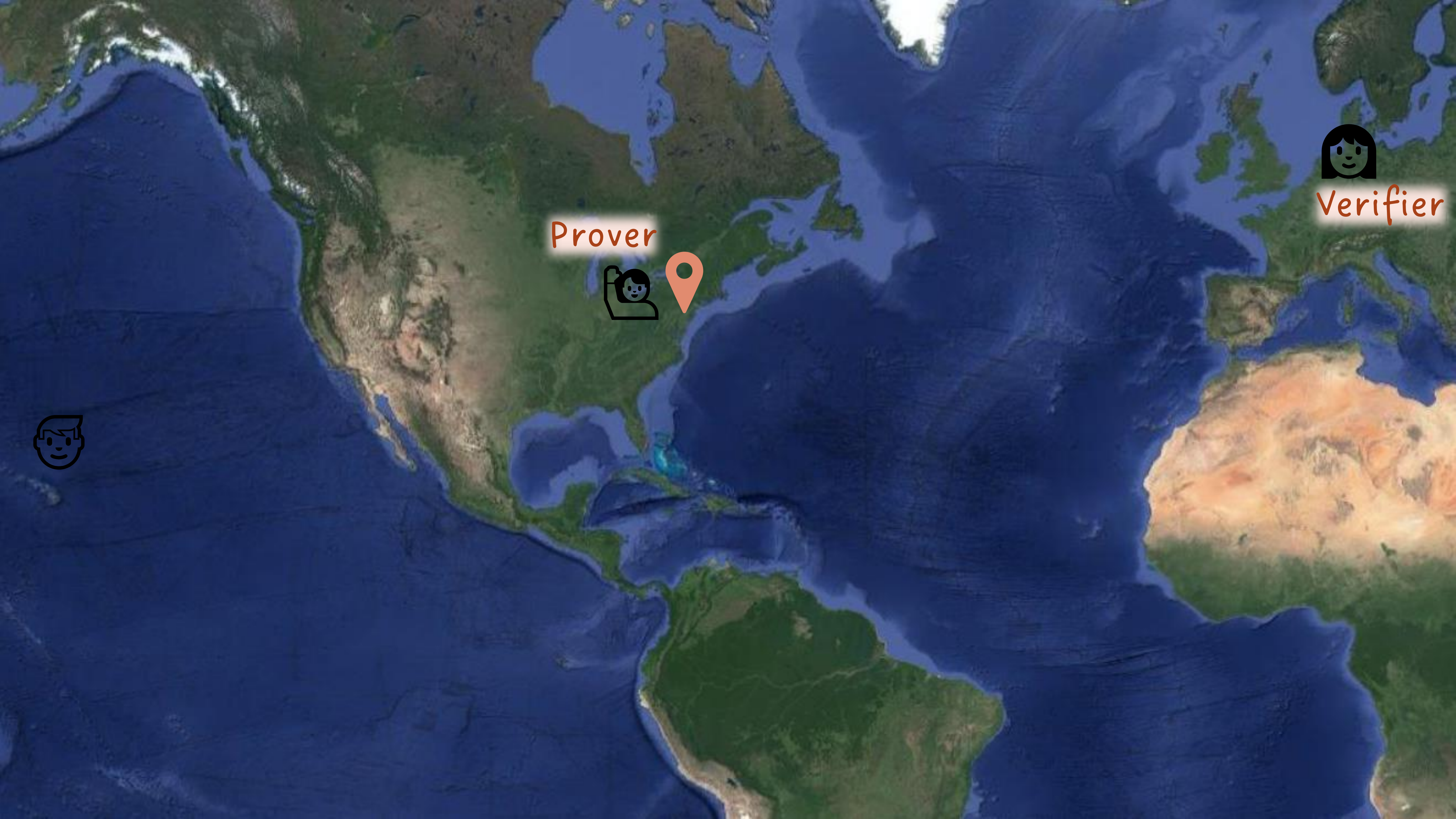
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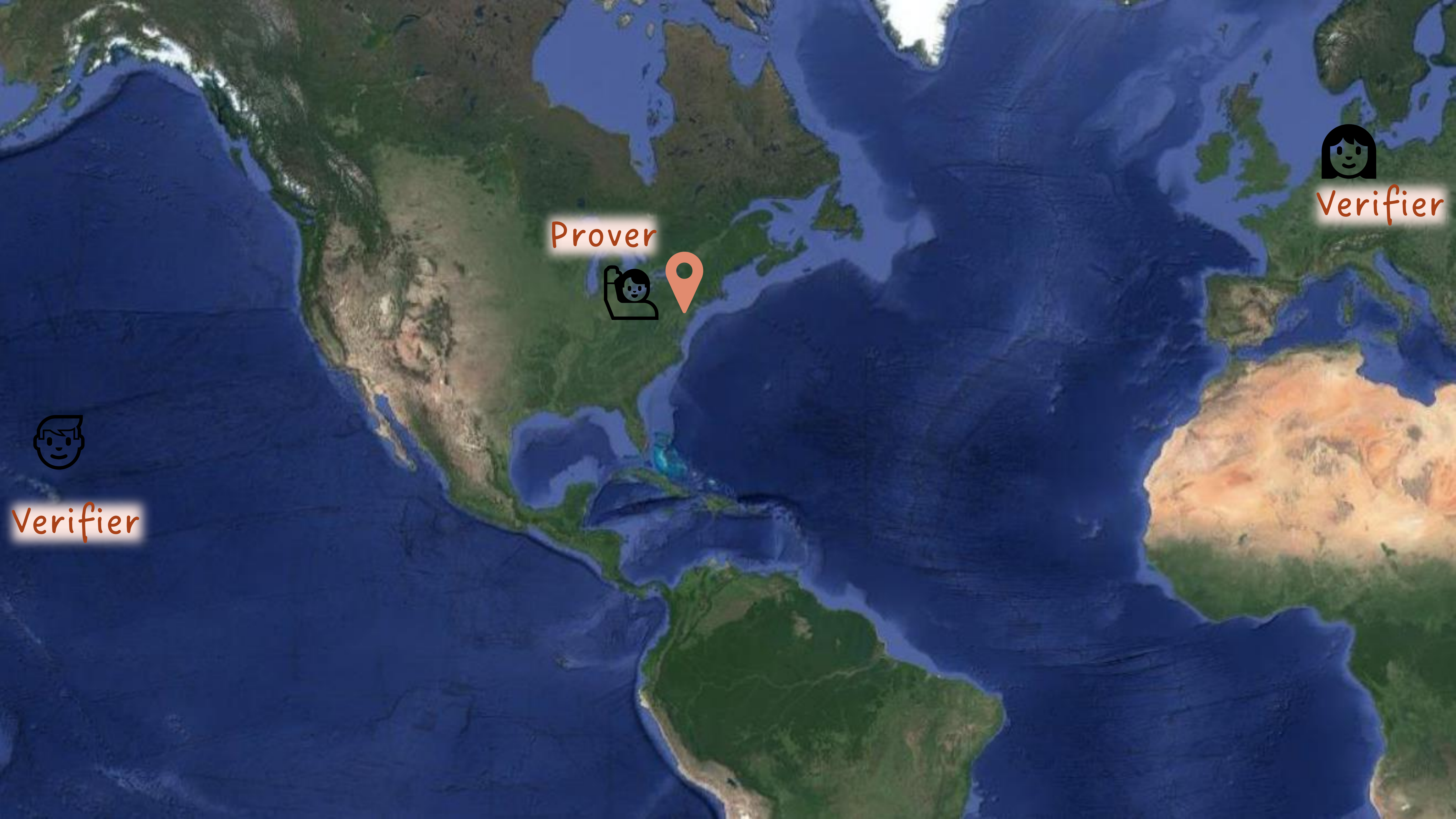
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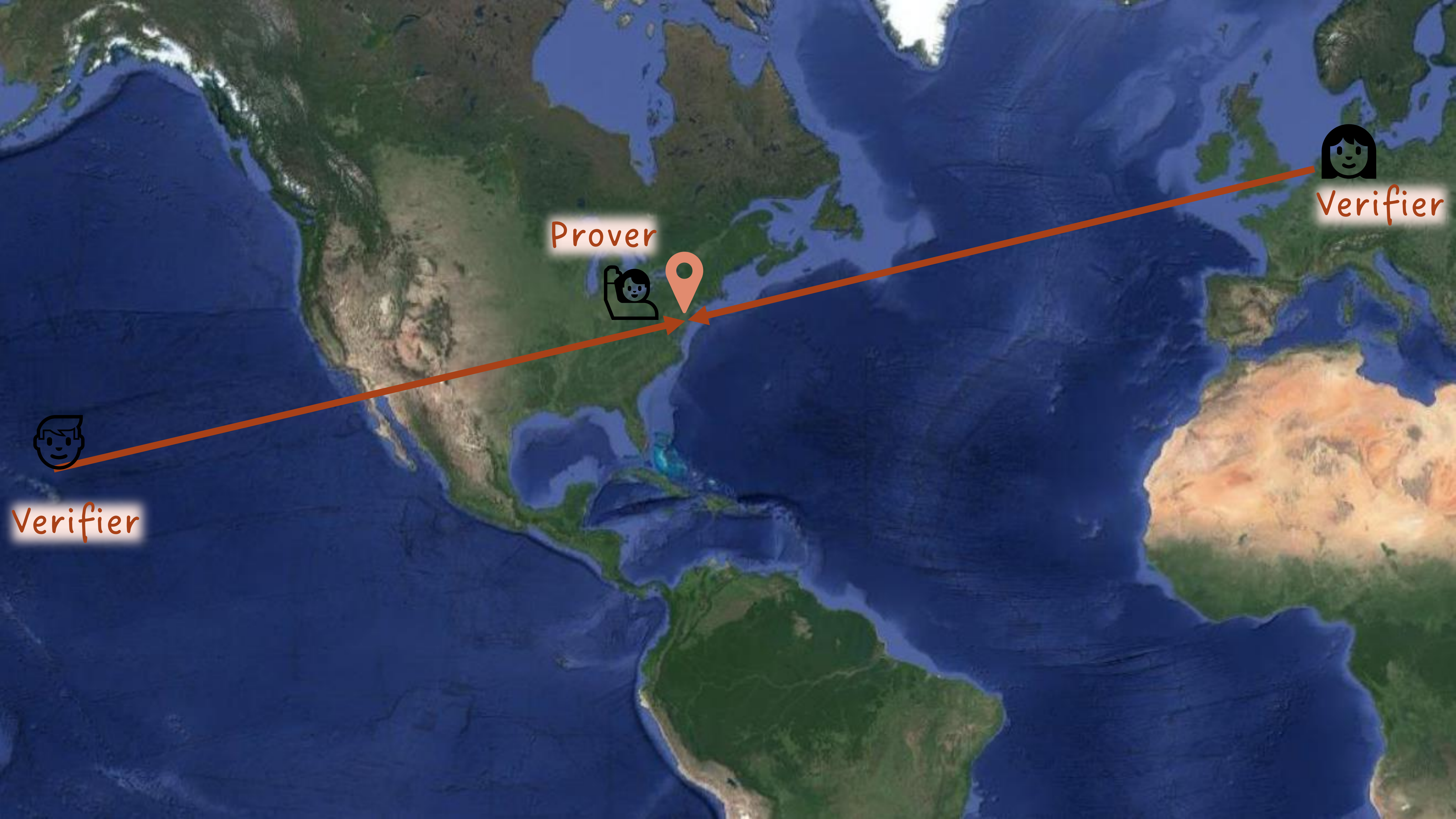
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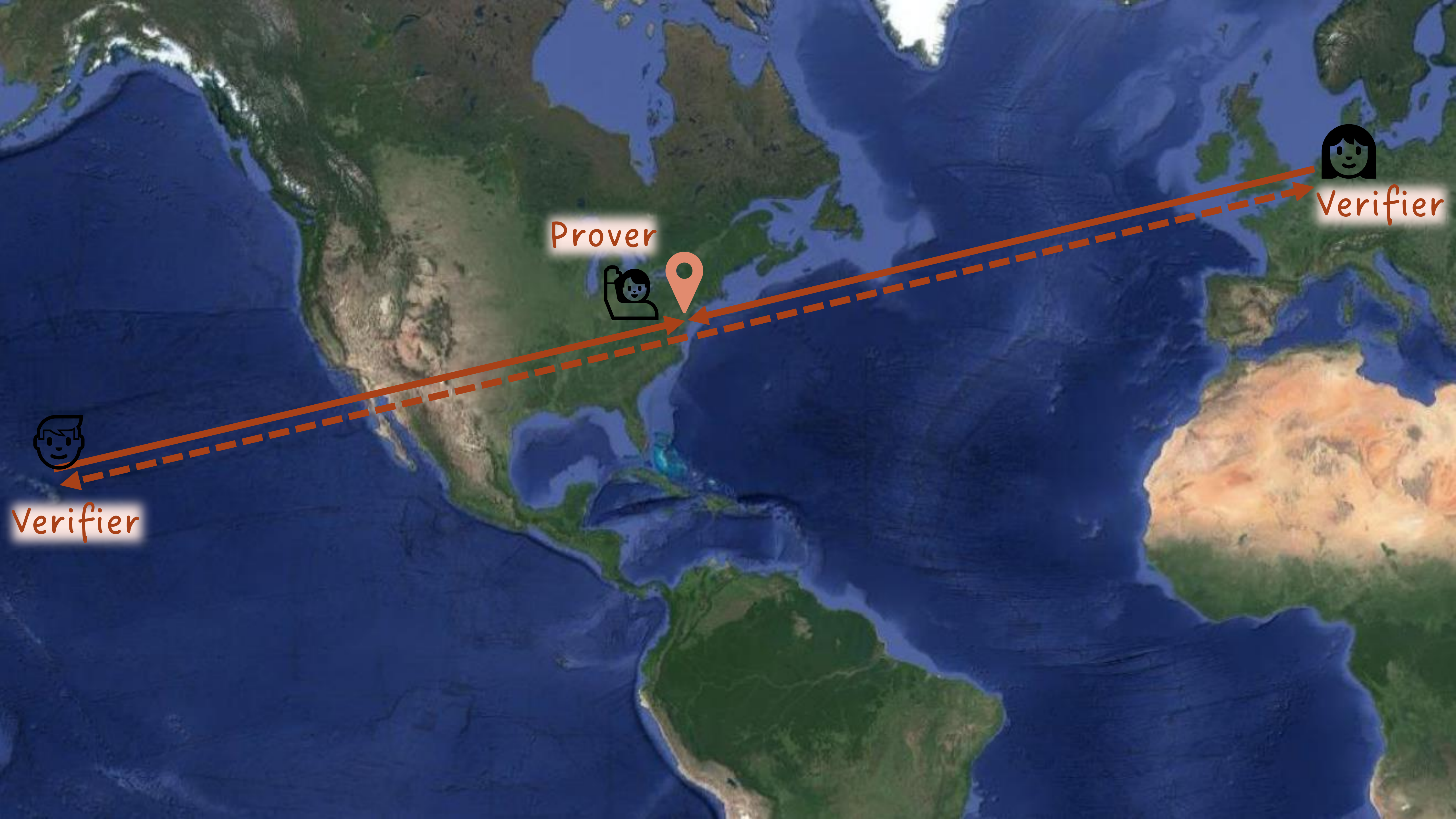
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Classical Position Verification

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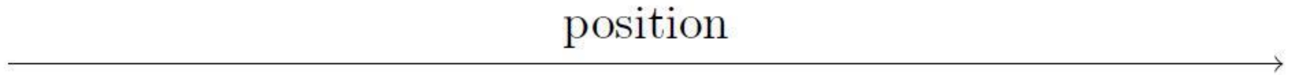

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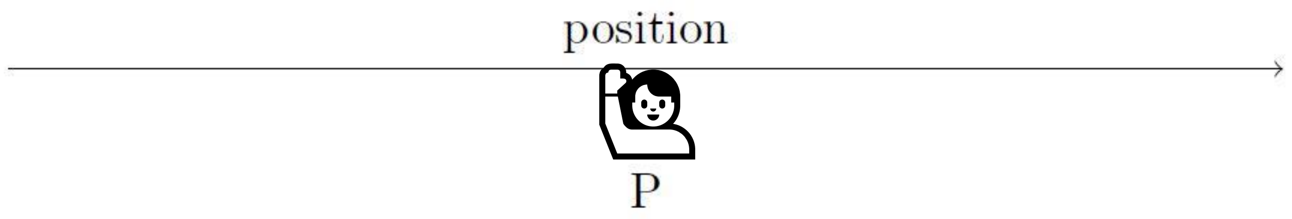

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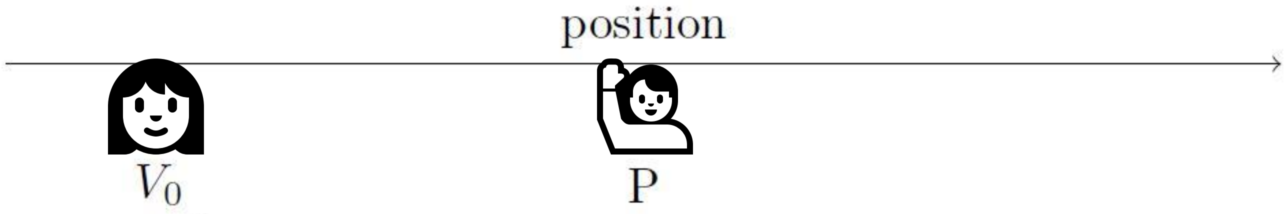

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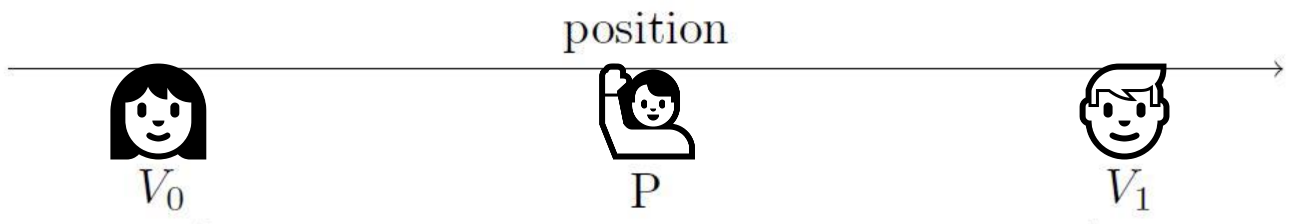

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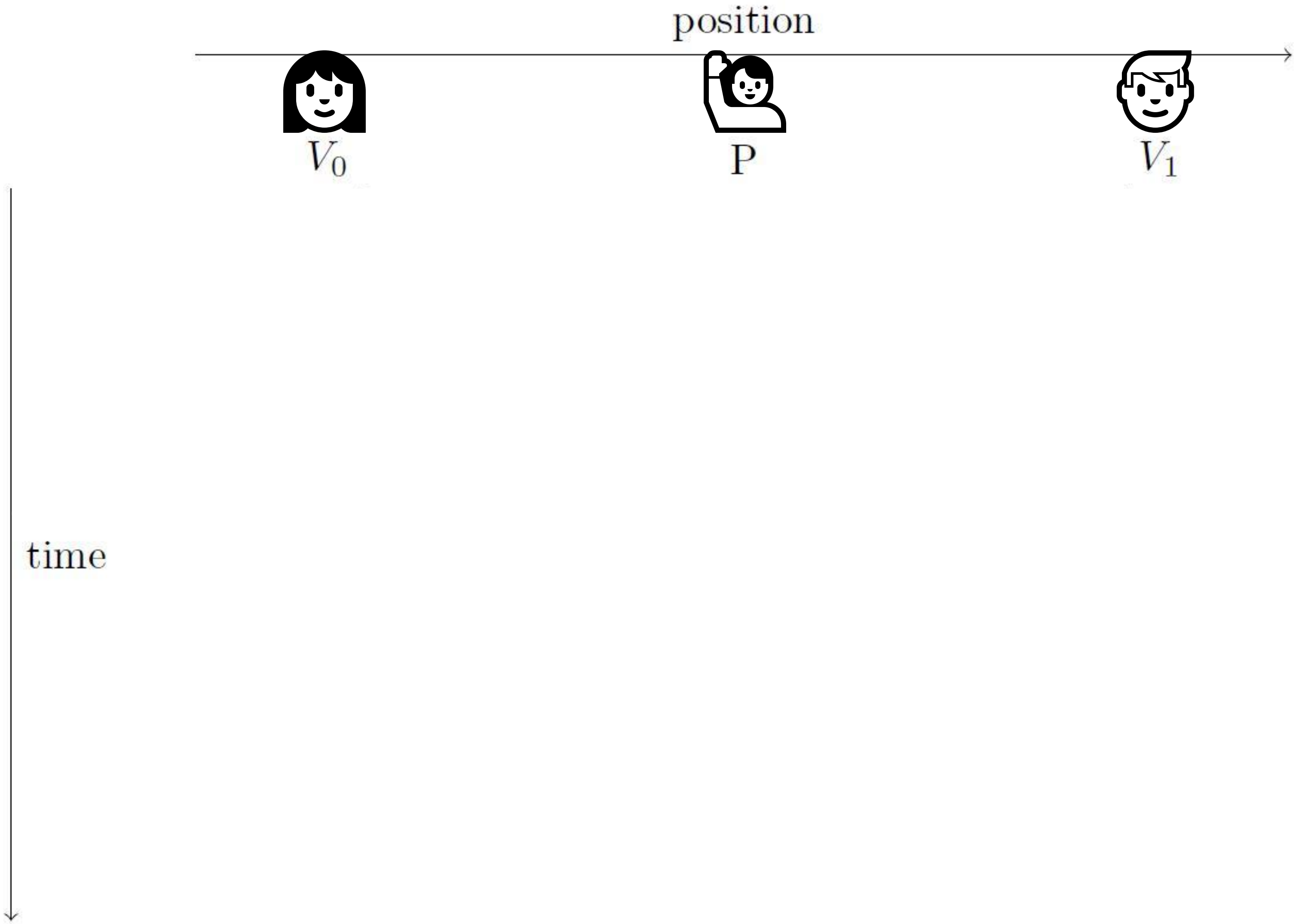

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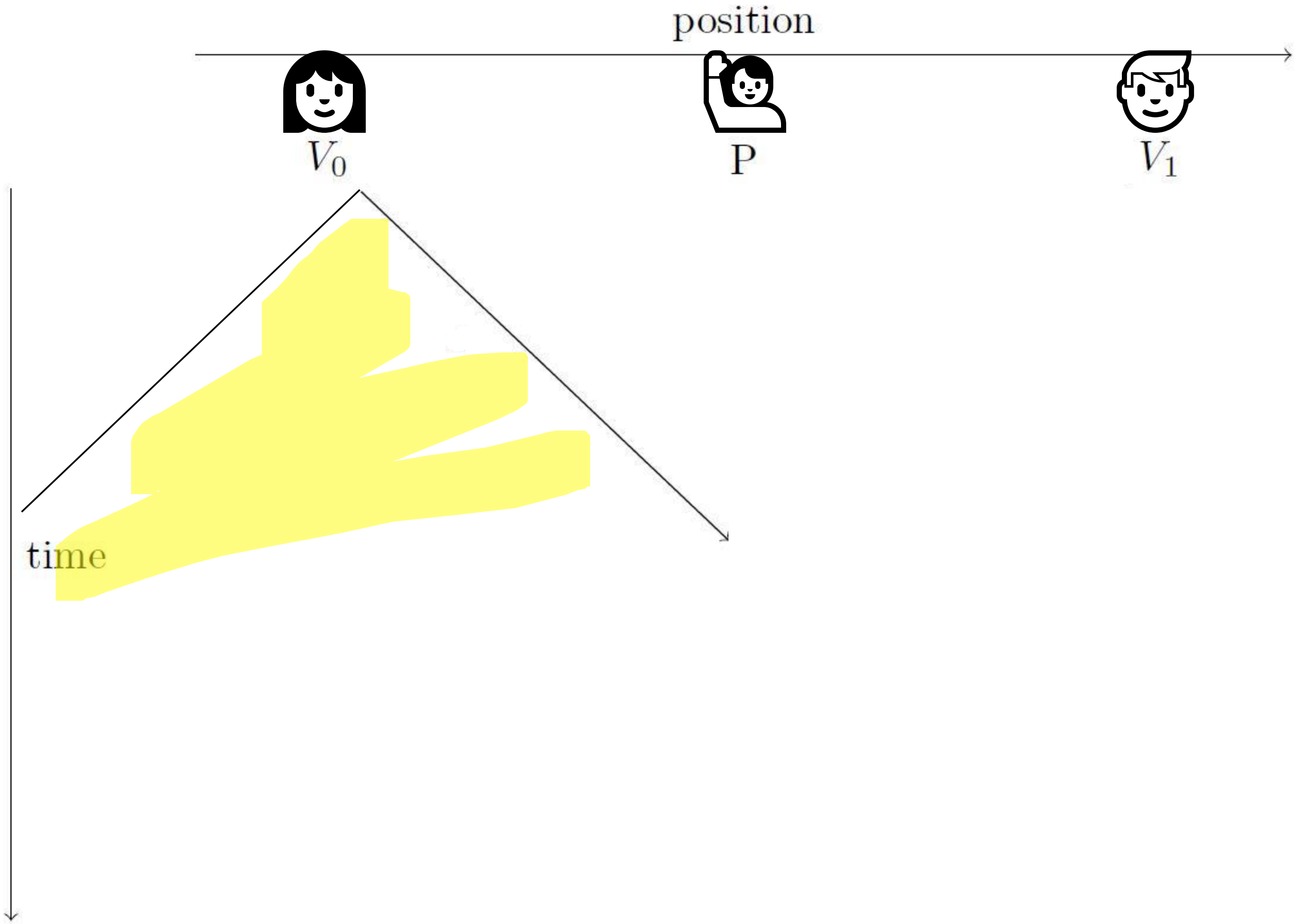

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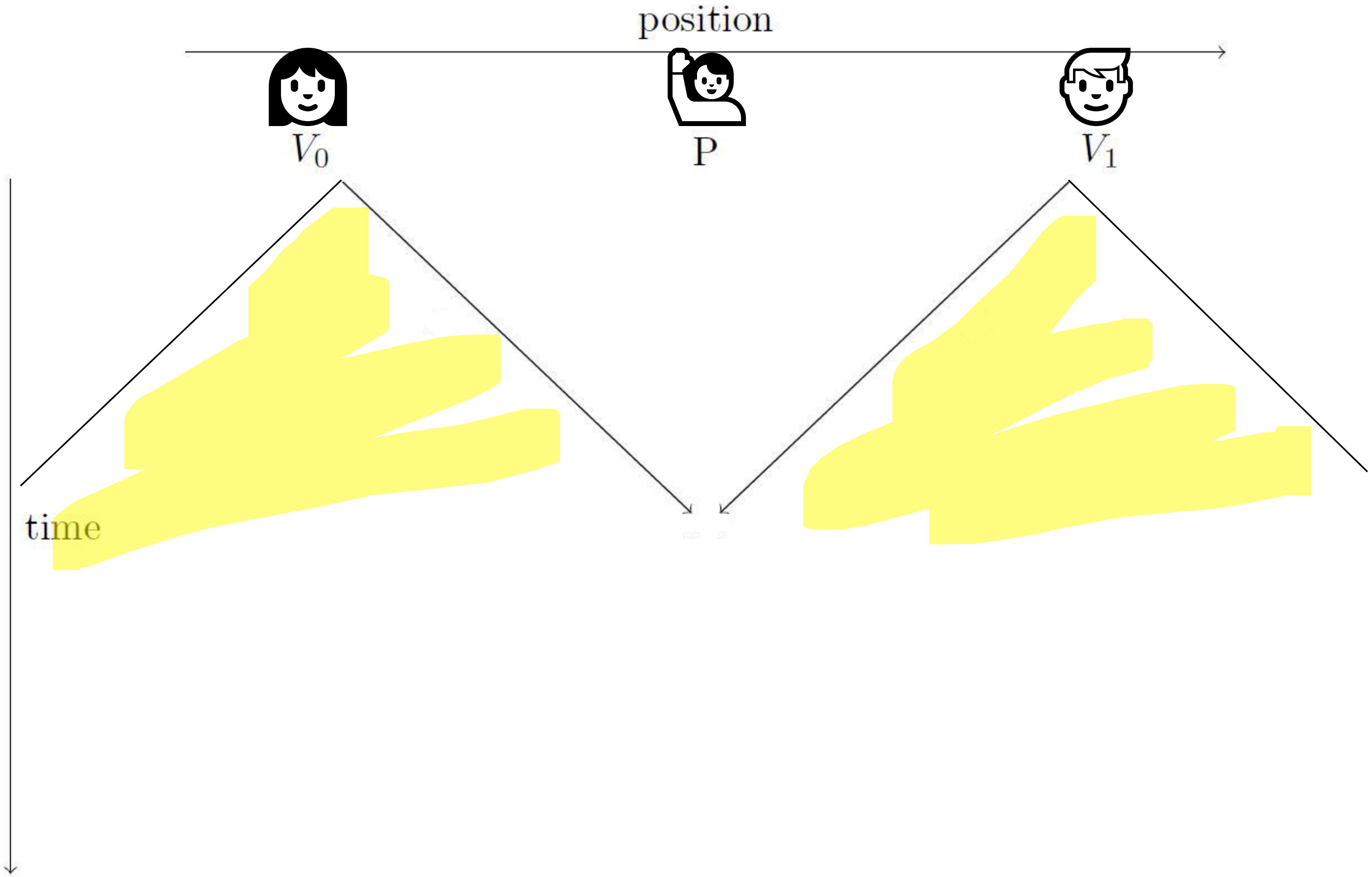

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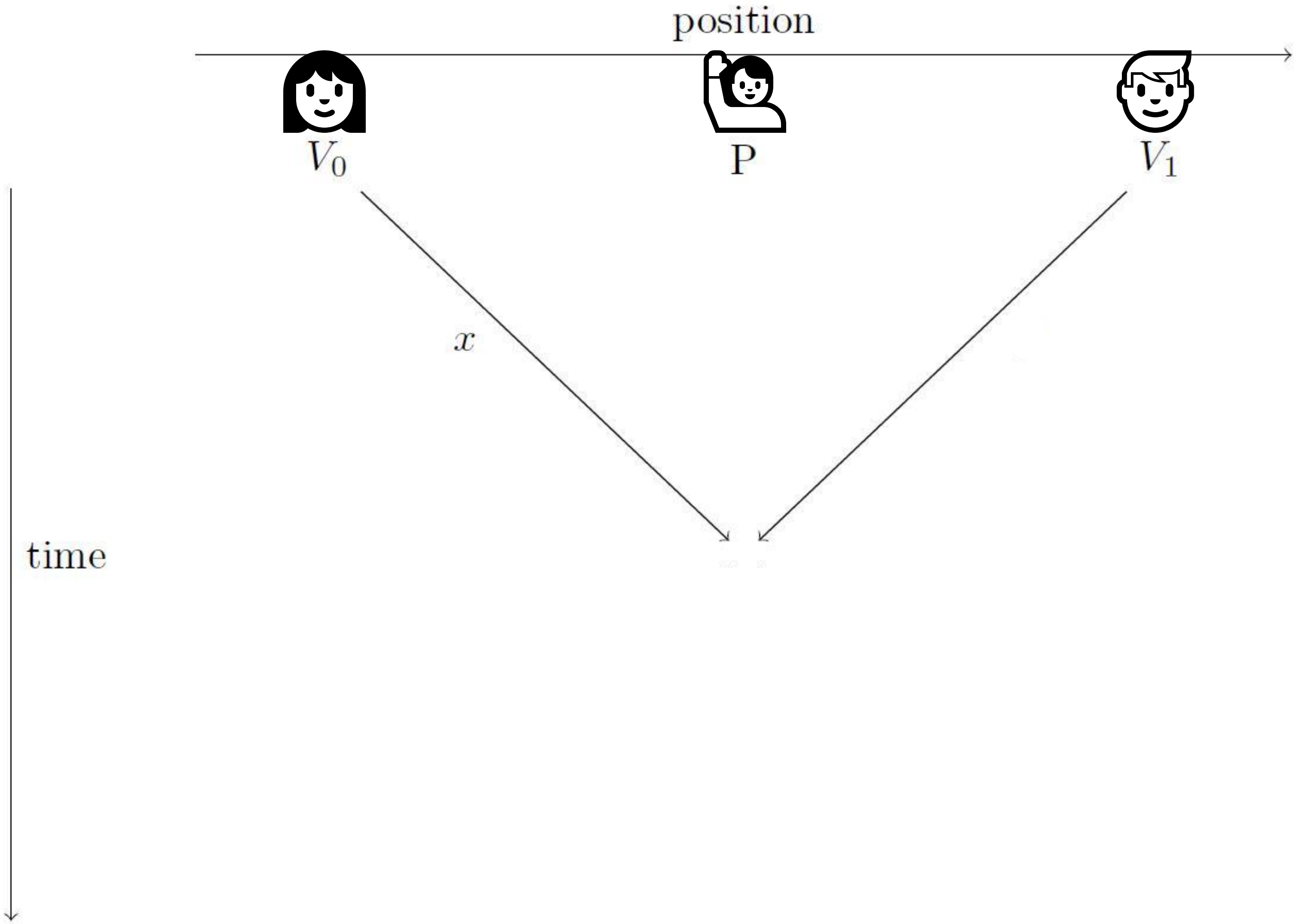

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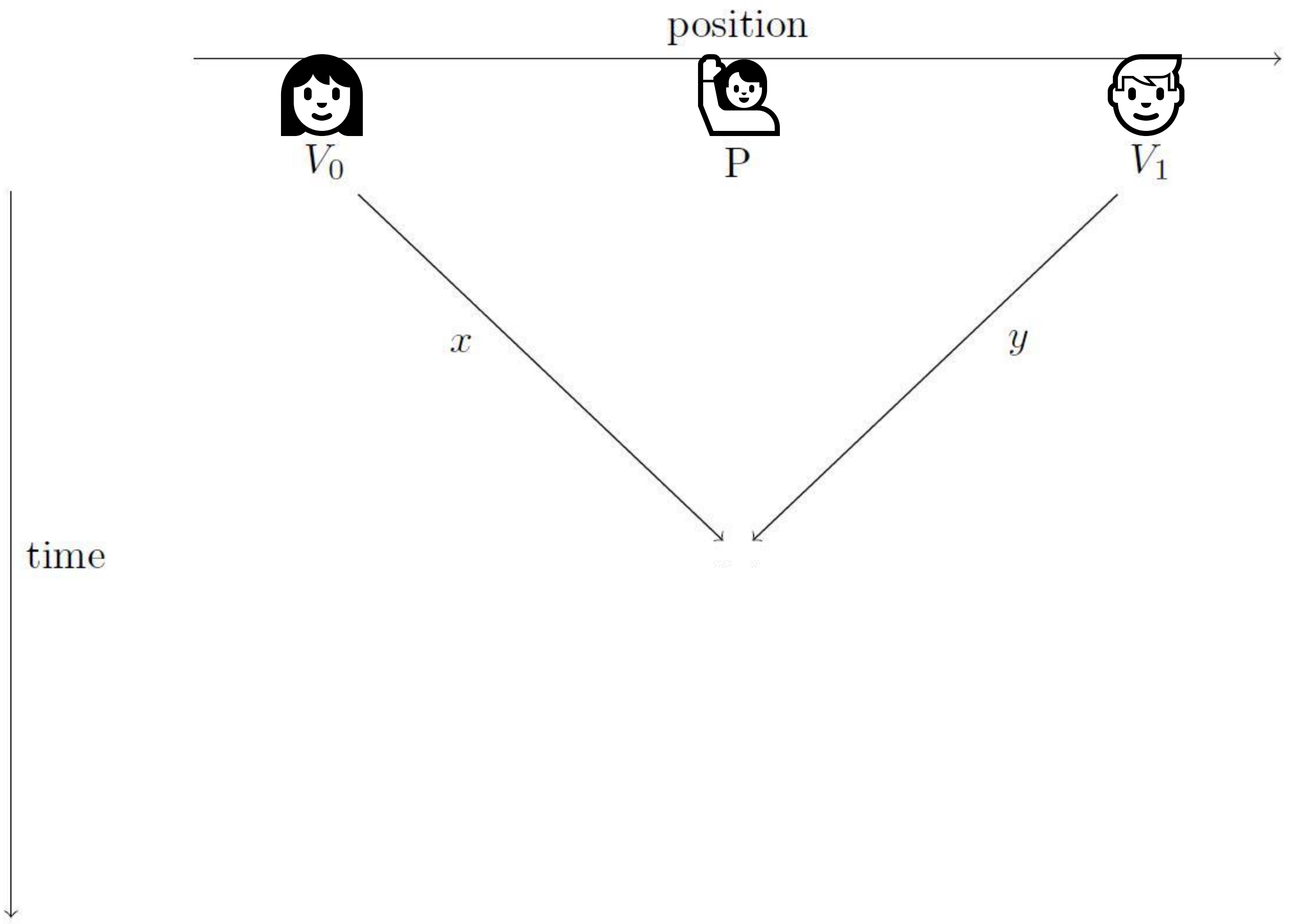

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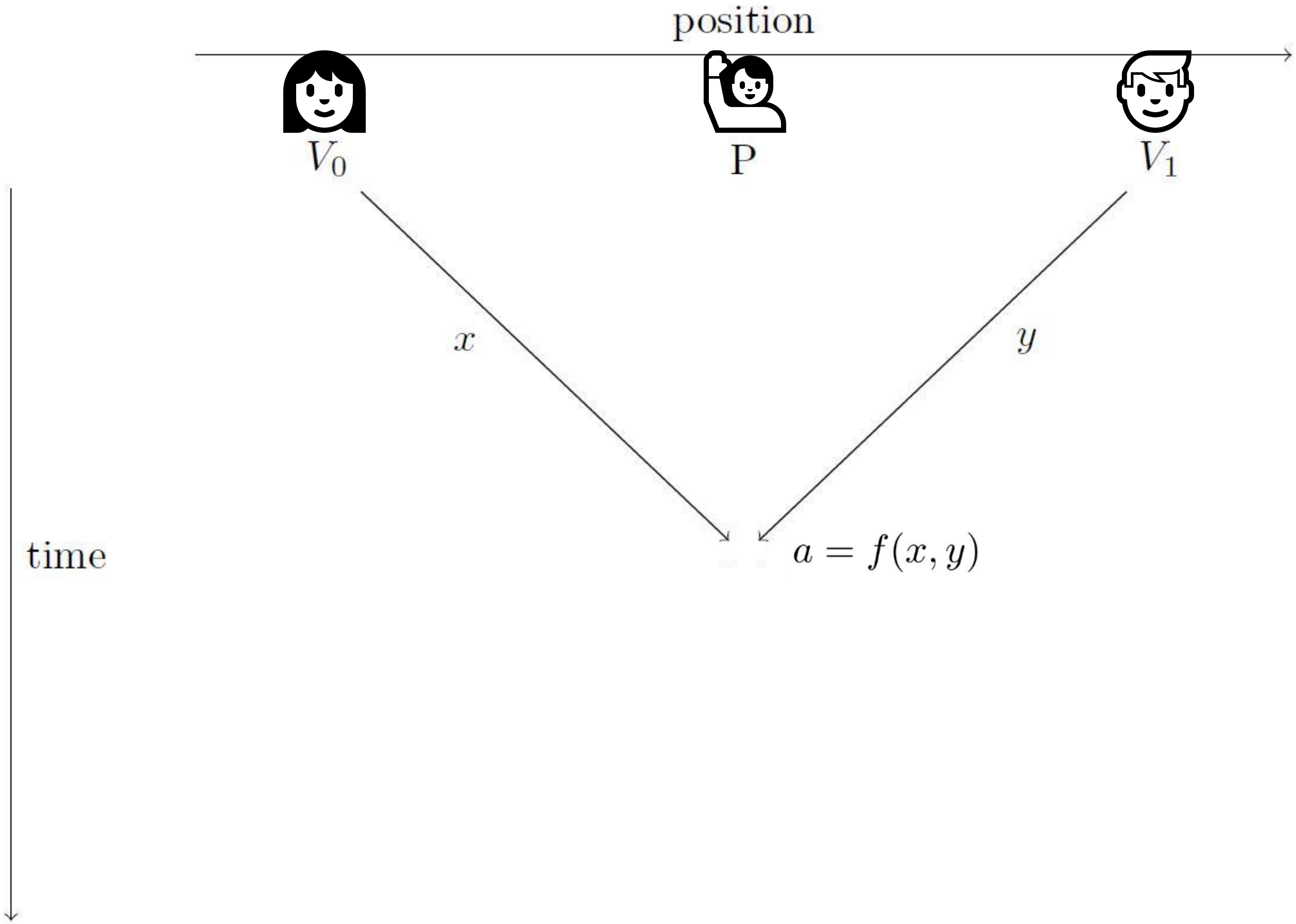

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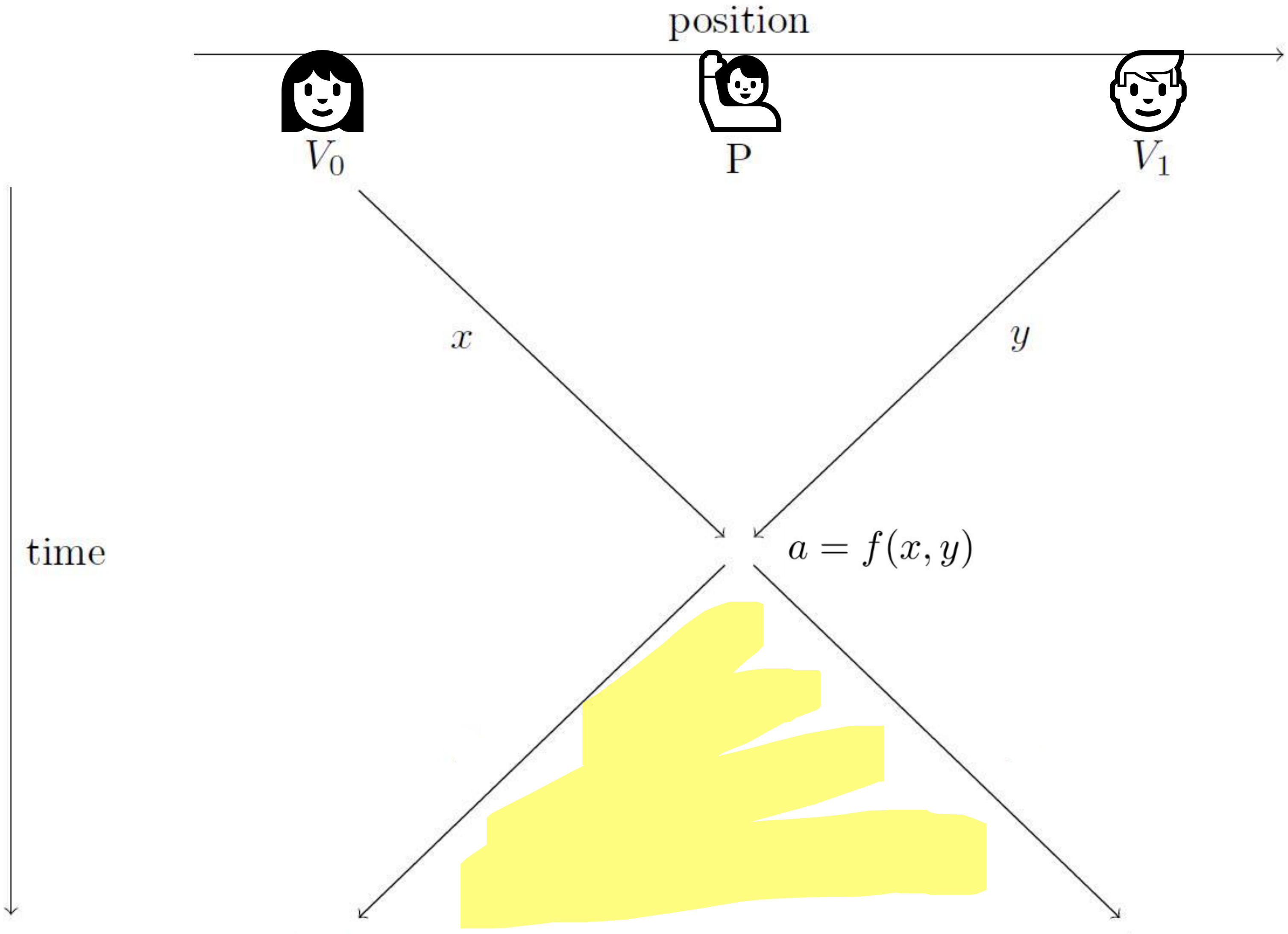

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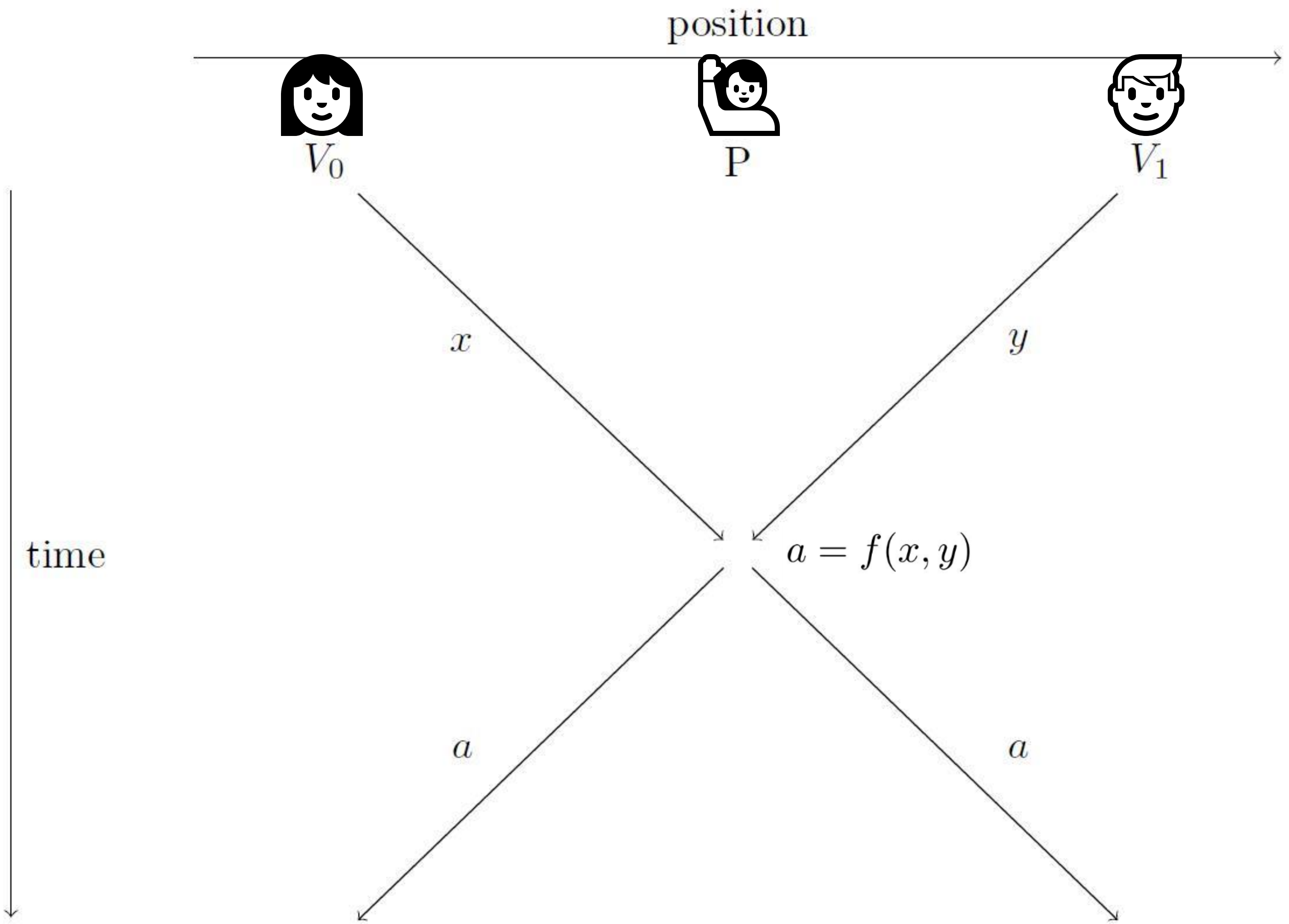

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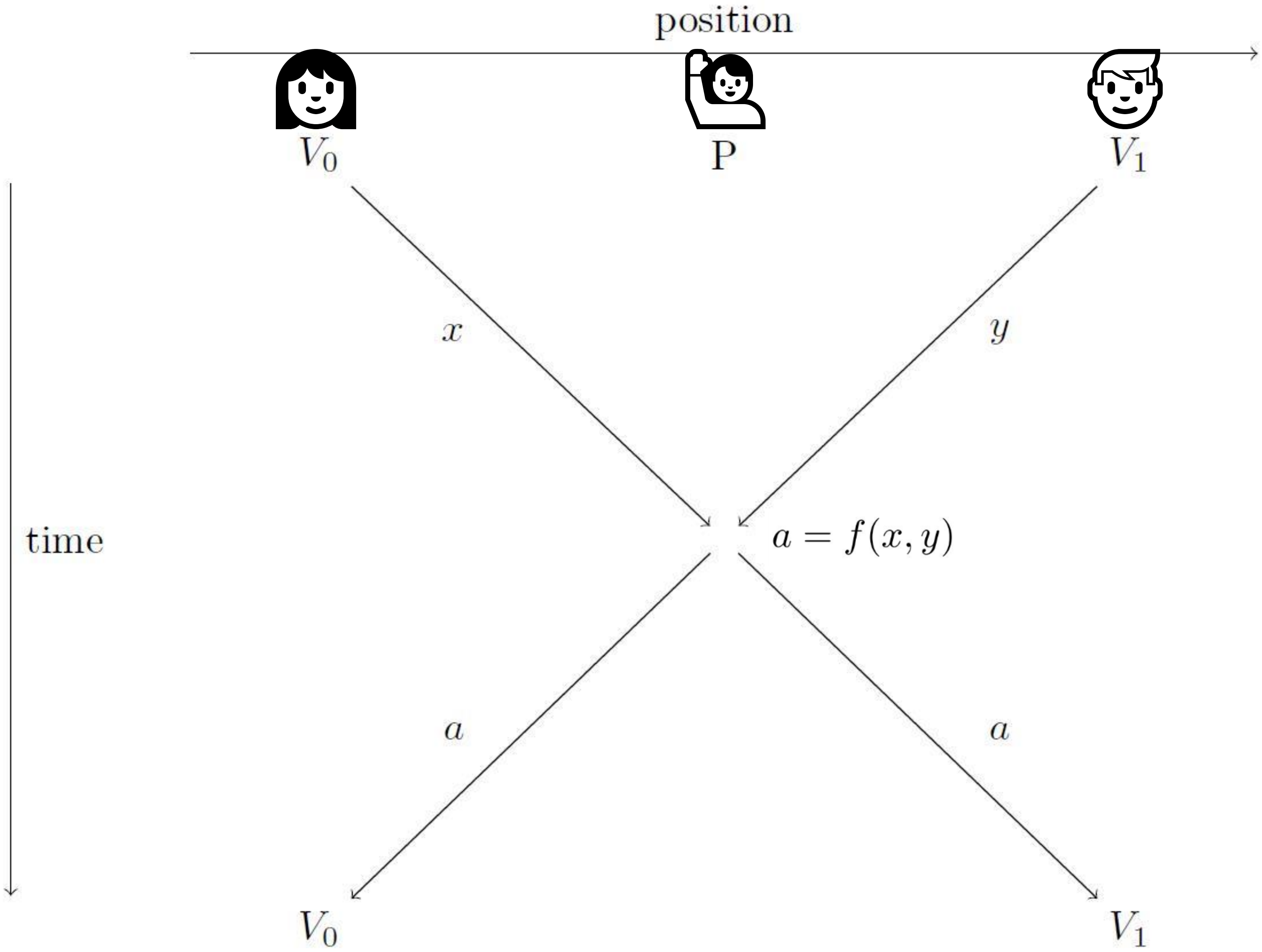

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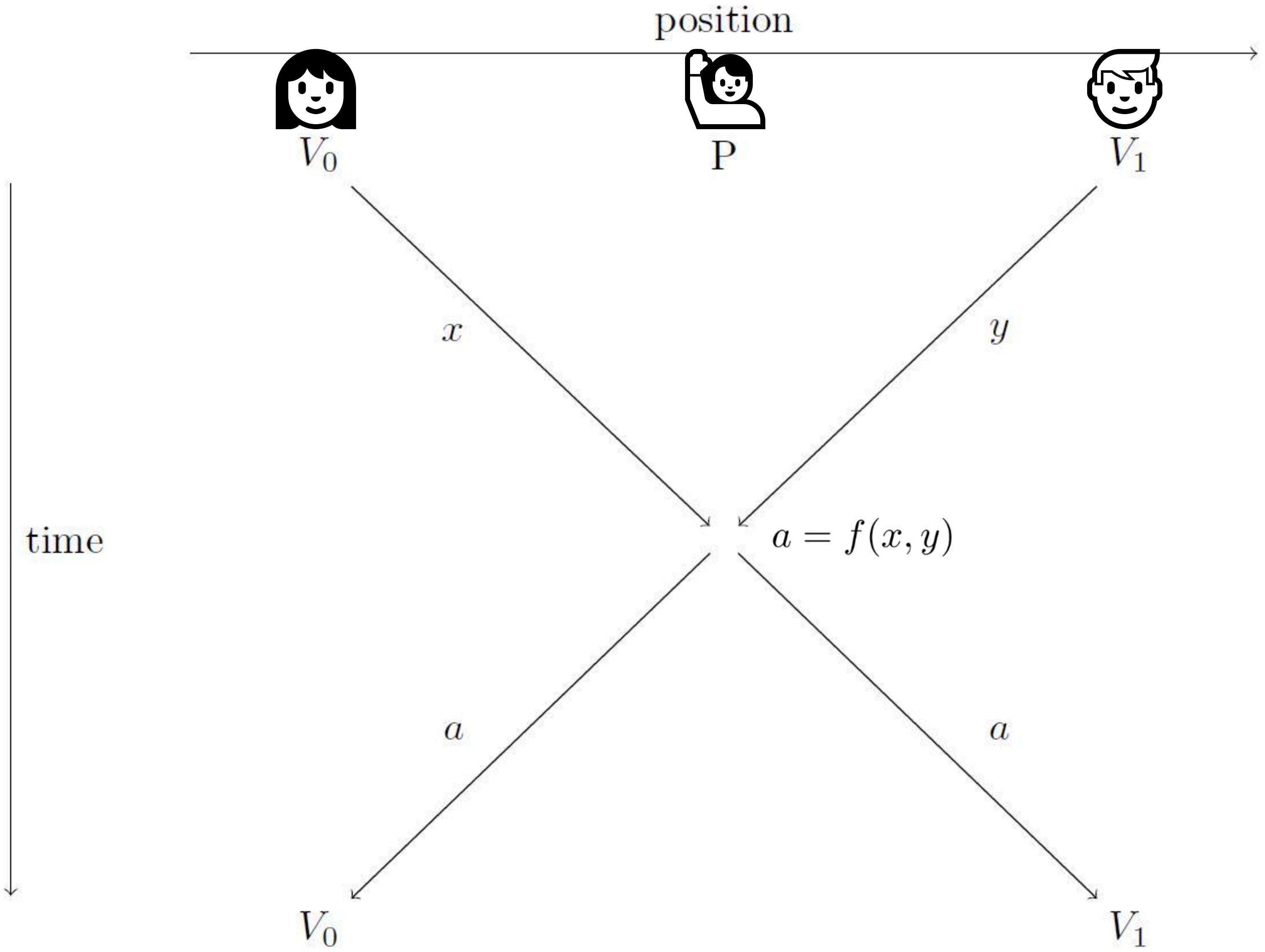

If both a are correct and arrive on time

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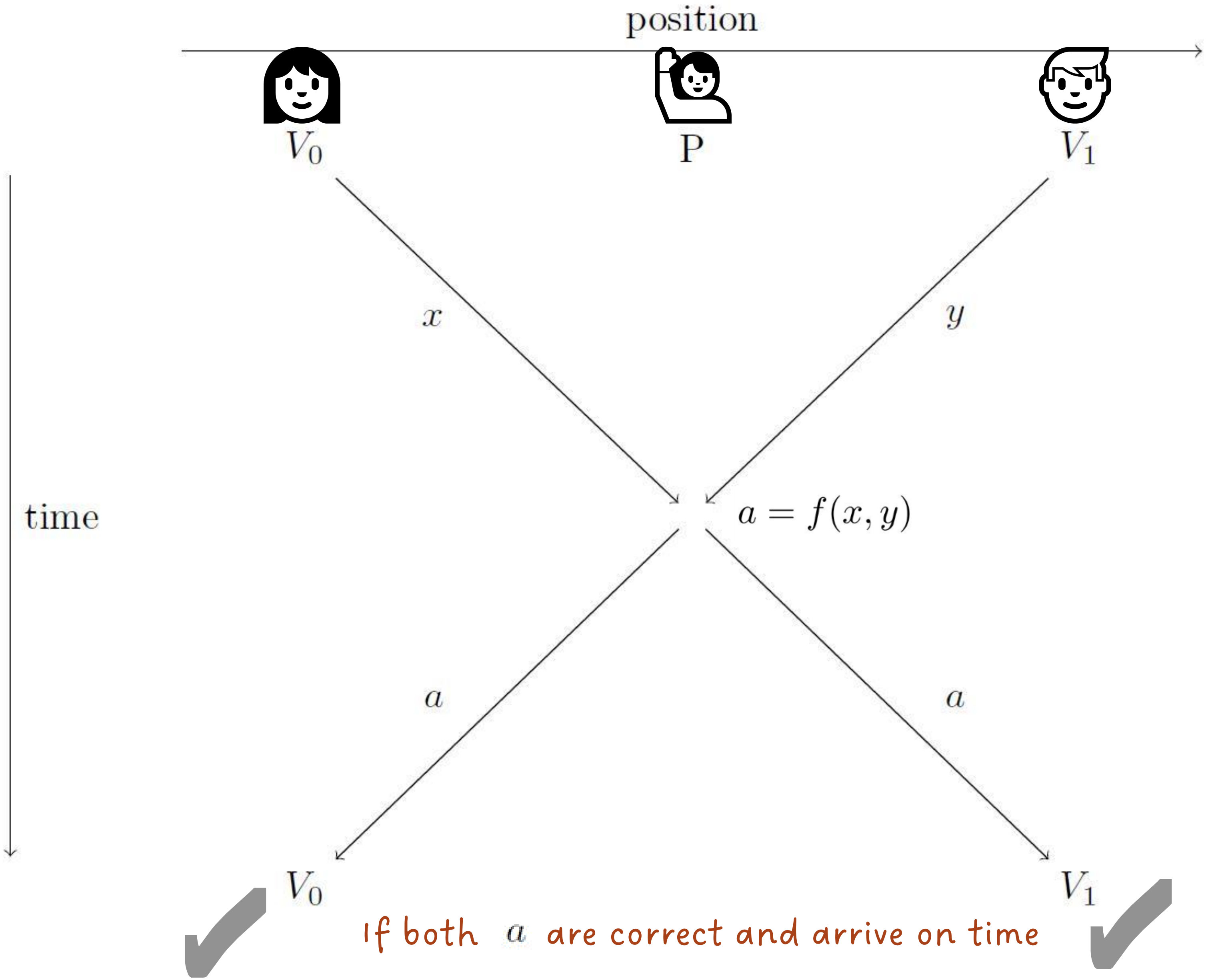

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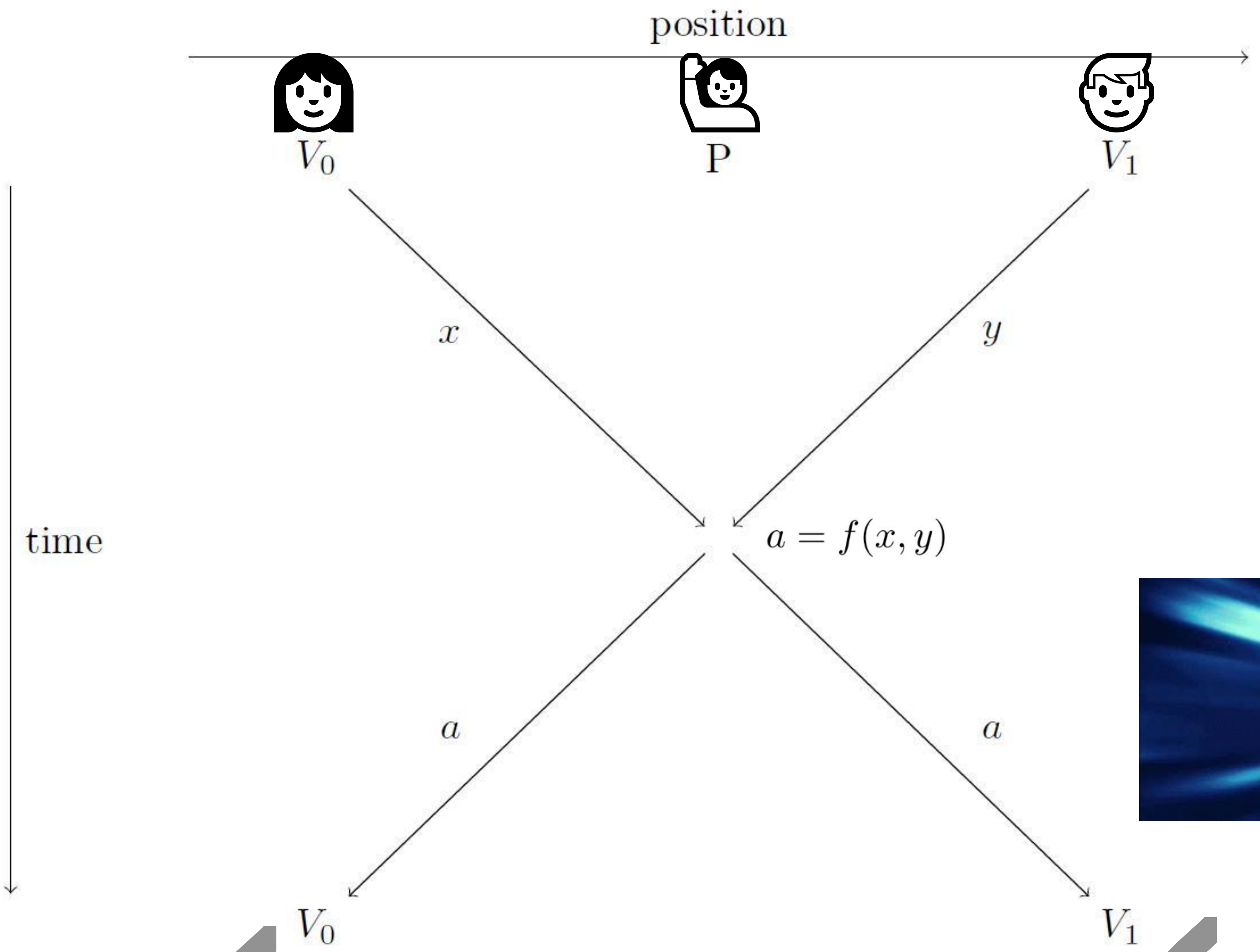

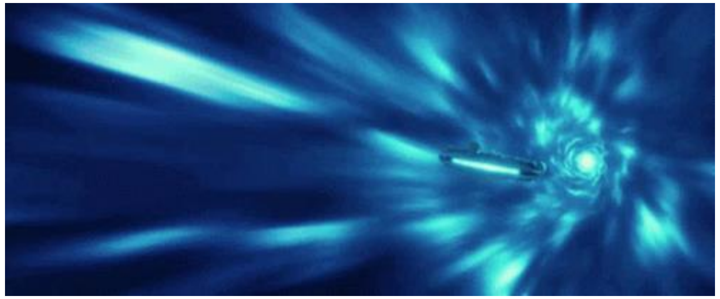

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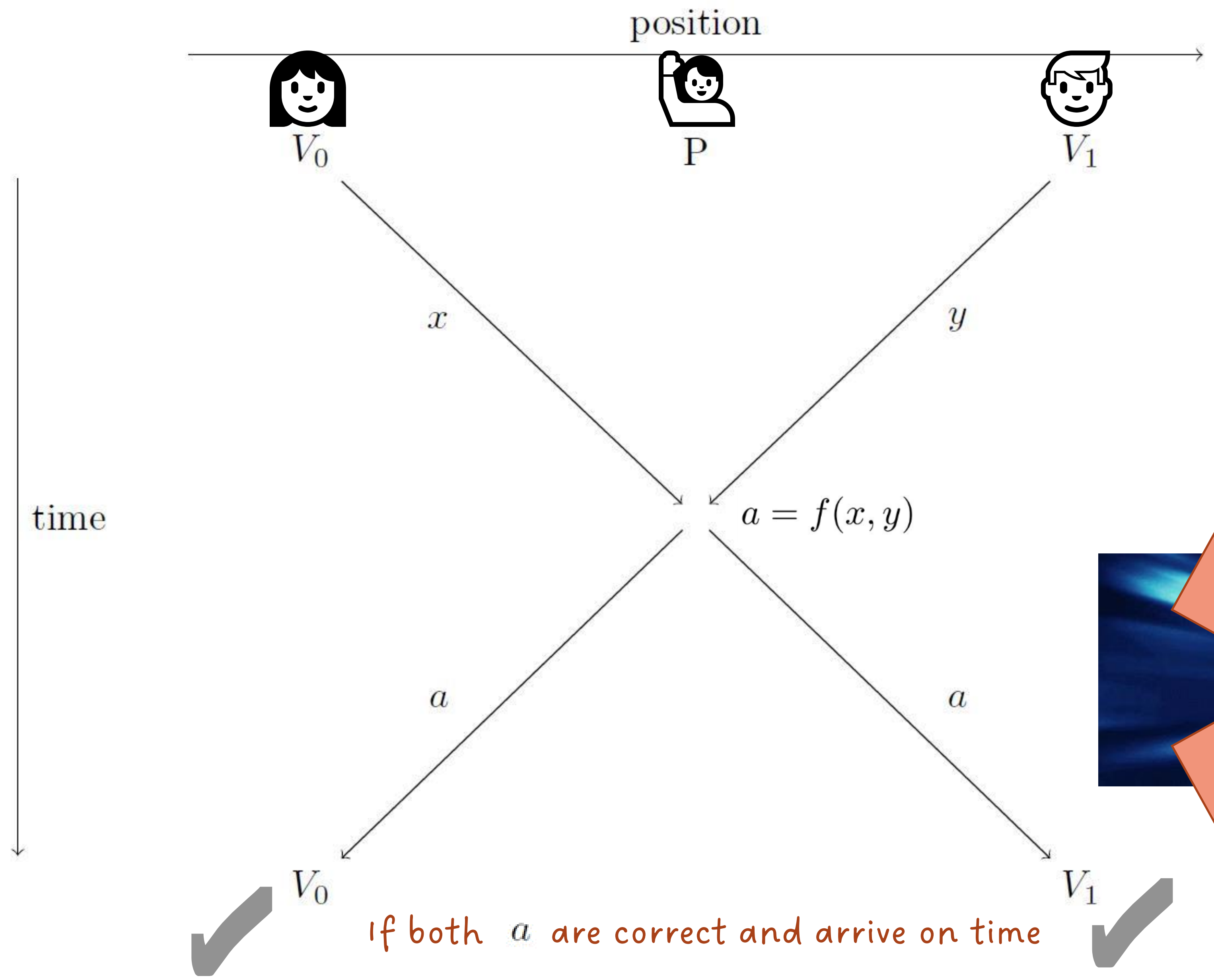
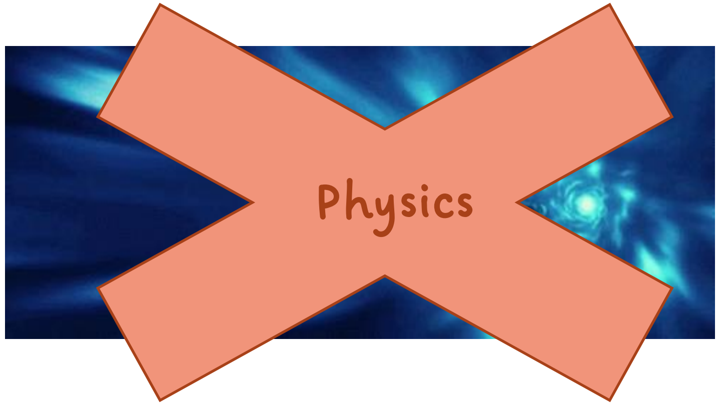


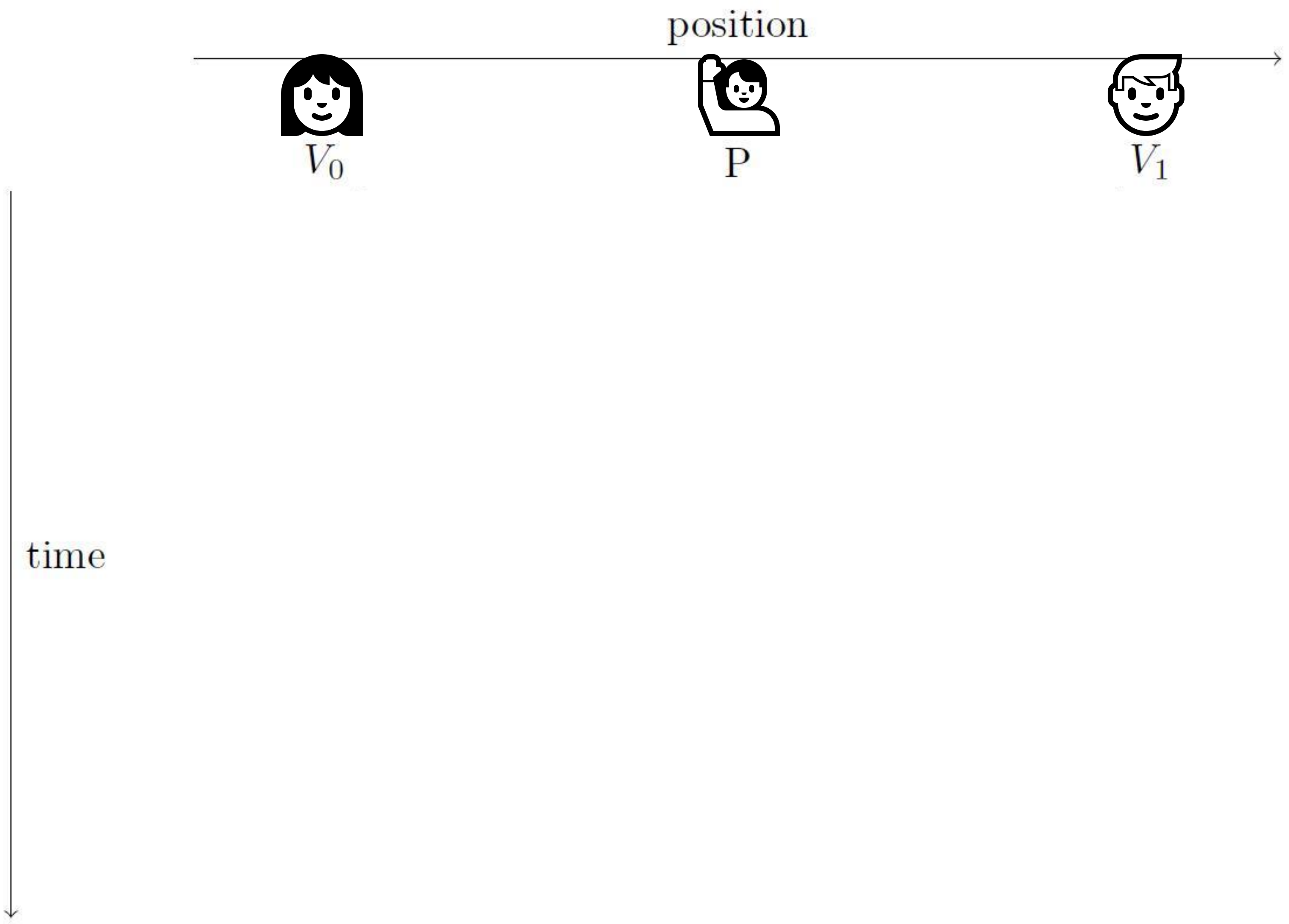


If both a are correct and arrive on time

But…

Universal attack

Classical universal attack



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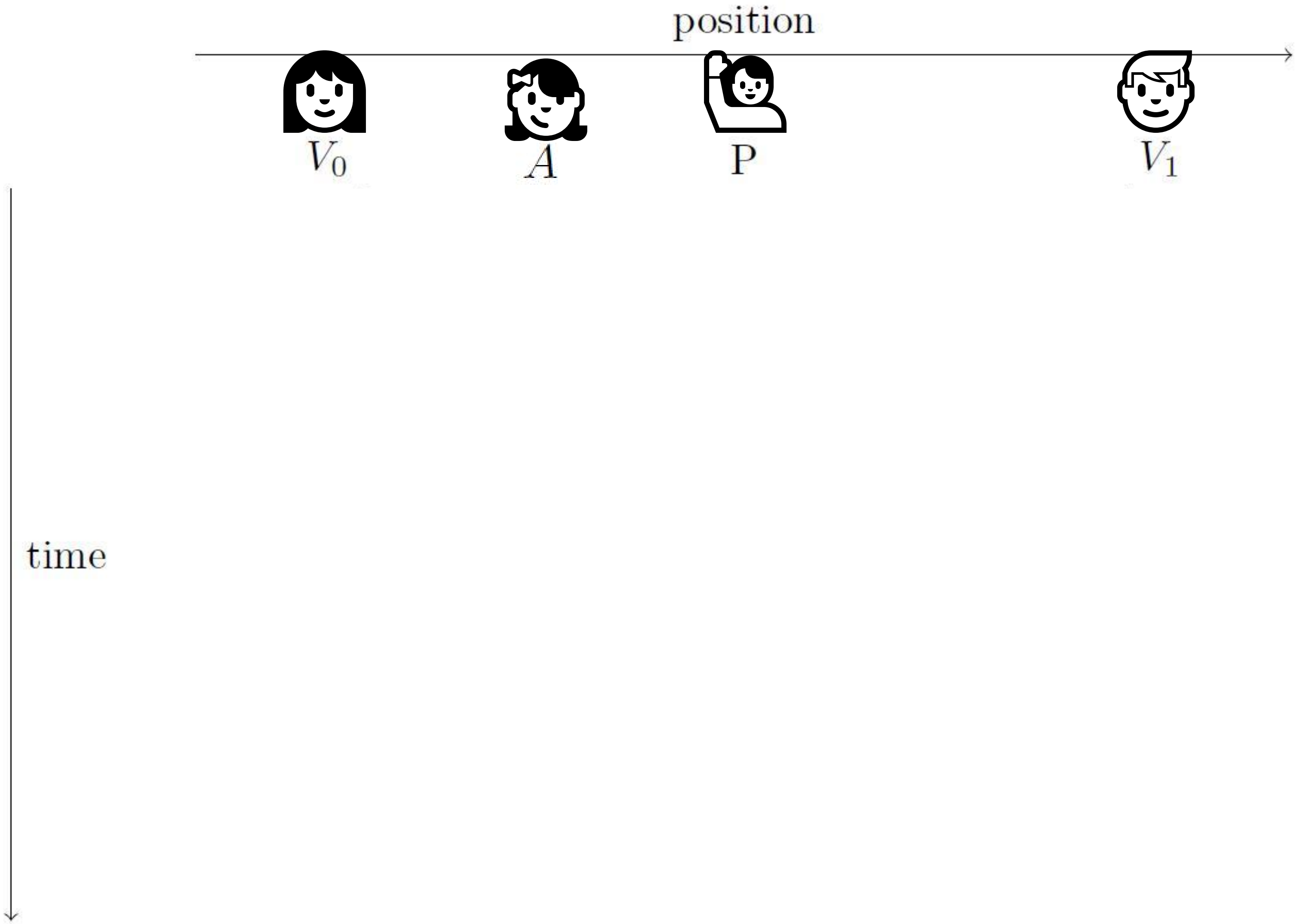

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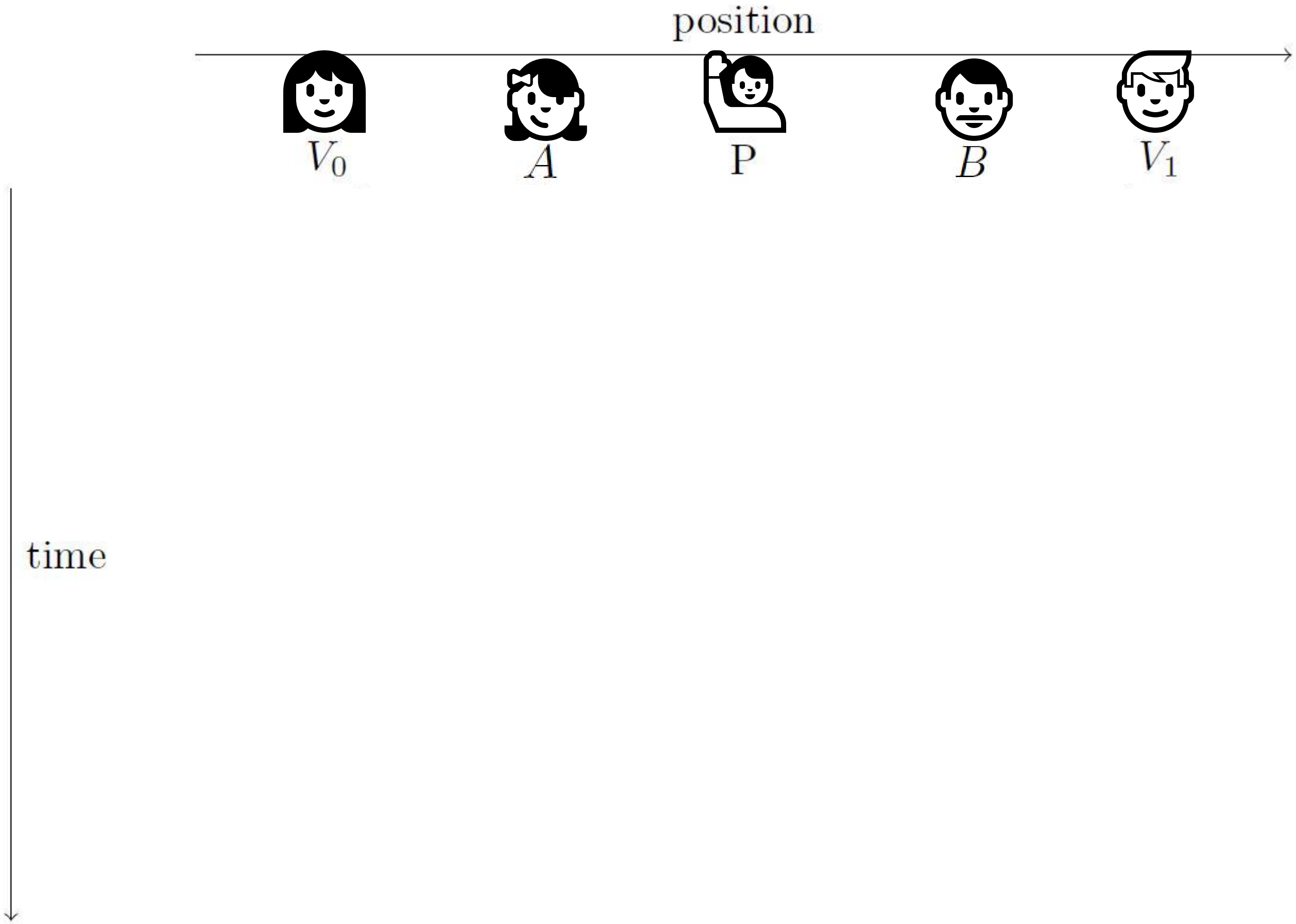

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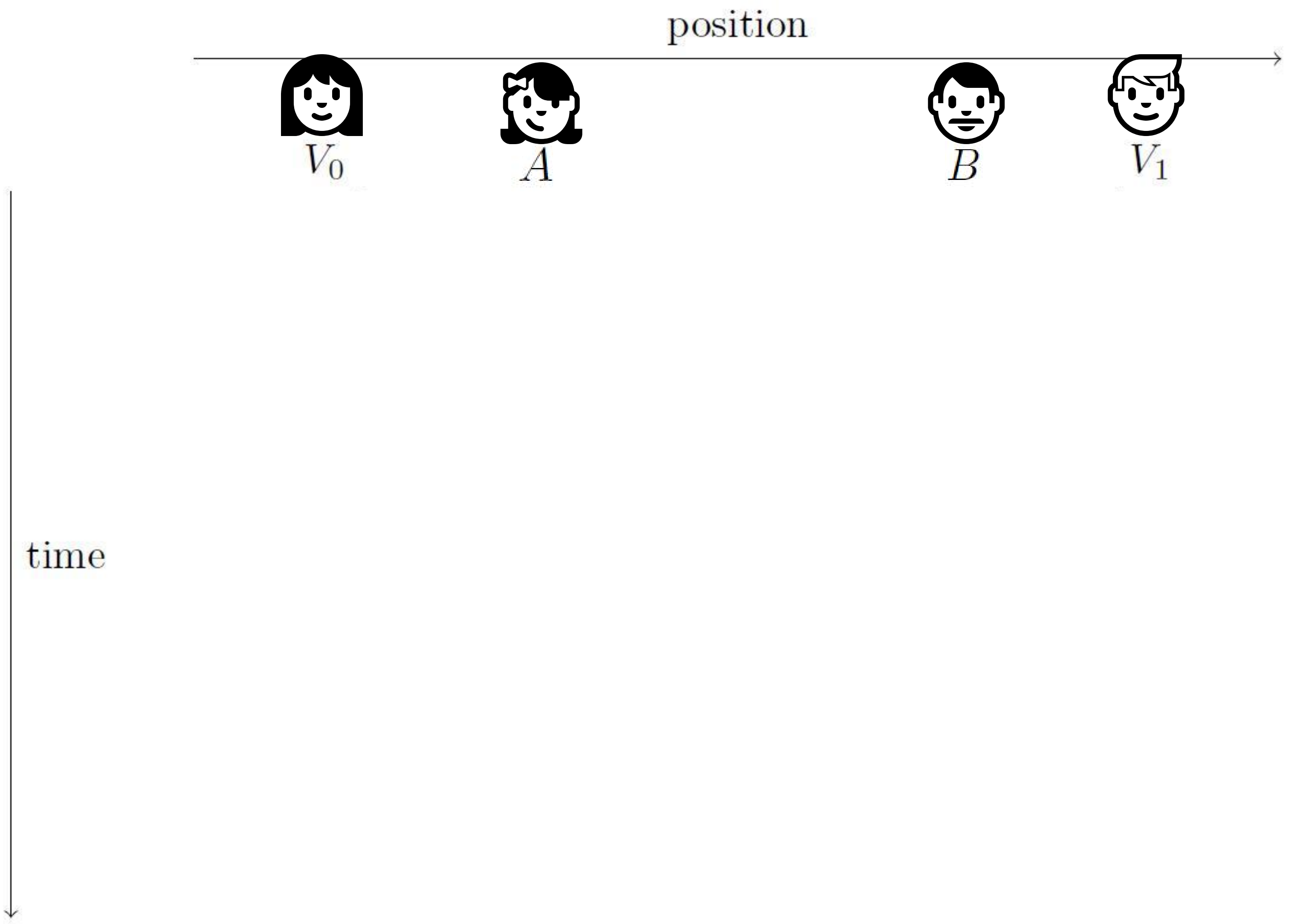

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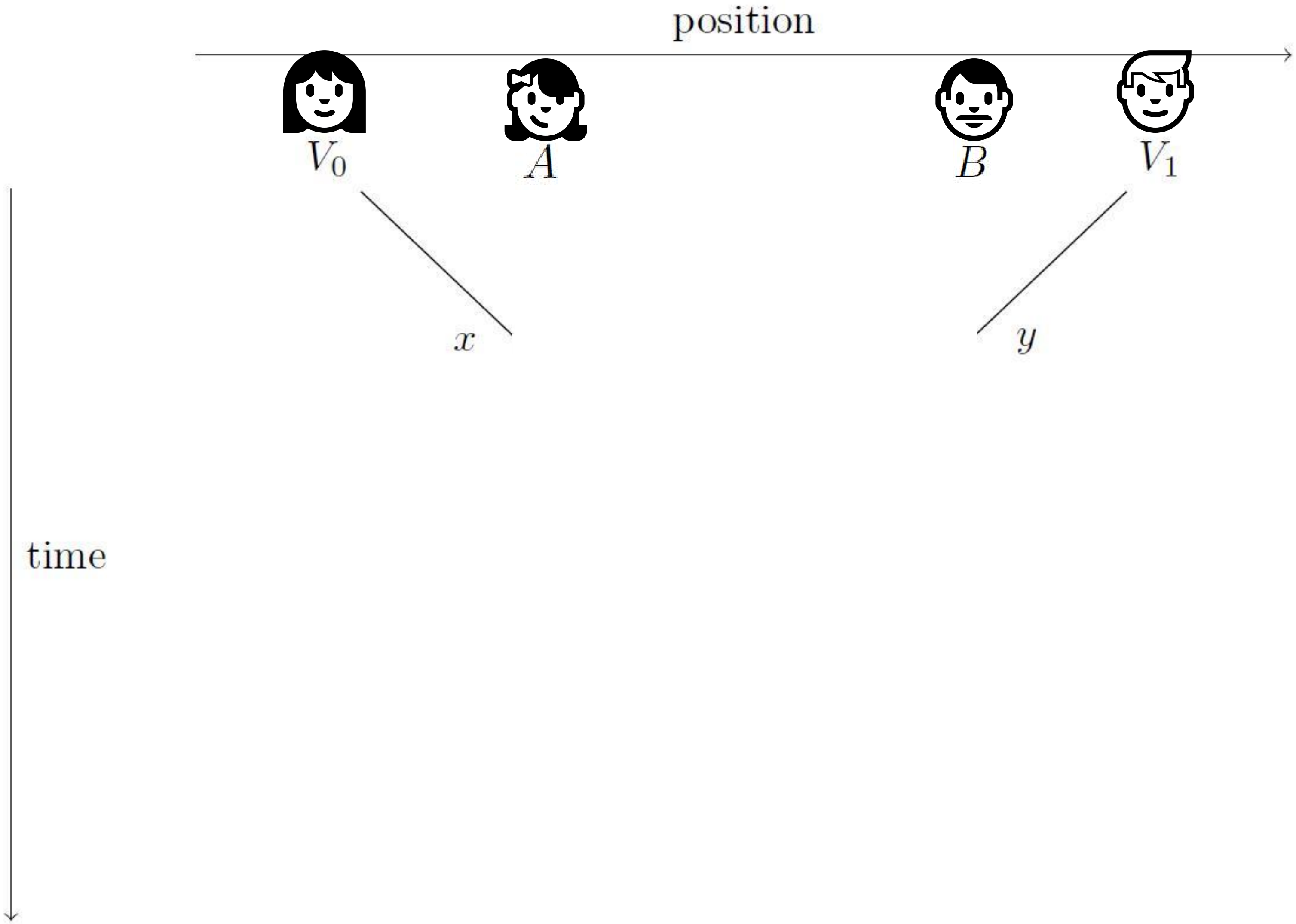

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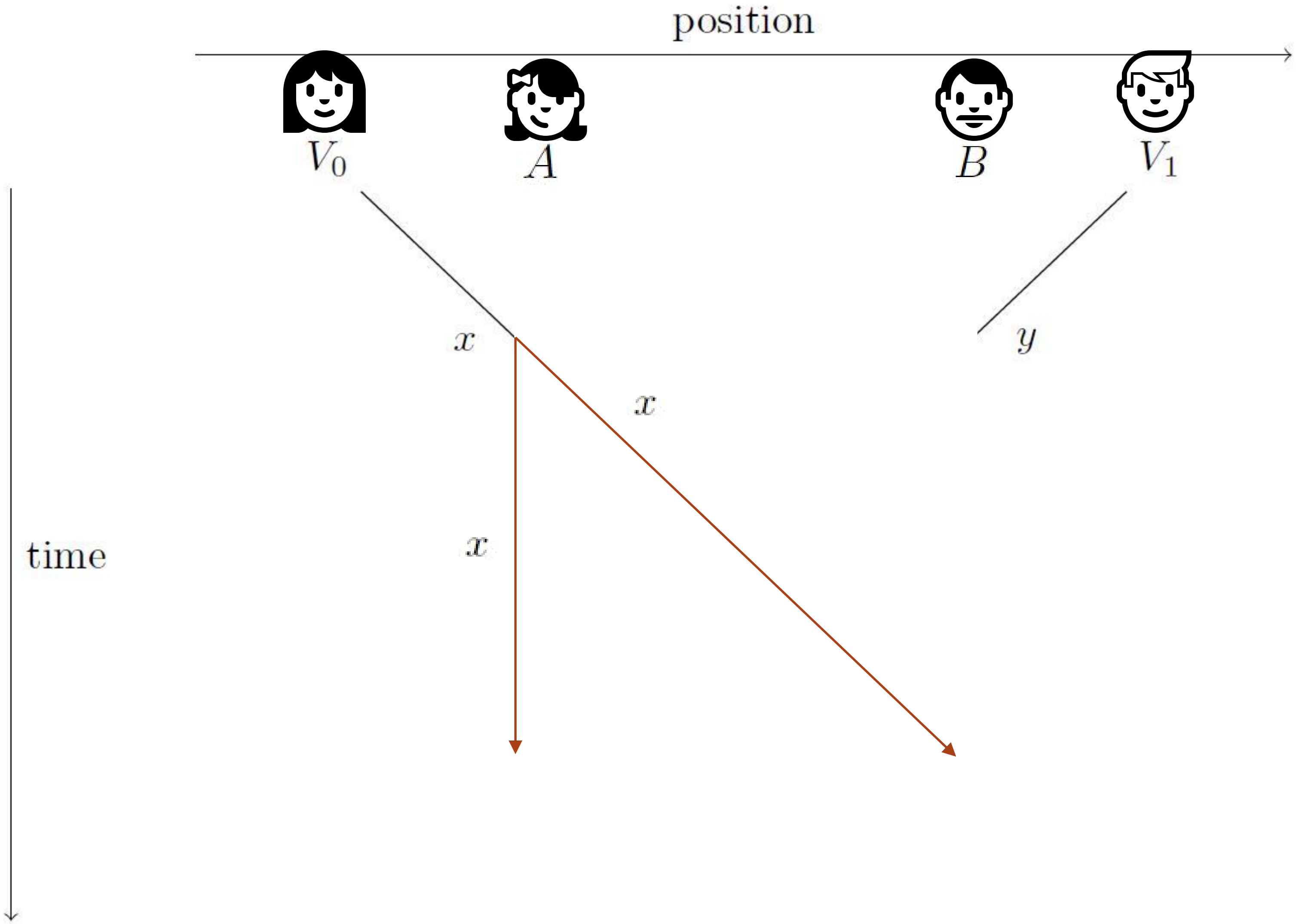

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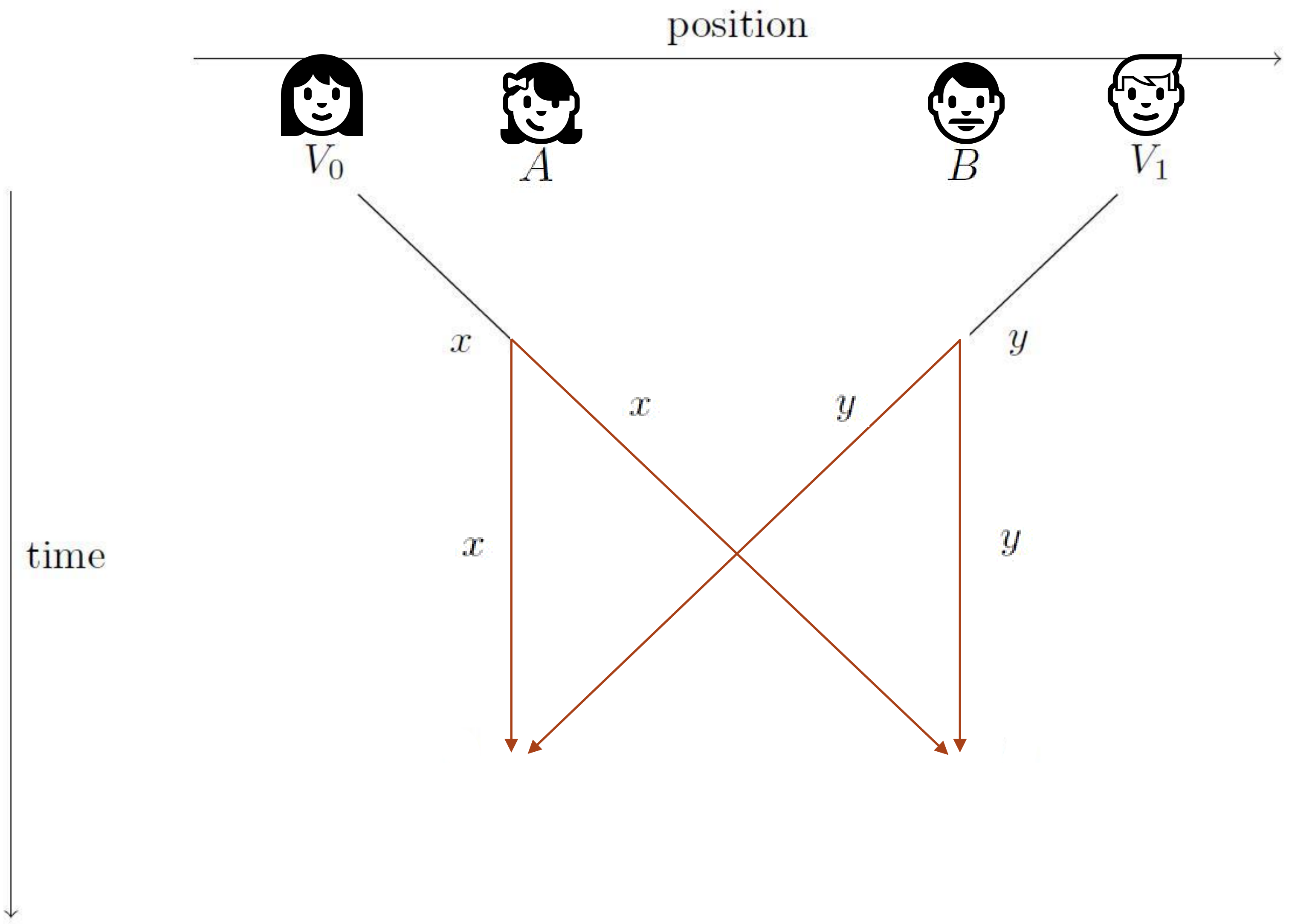

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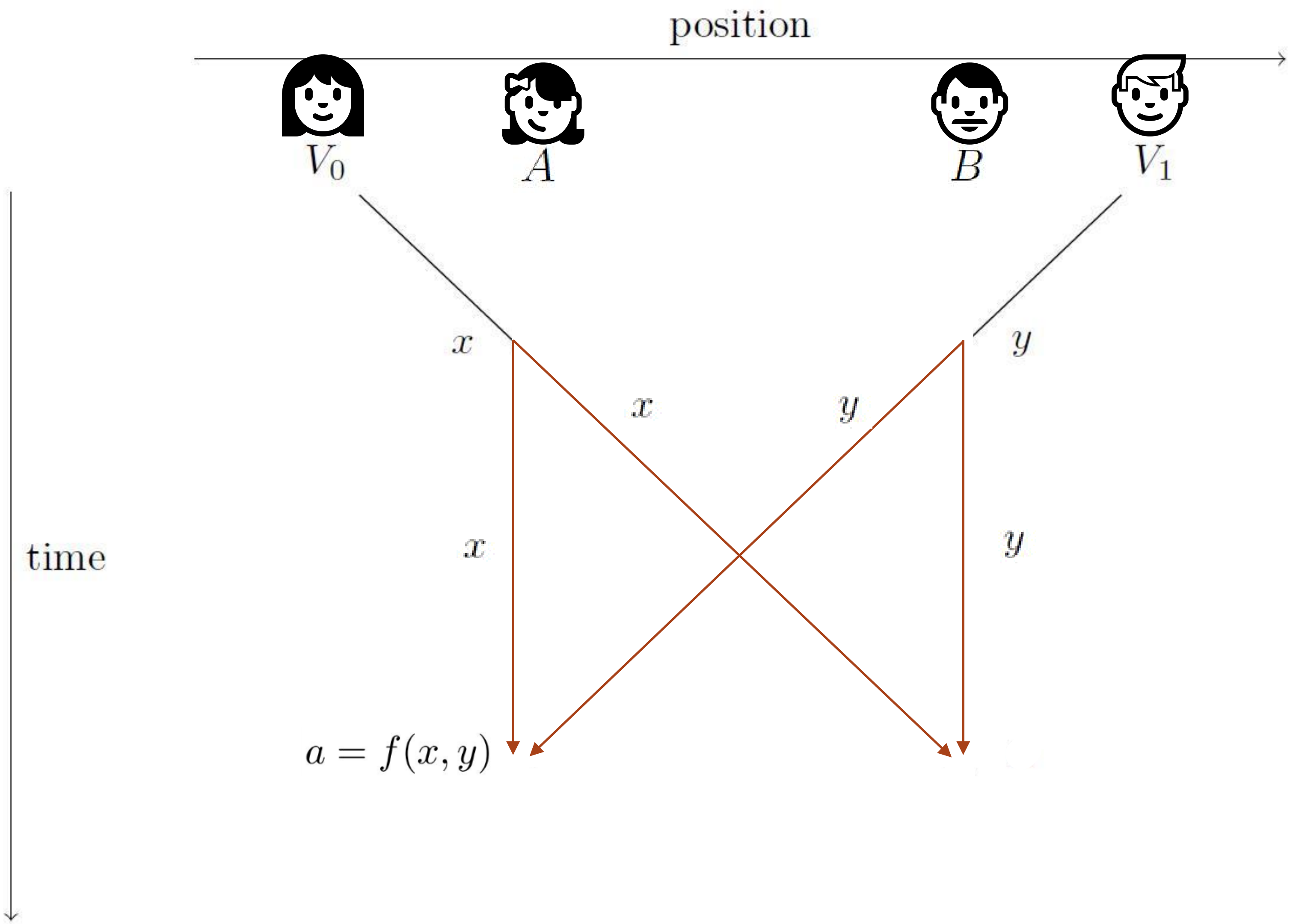

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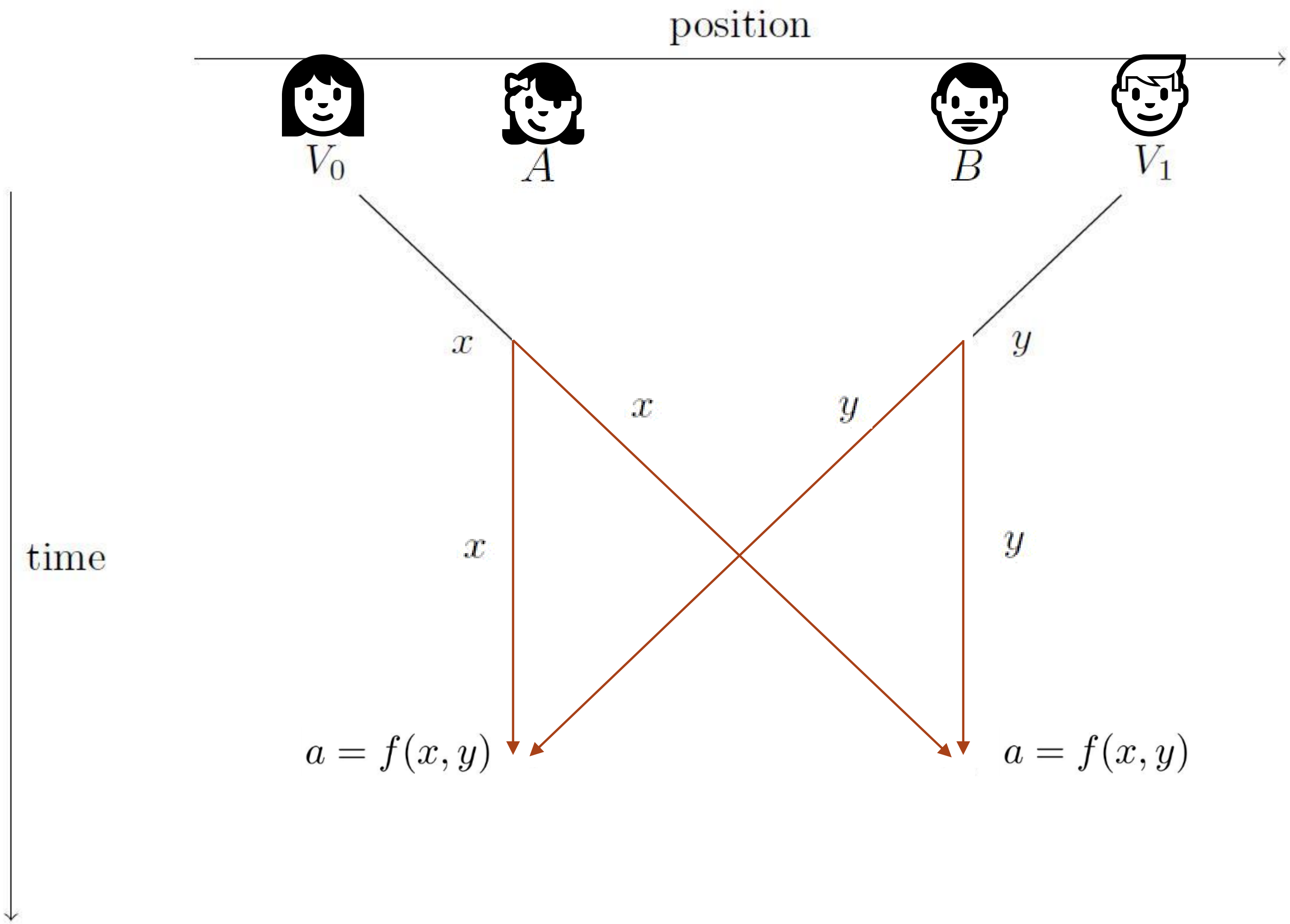

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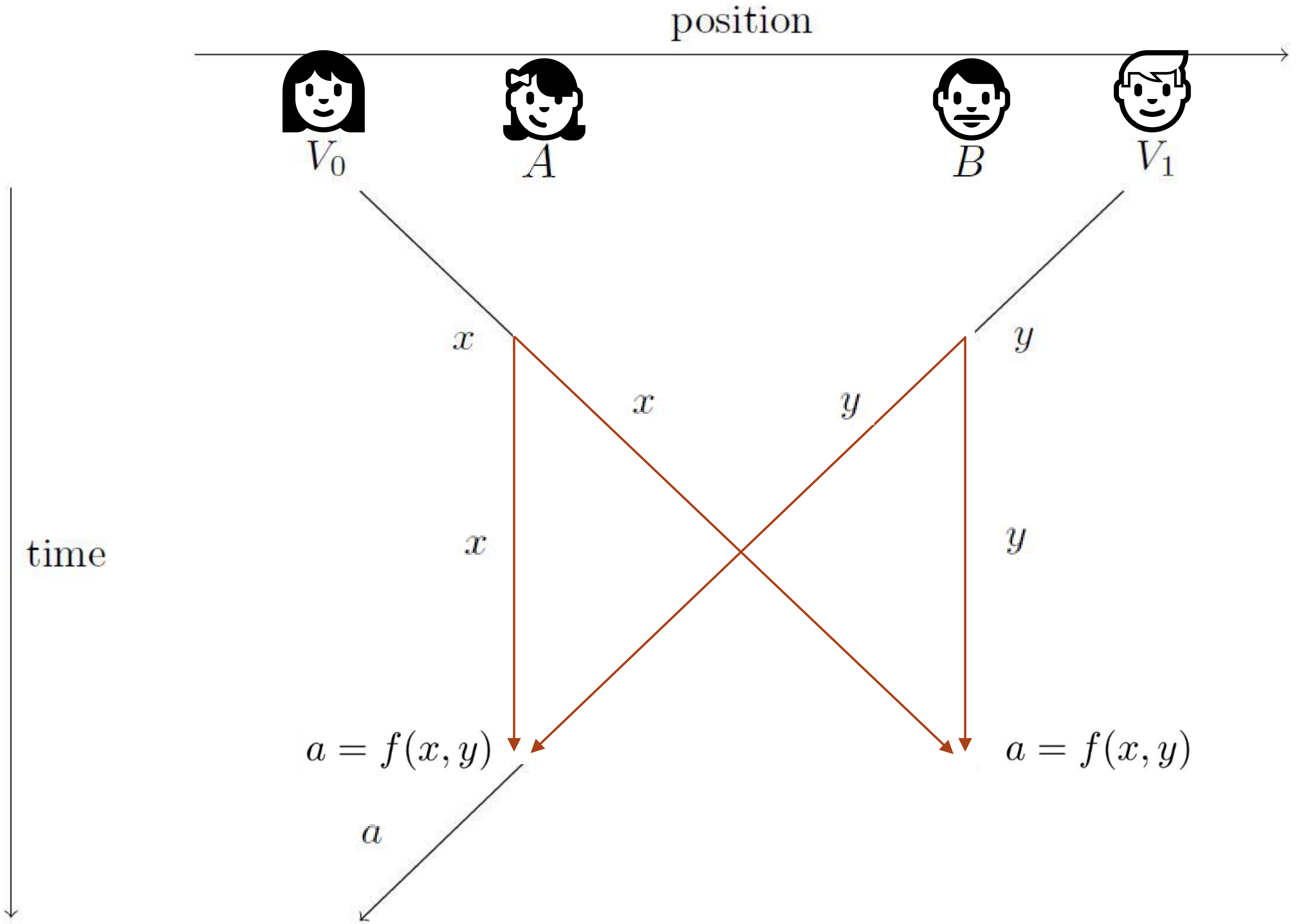

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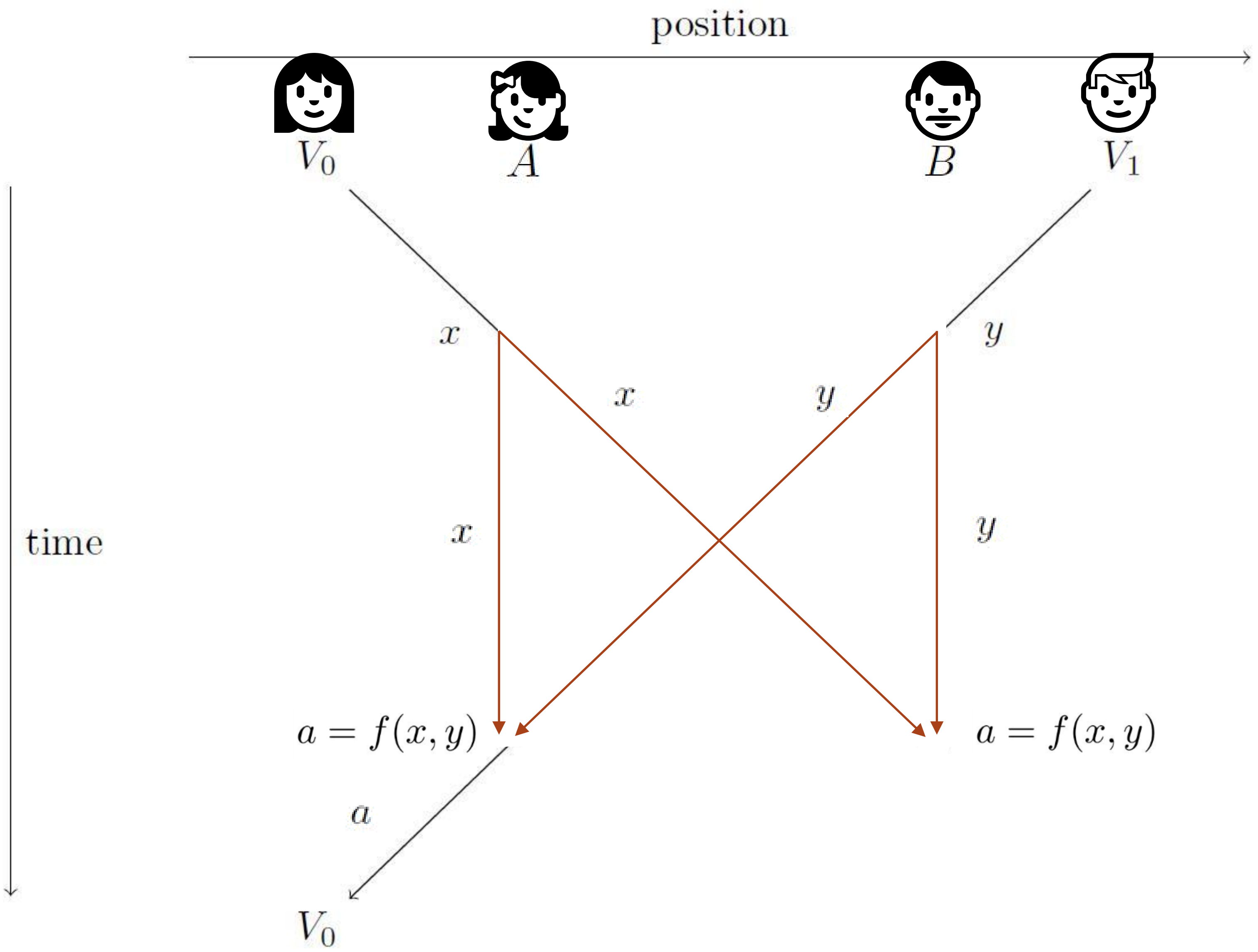

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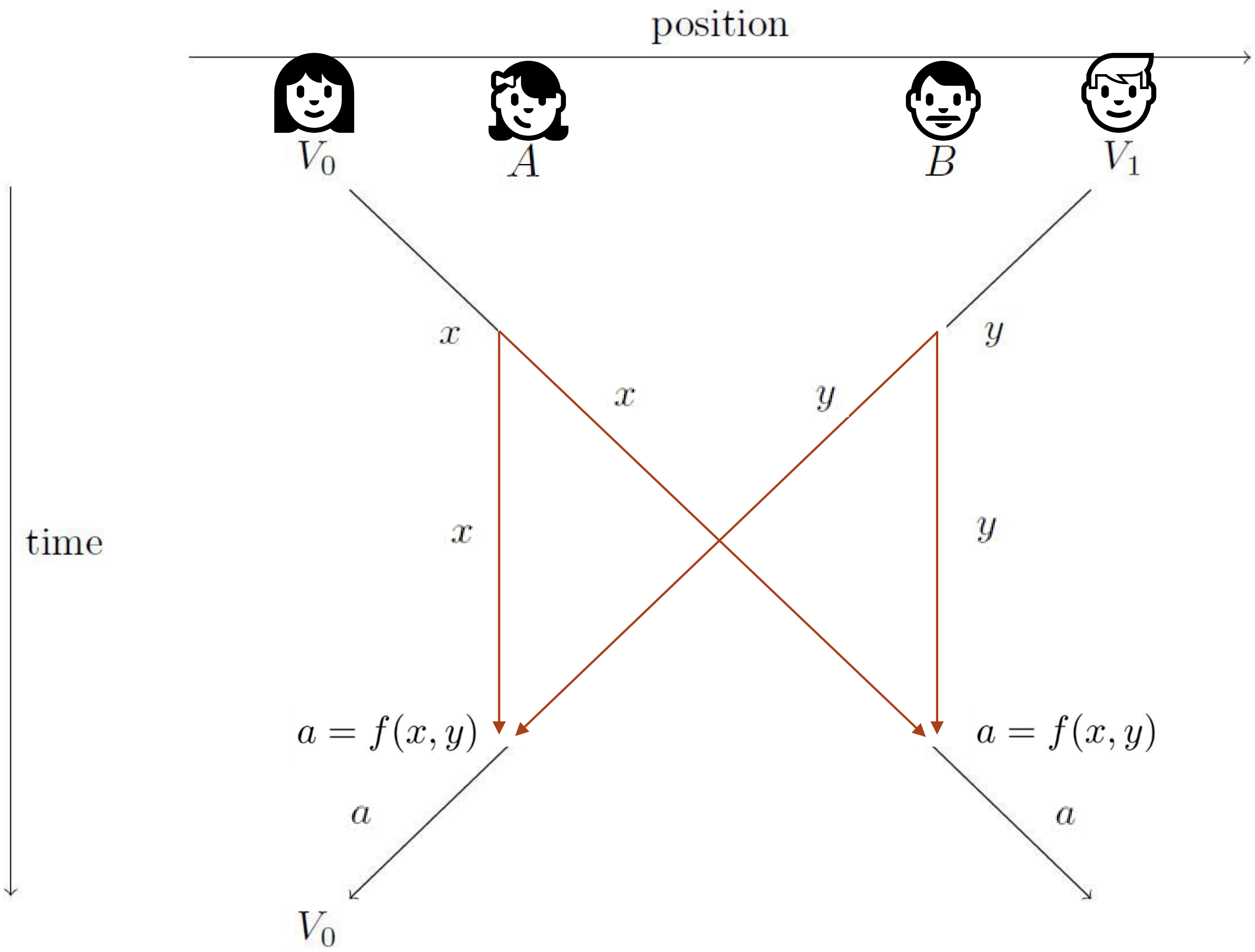

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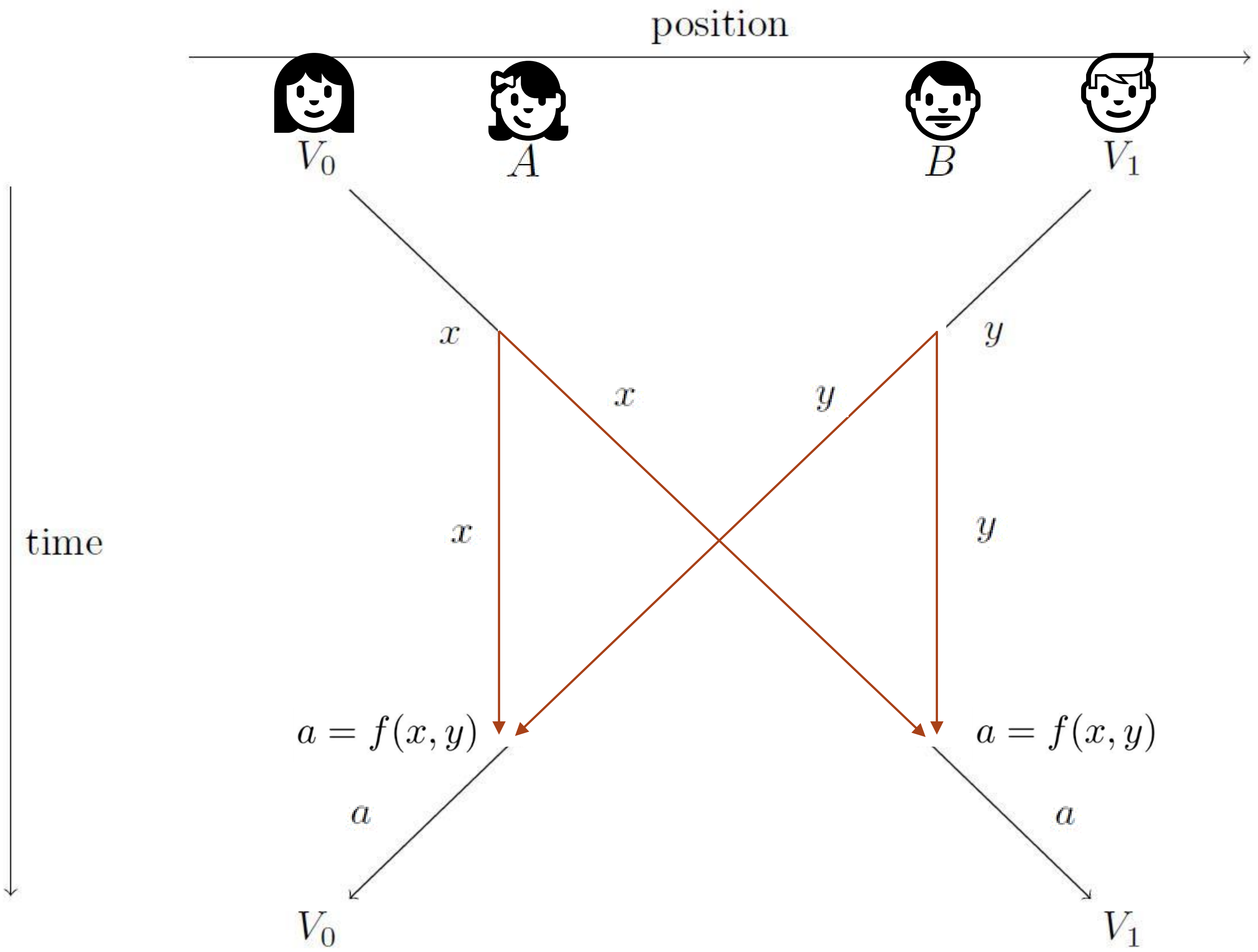

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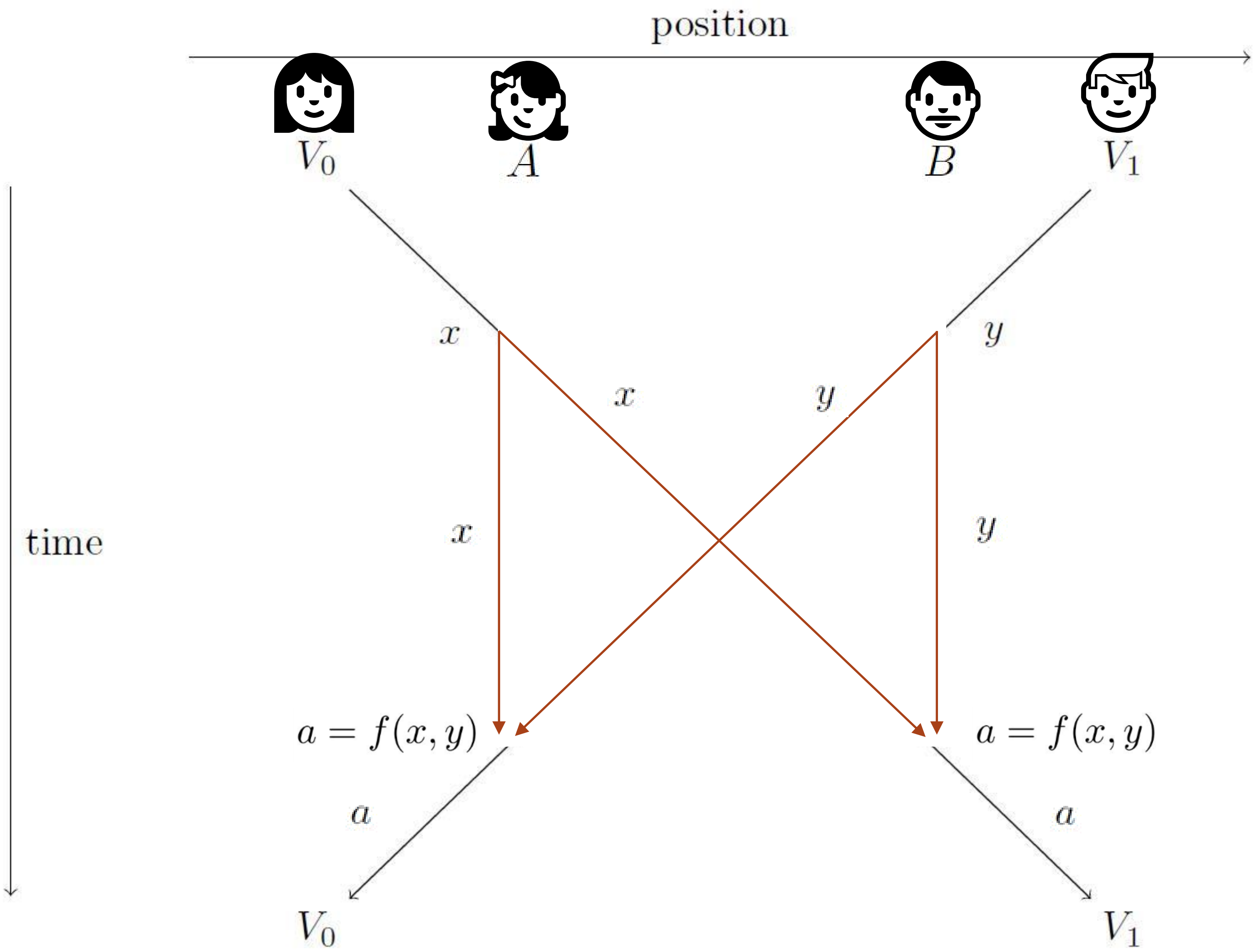

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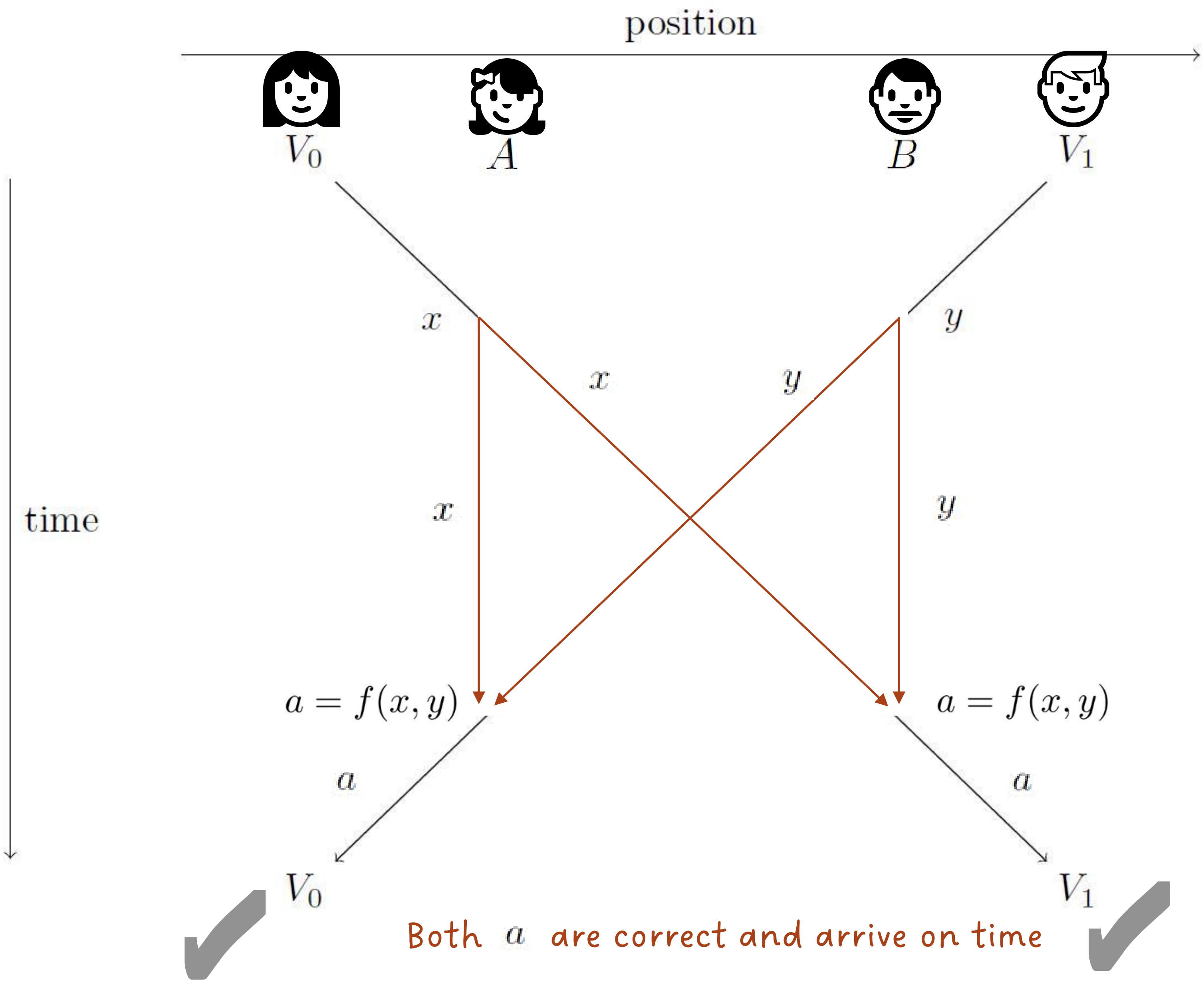

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No-cloning theorem

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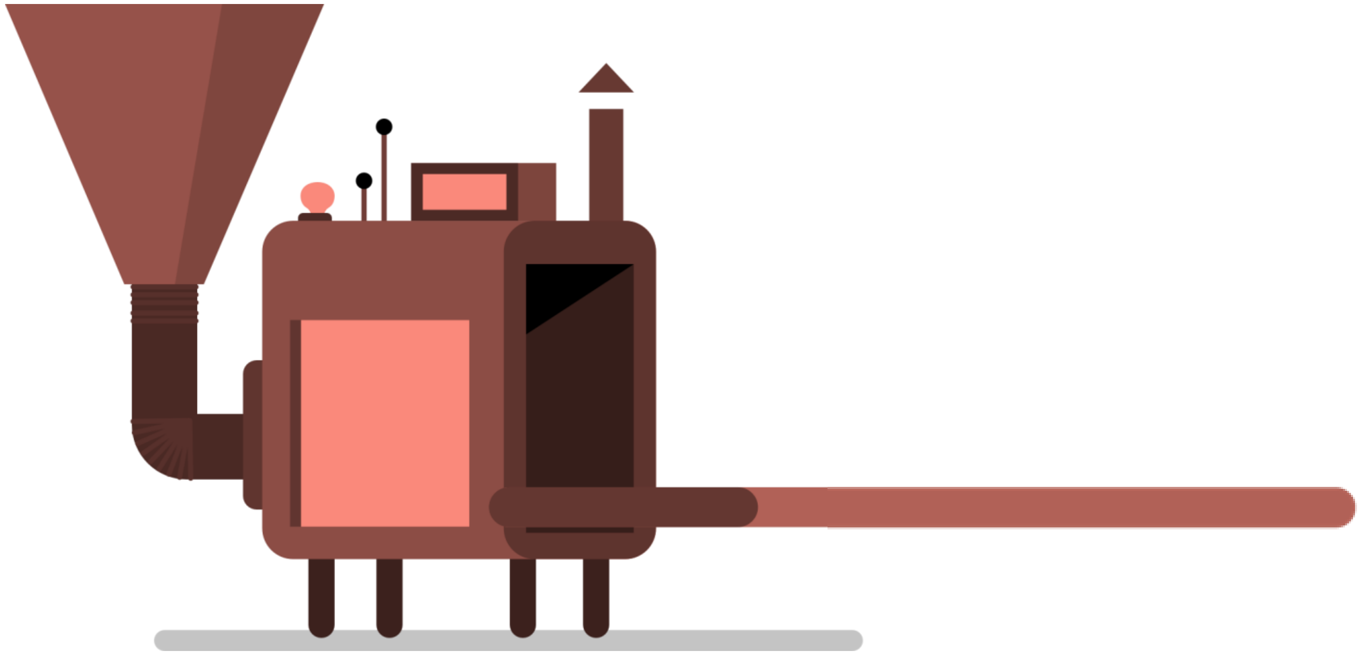
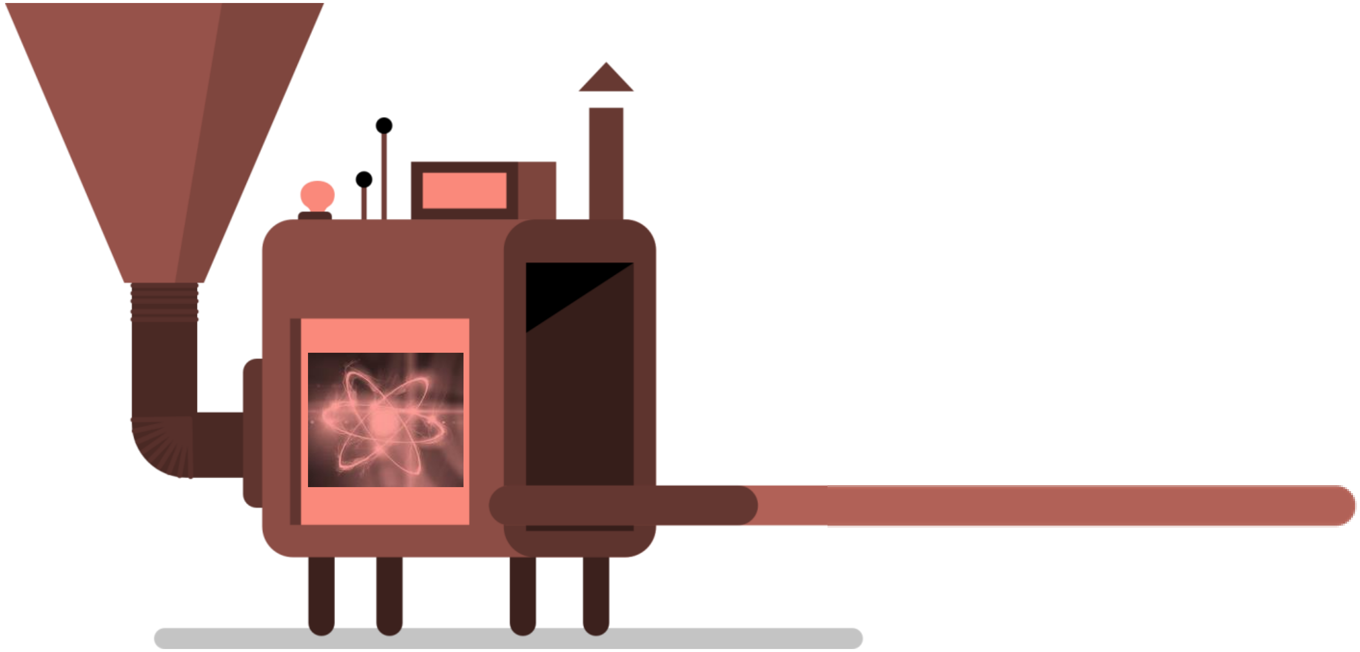


No-cloning theorem



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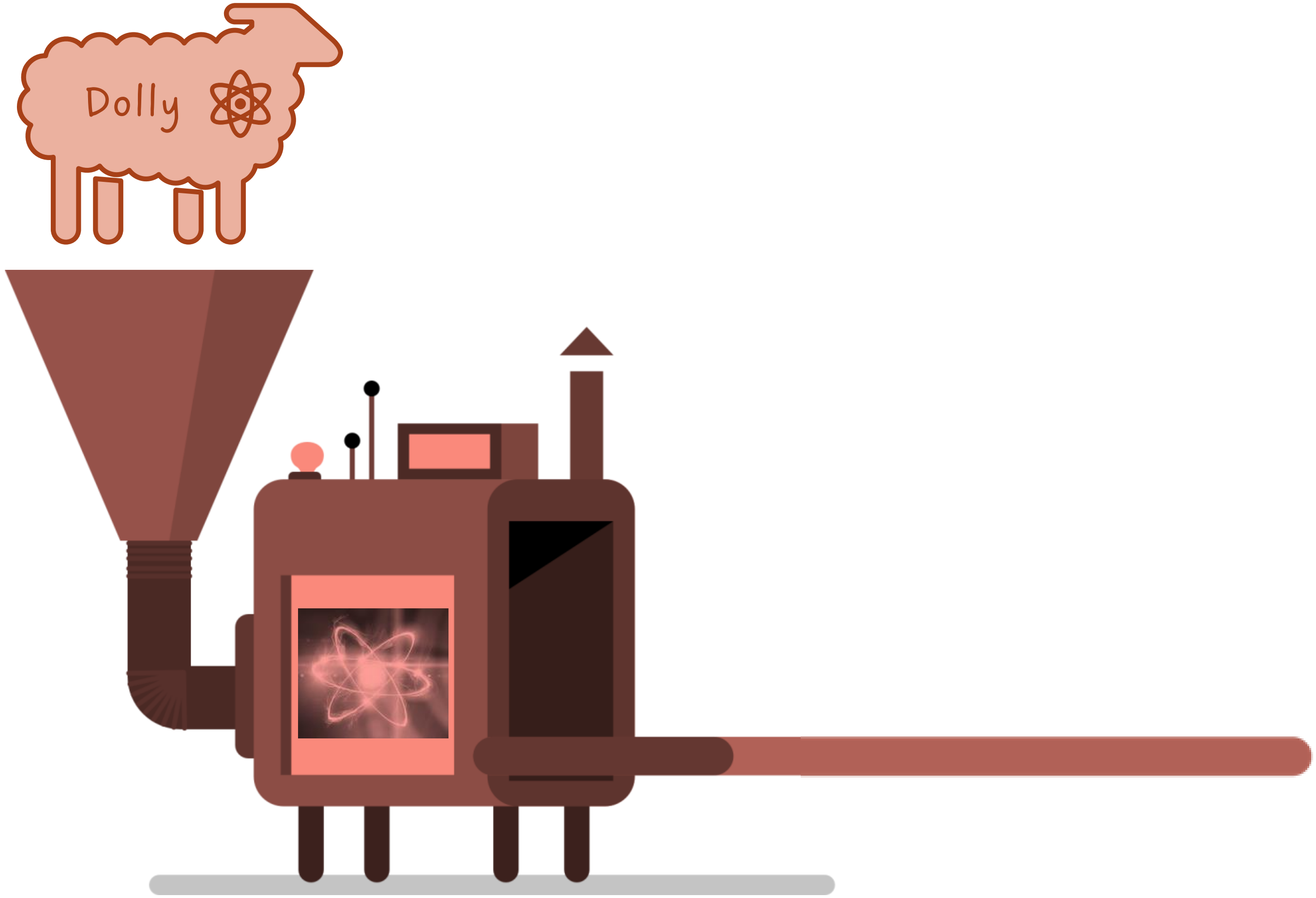

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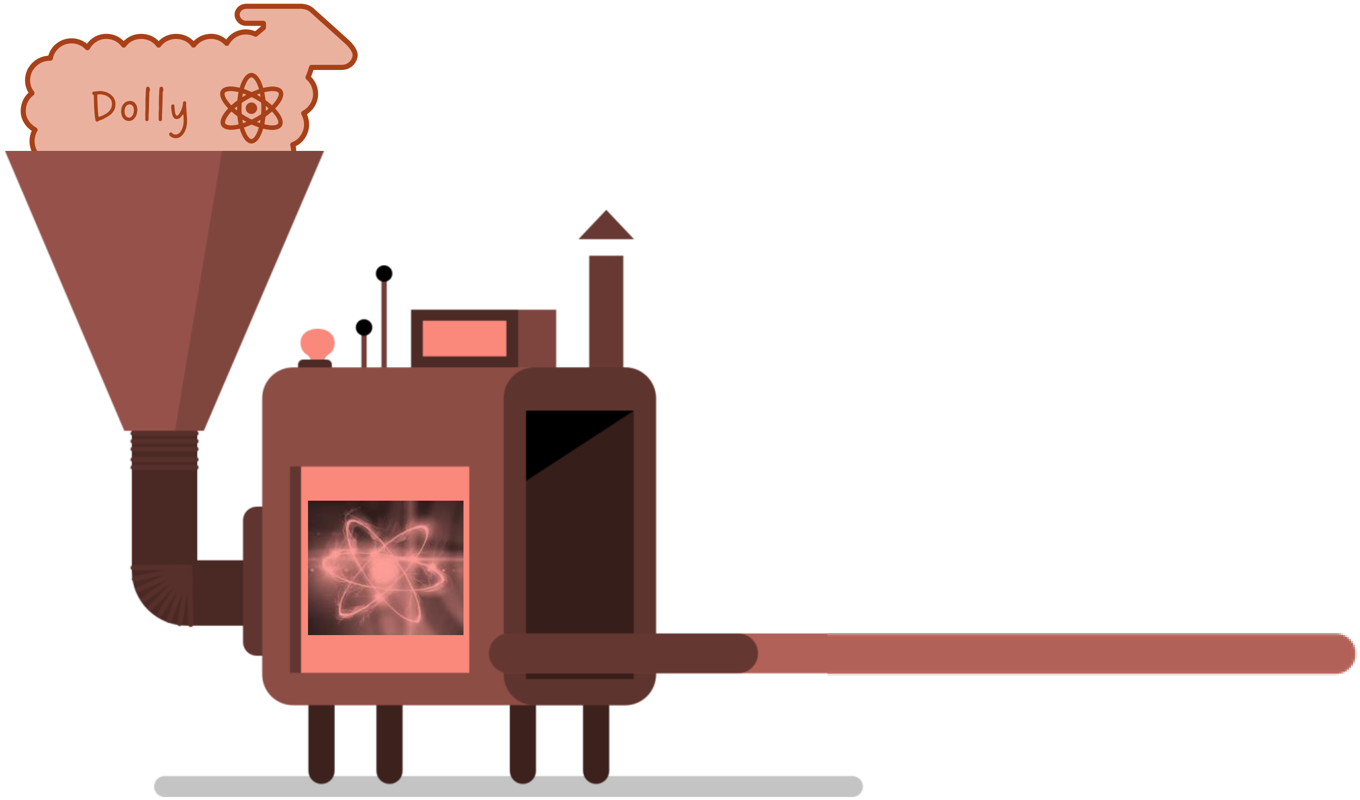

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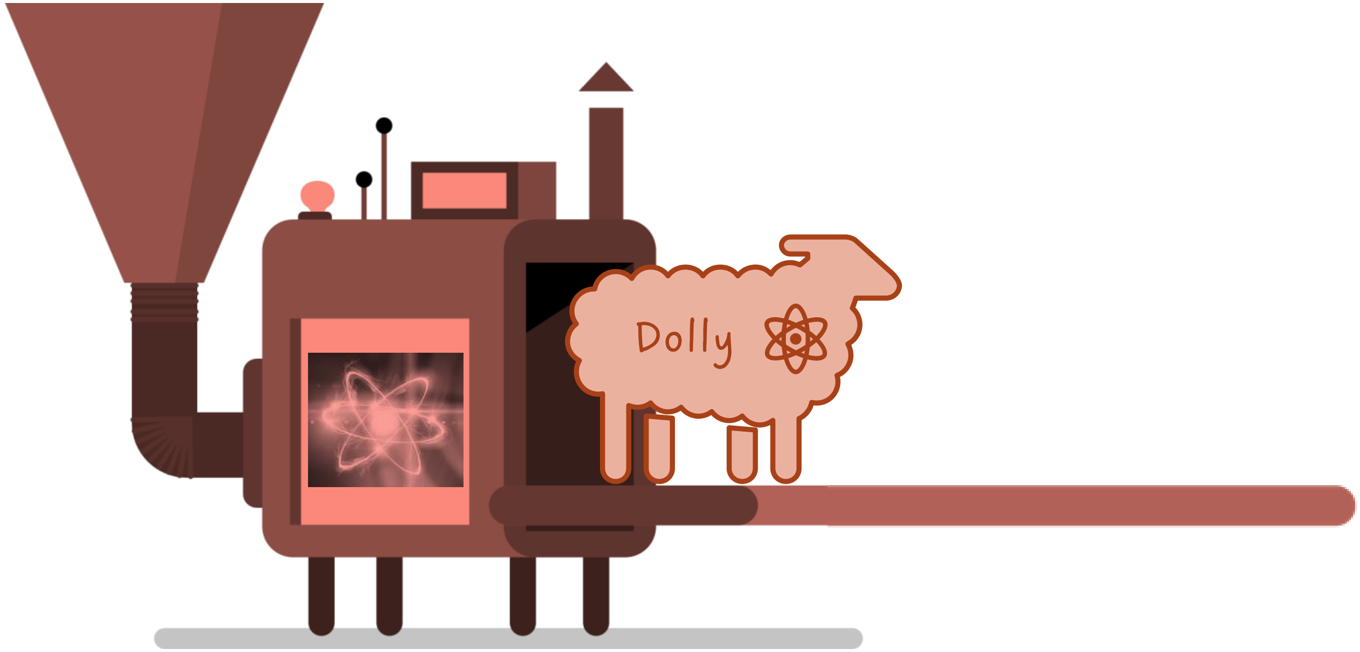

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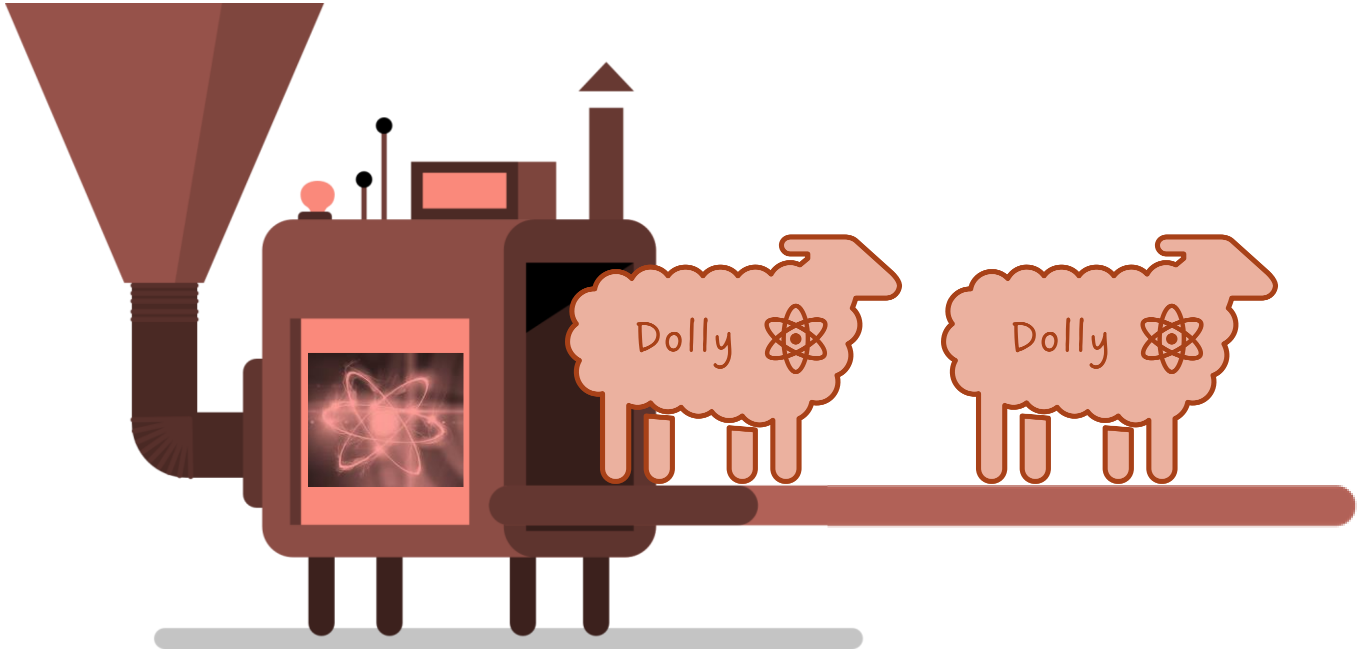

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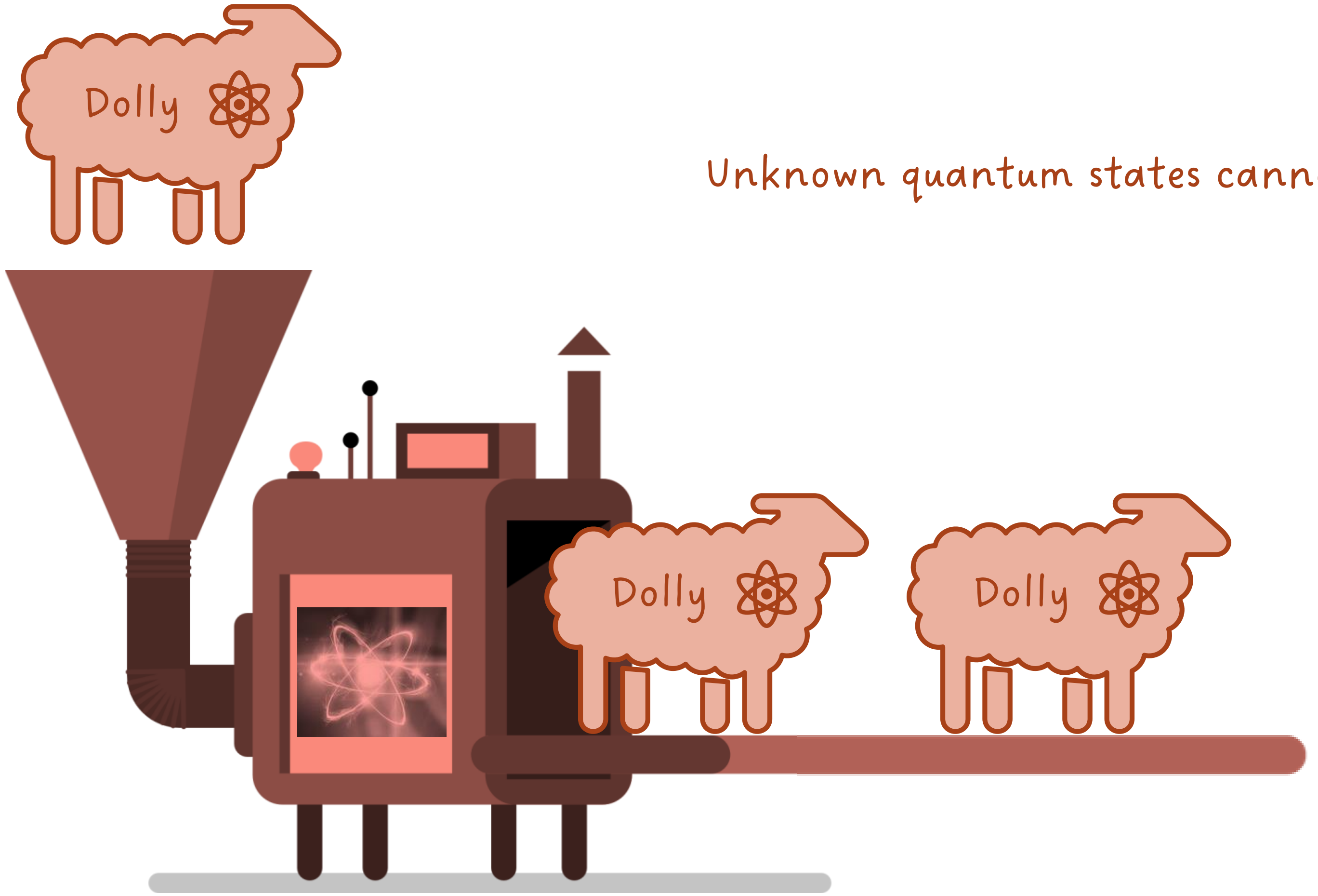

Unknown quantum states cannot be copied

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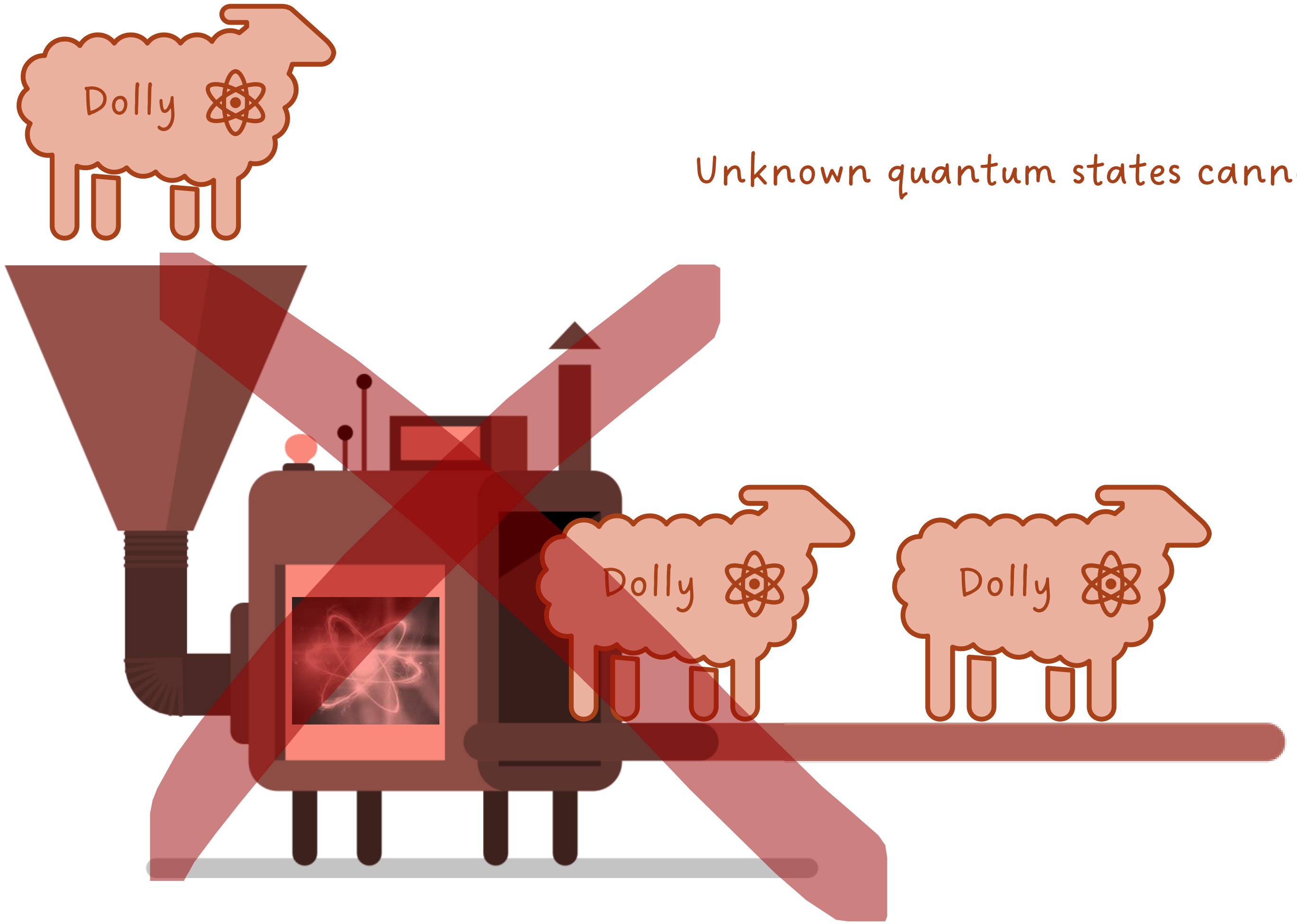
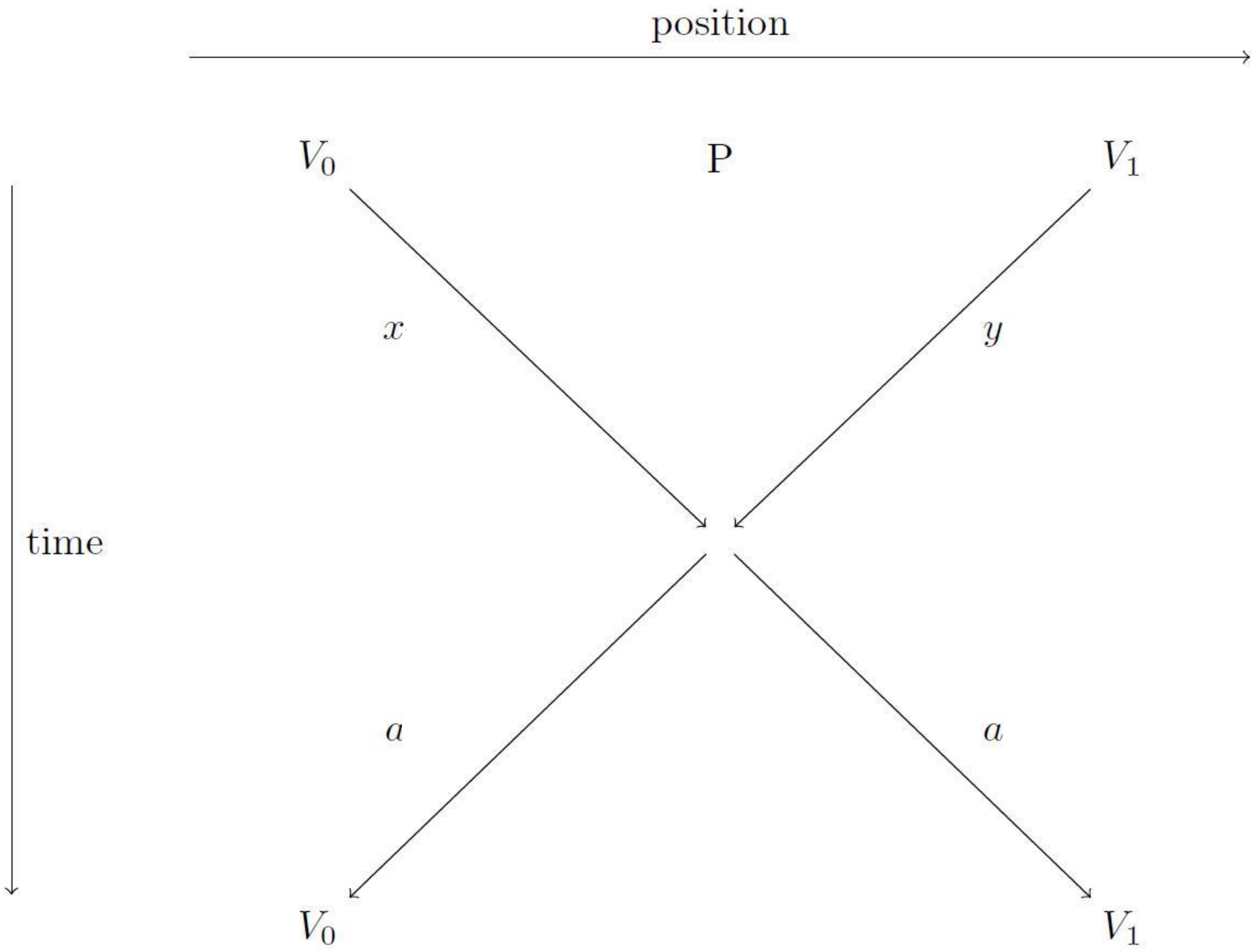
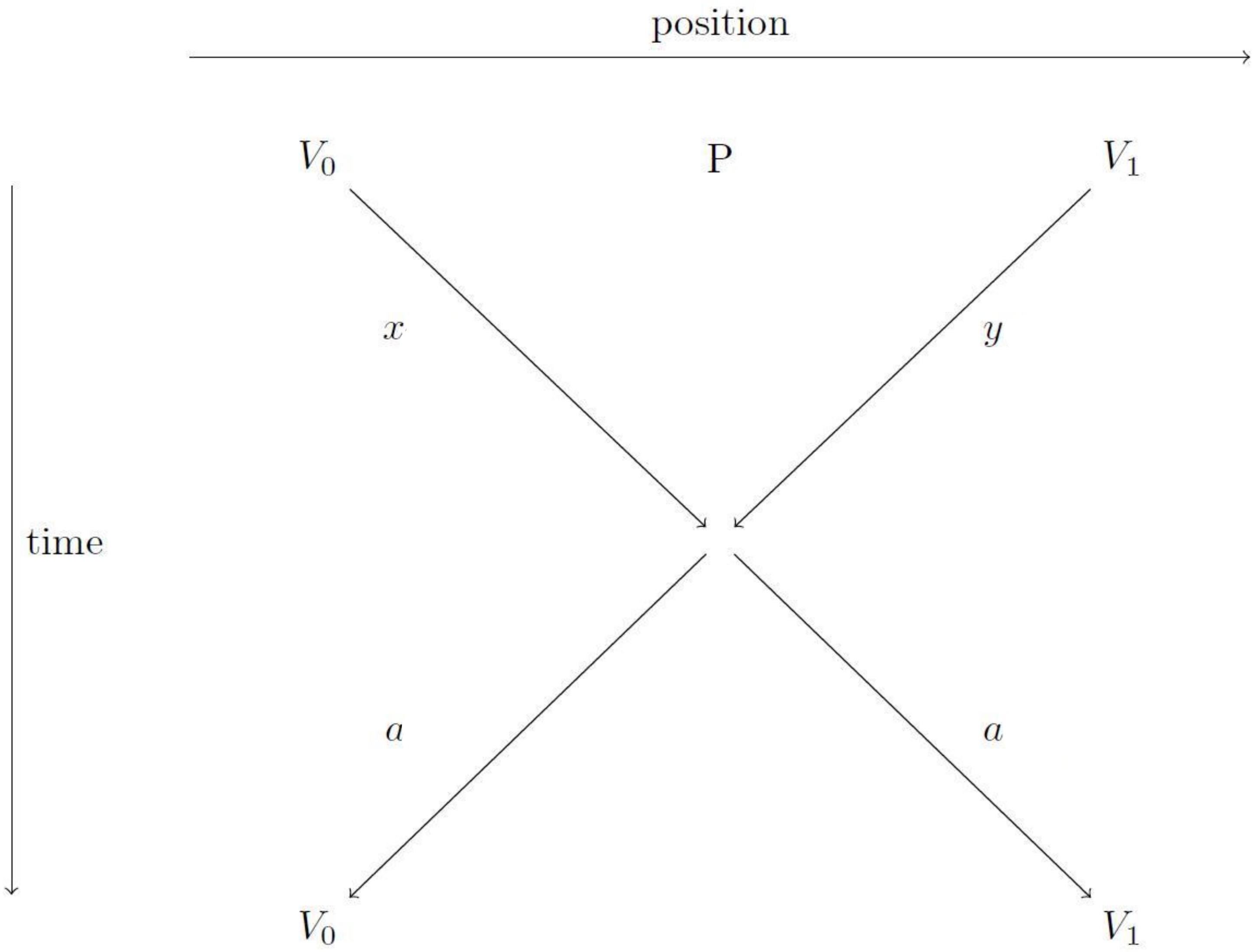


Position Verification (PV)



Quantum Position Verification (QPV)



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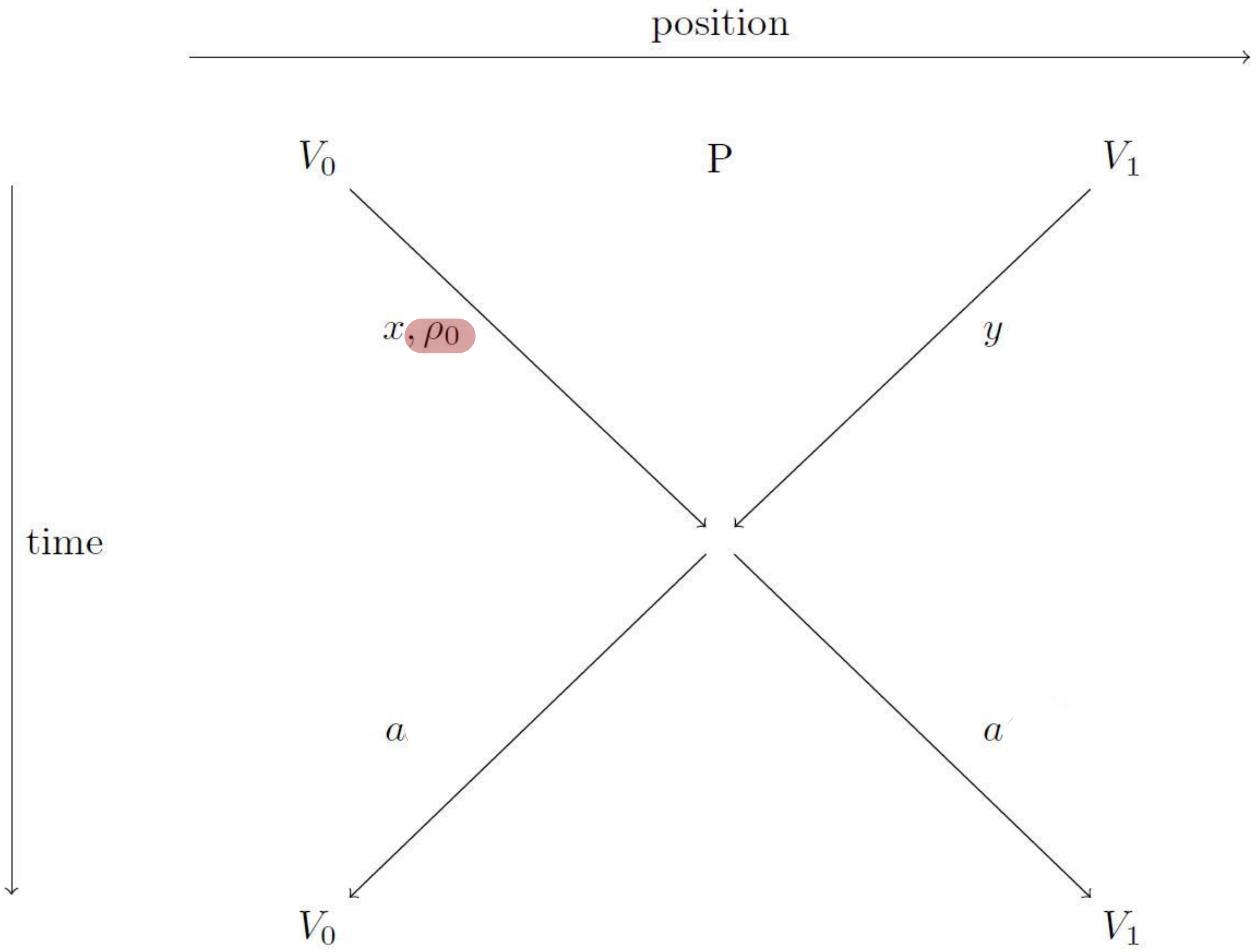

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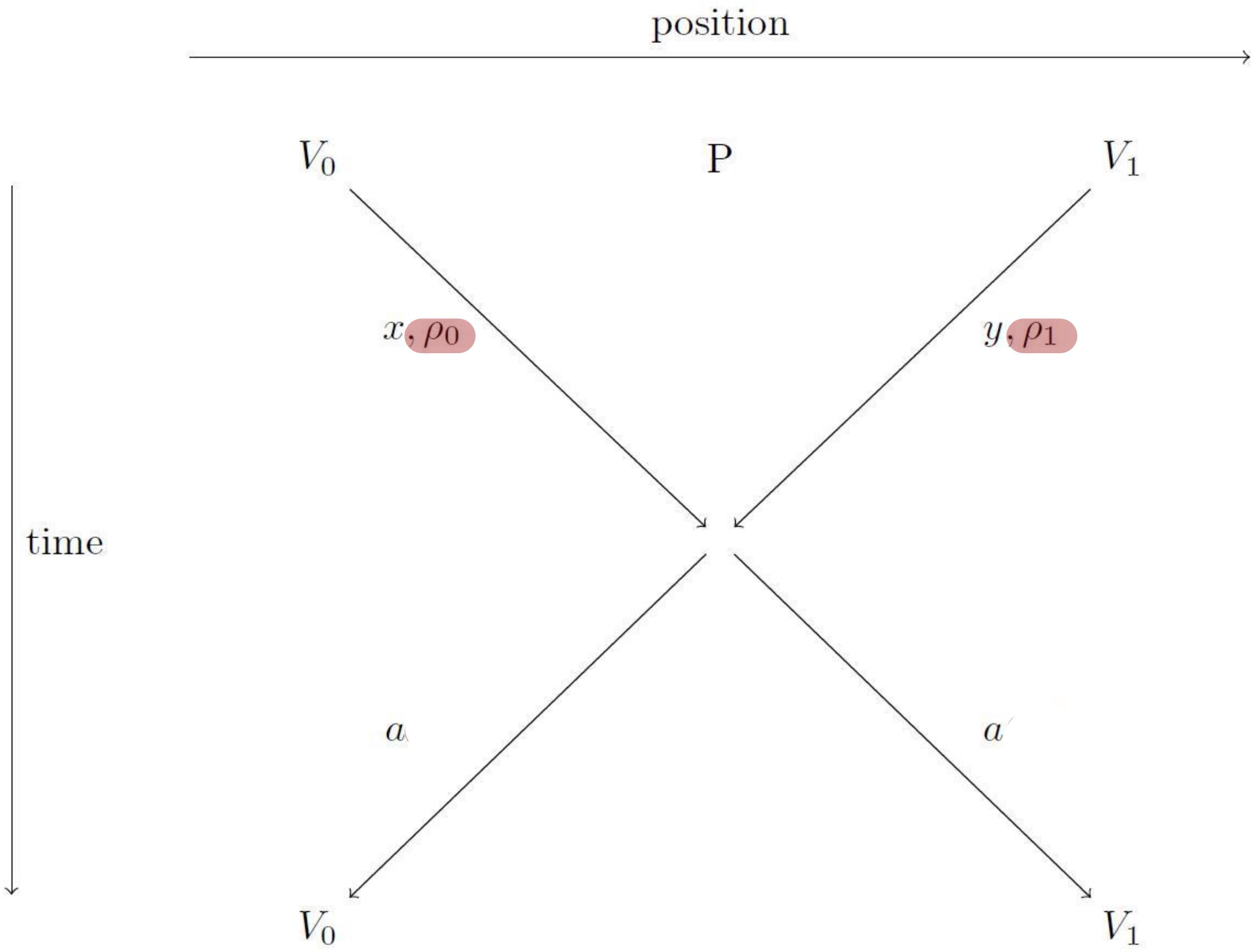

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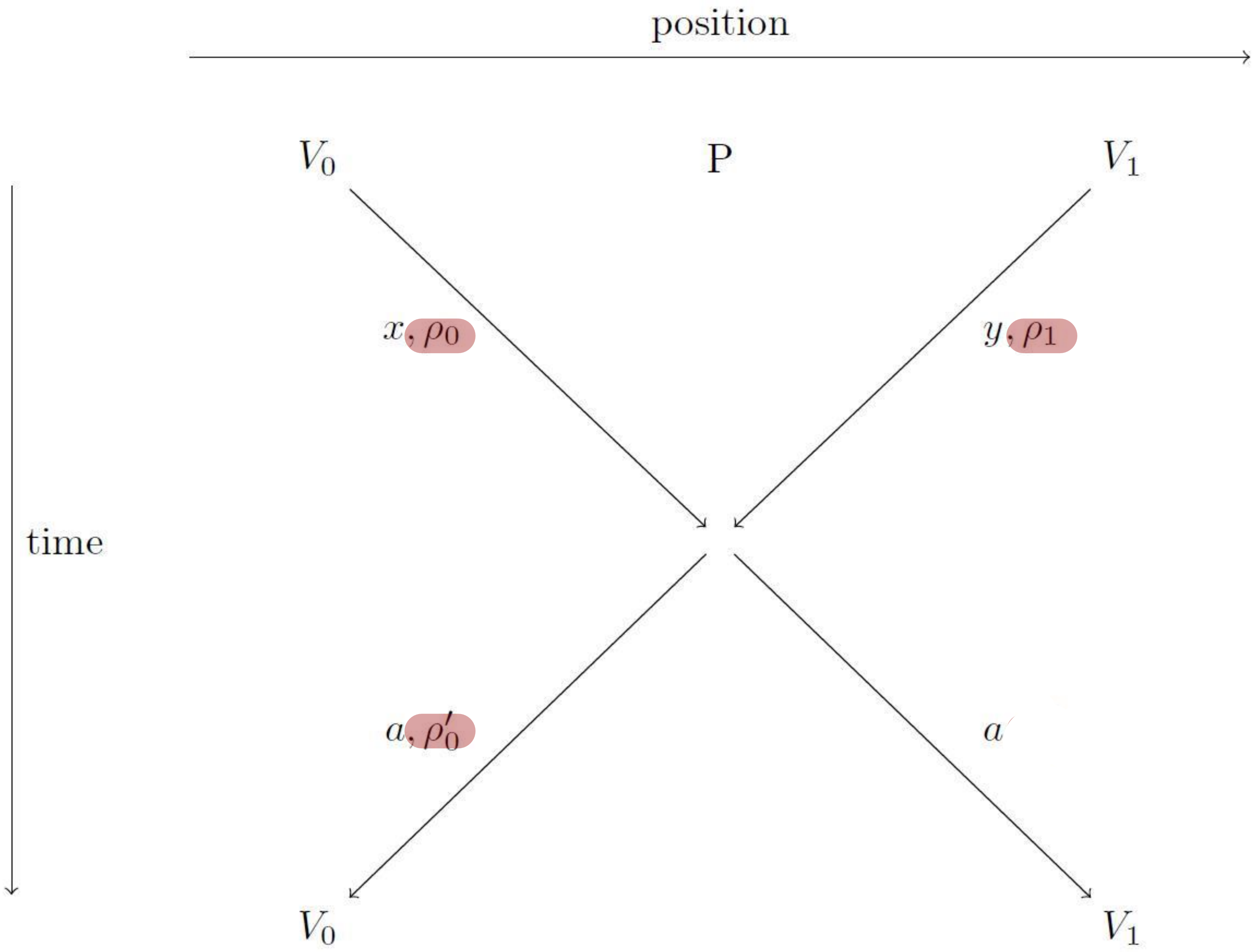

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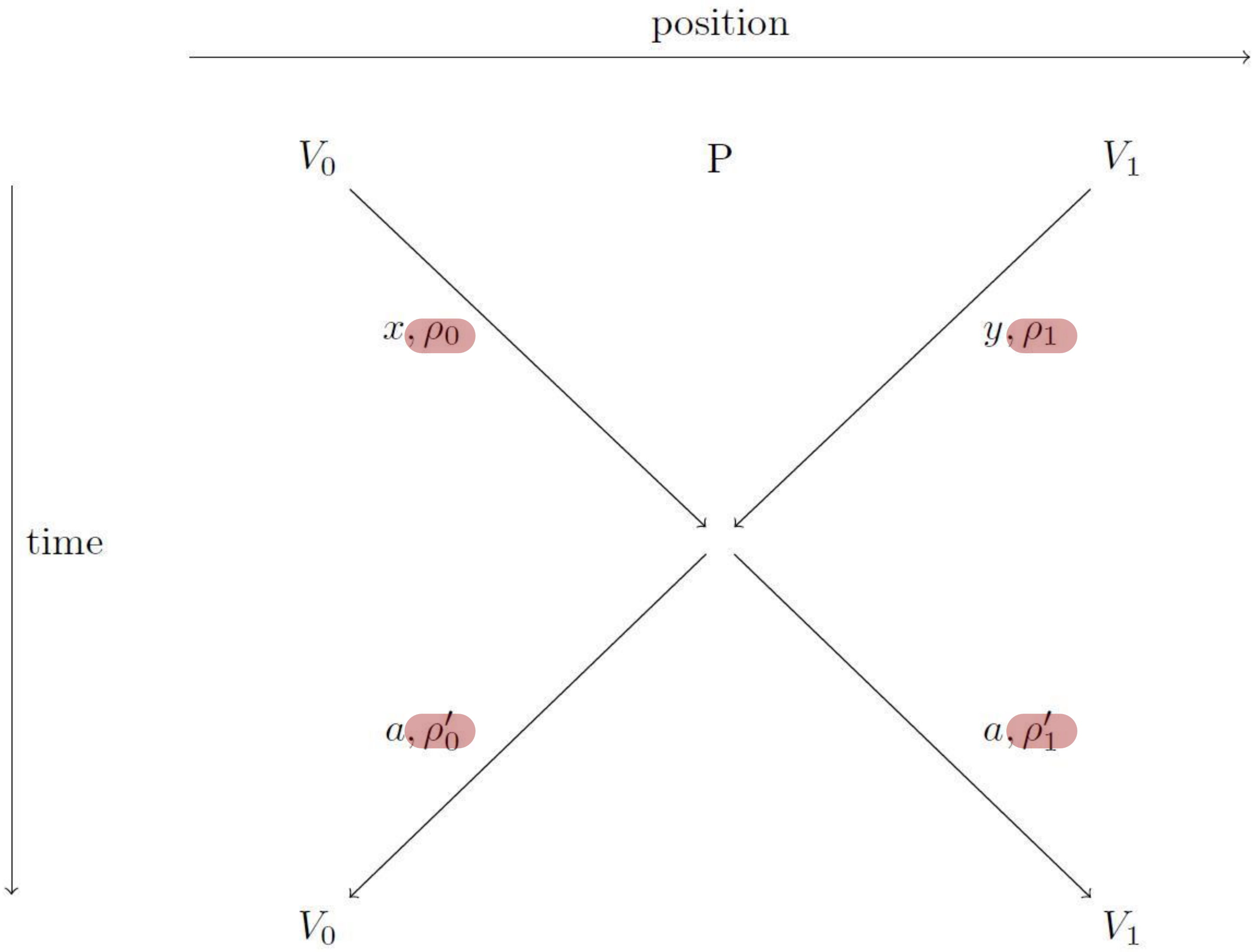

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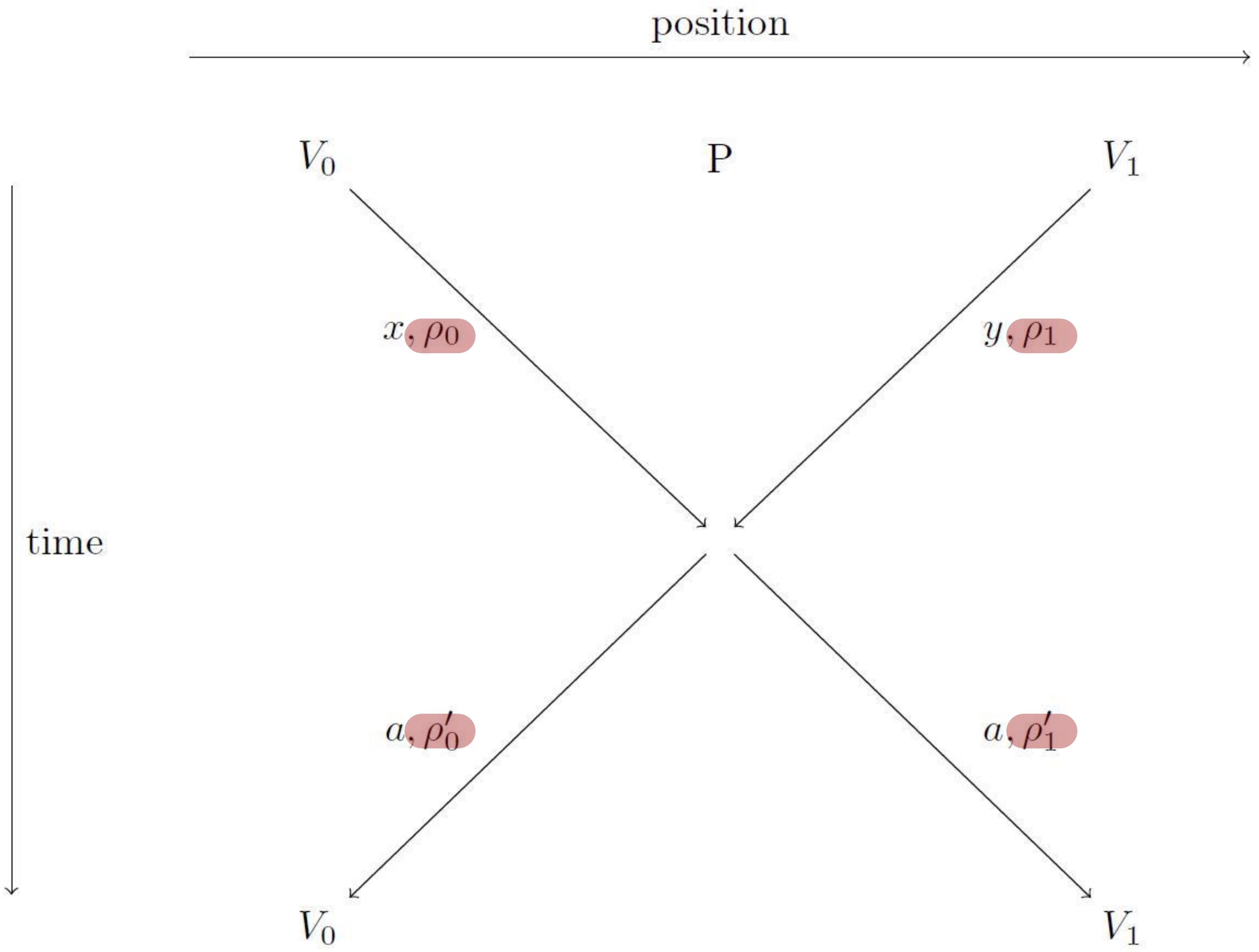

This prevents copying attacks

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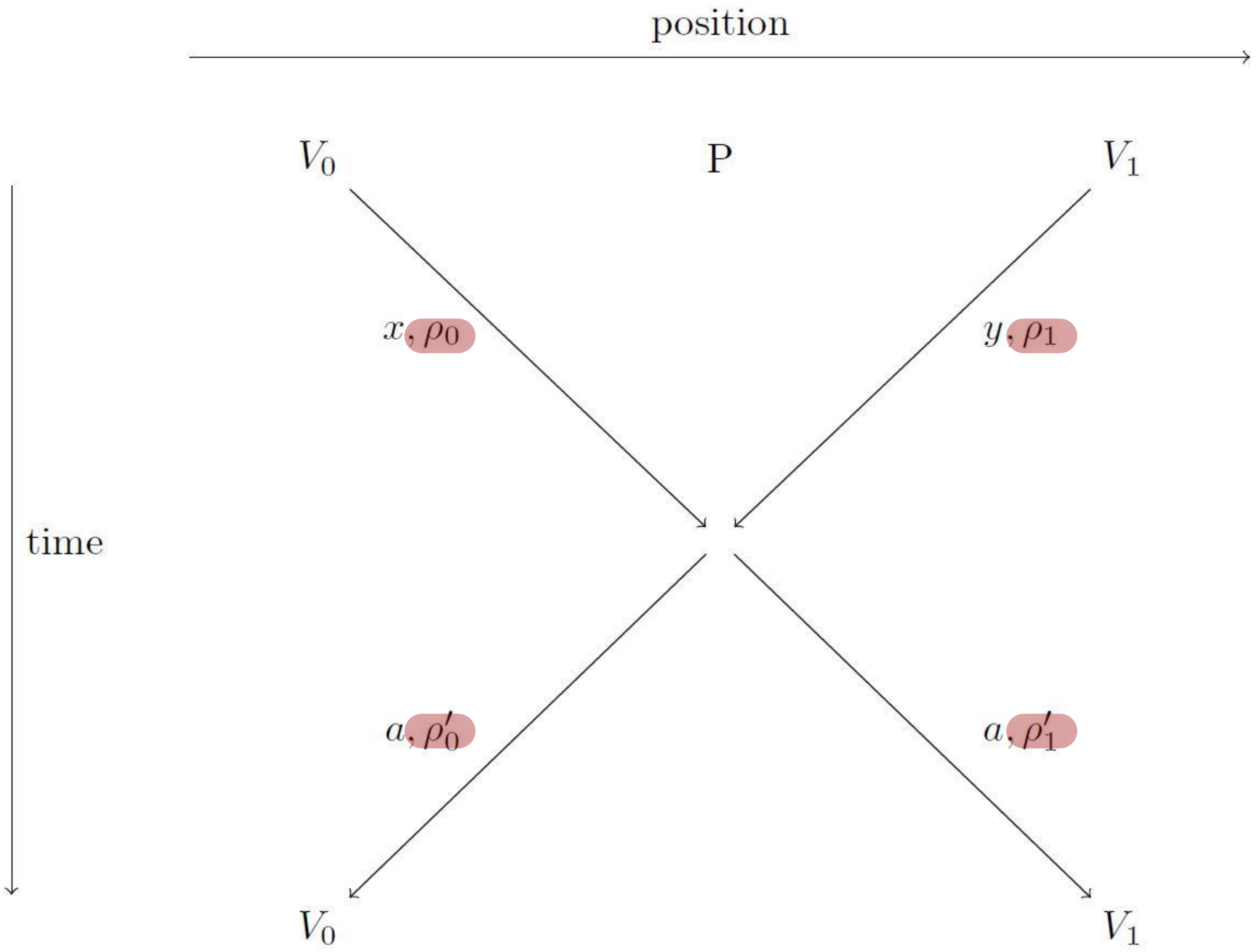

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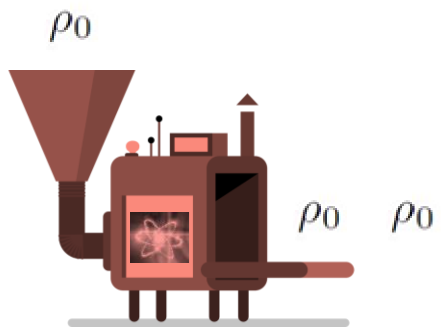

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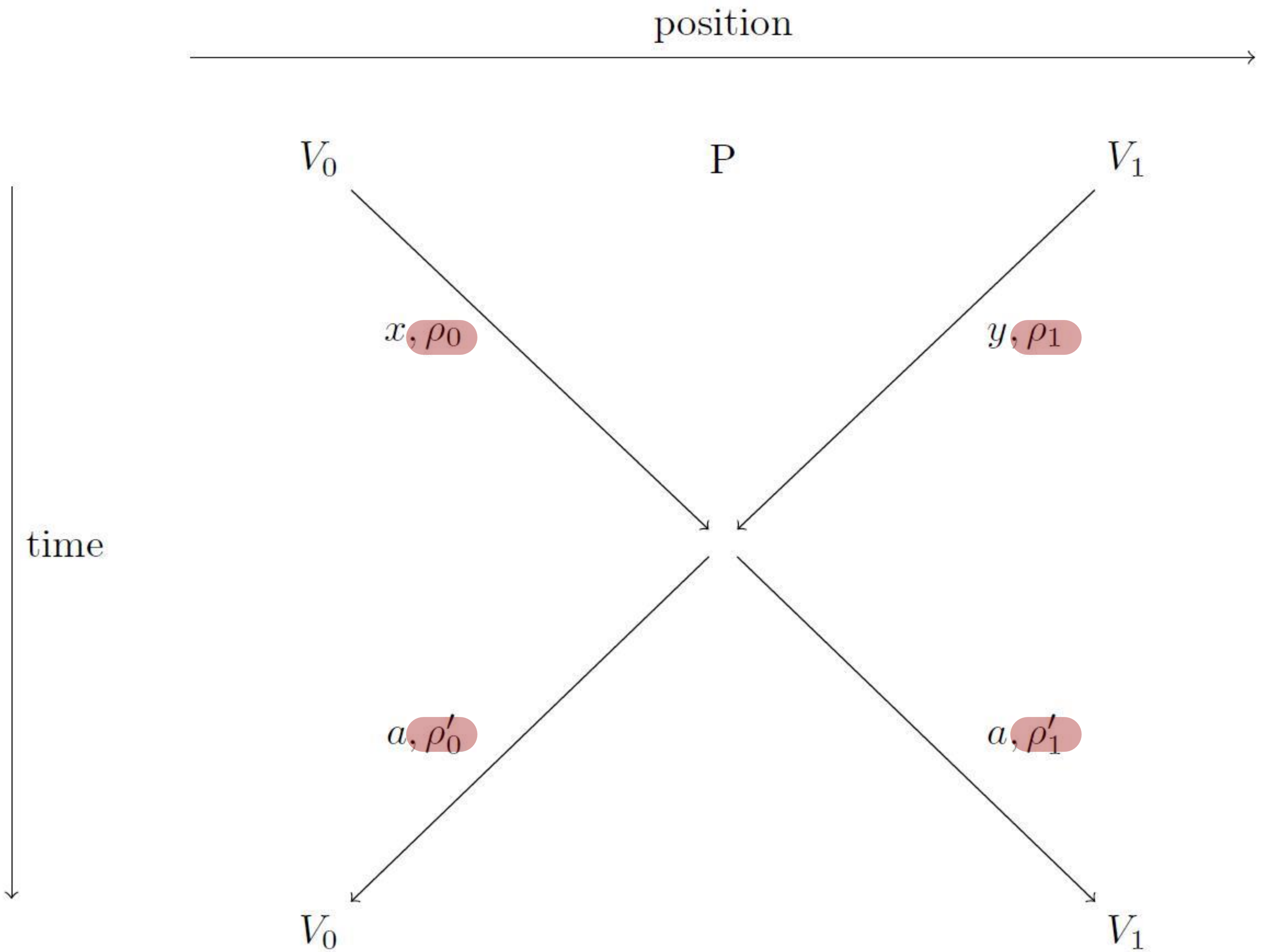

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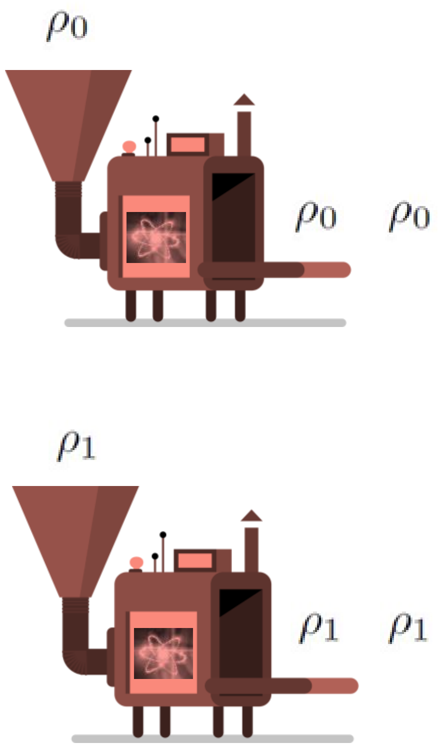

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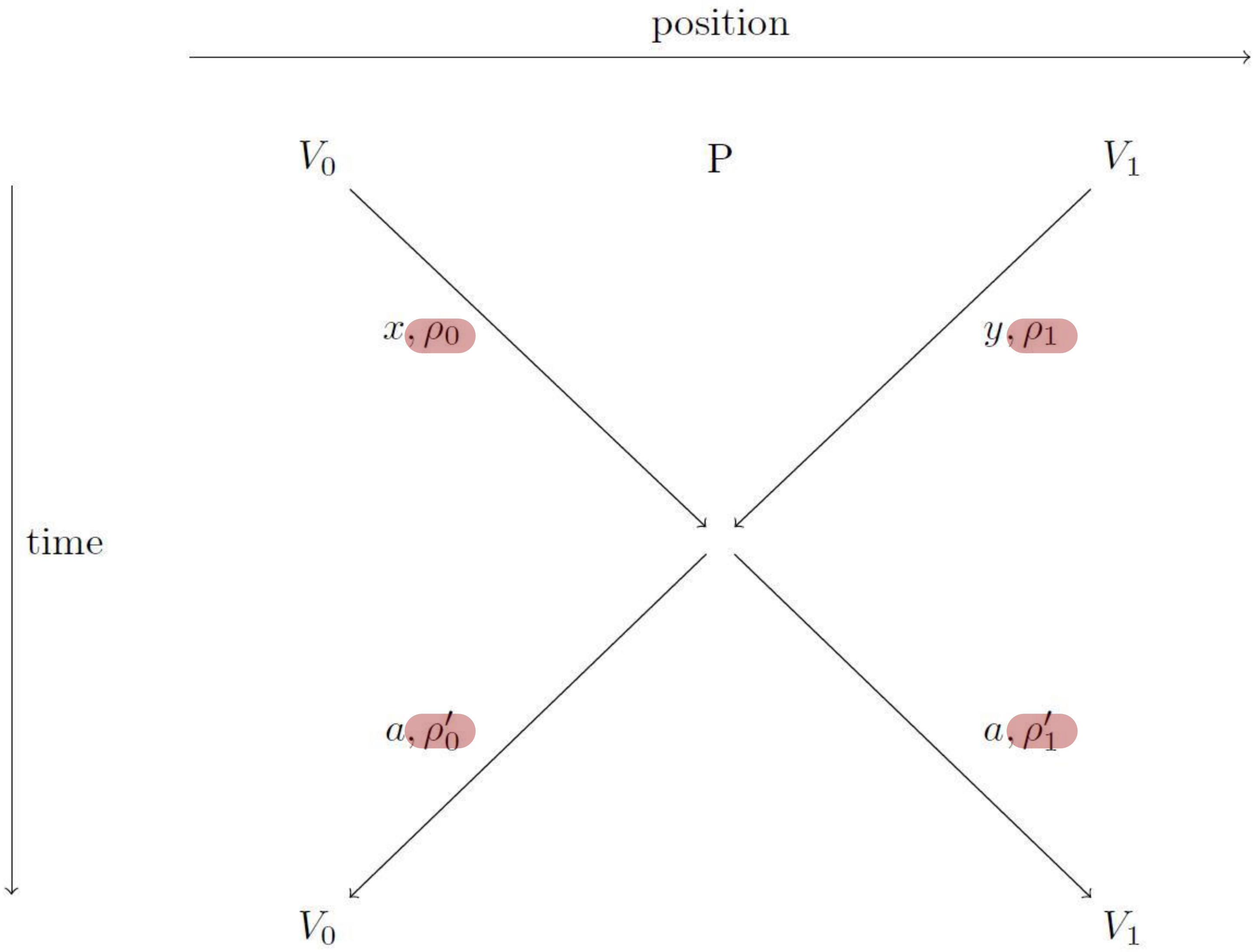

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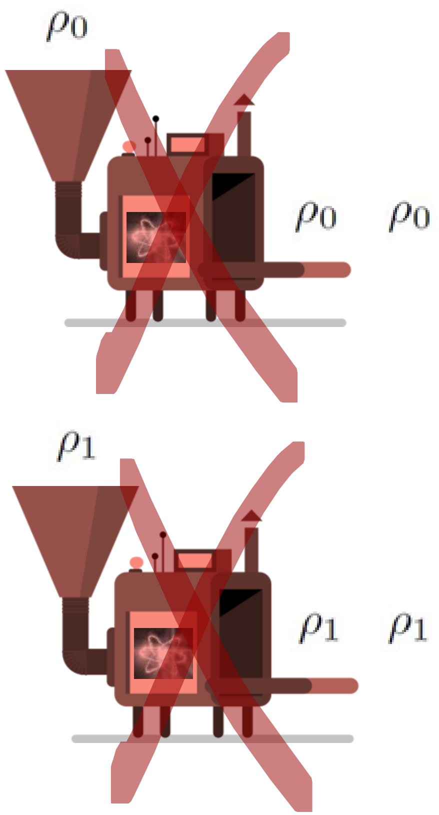
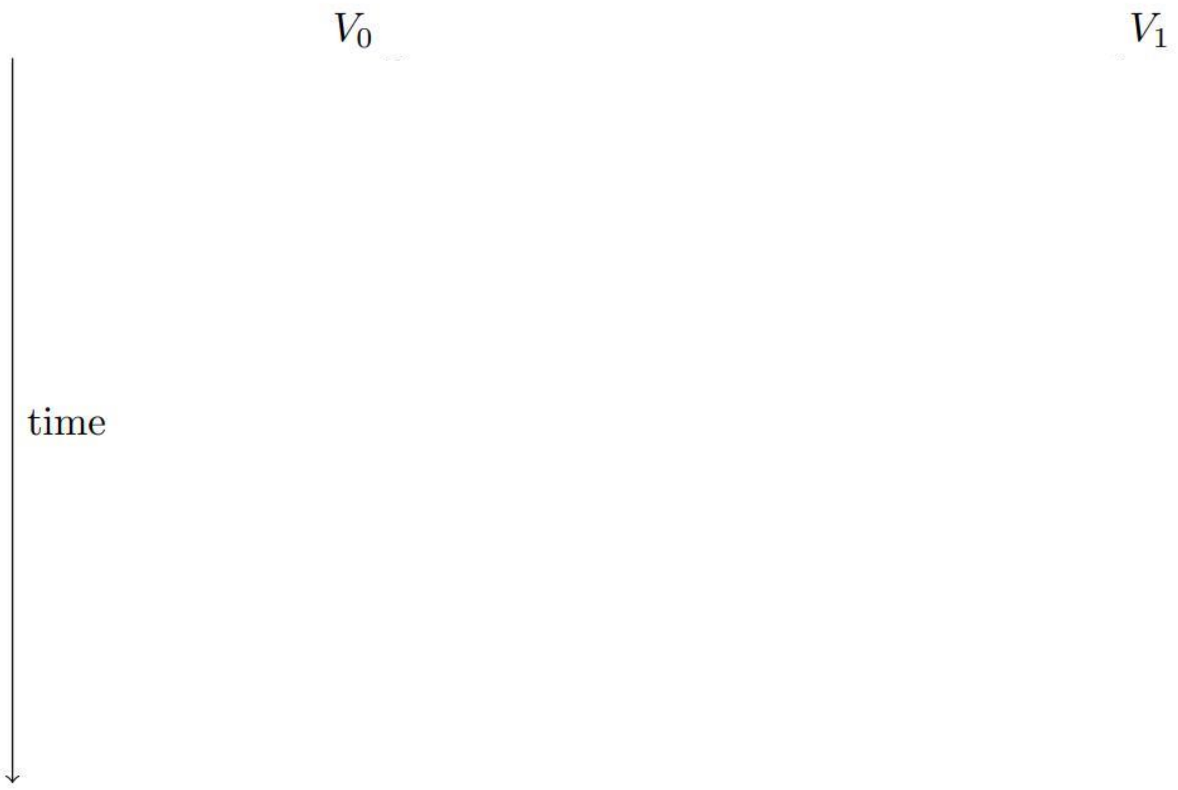


A concrete QPV protocol

$\mathrm{QPV}_{\mathrm{BB84}}$

Kent, Munro, and Spiller [KMS11]

$|\phi\rangle \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$



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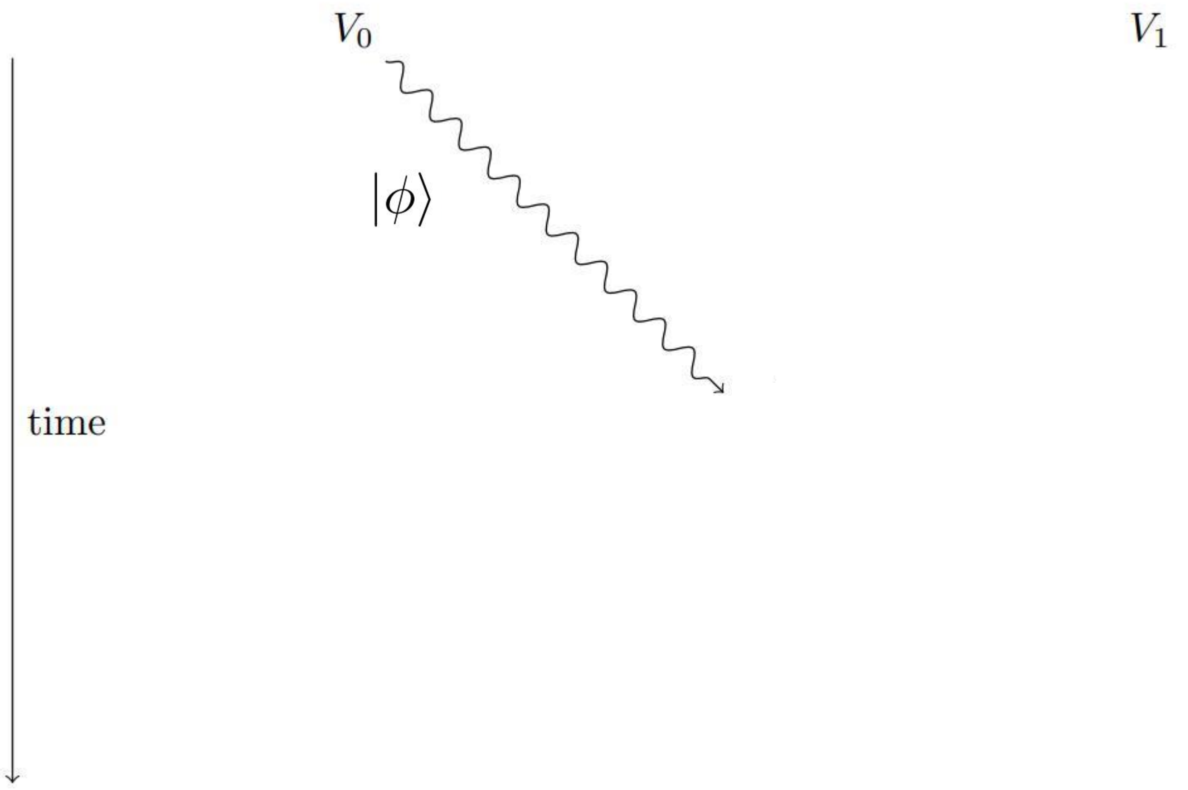

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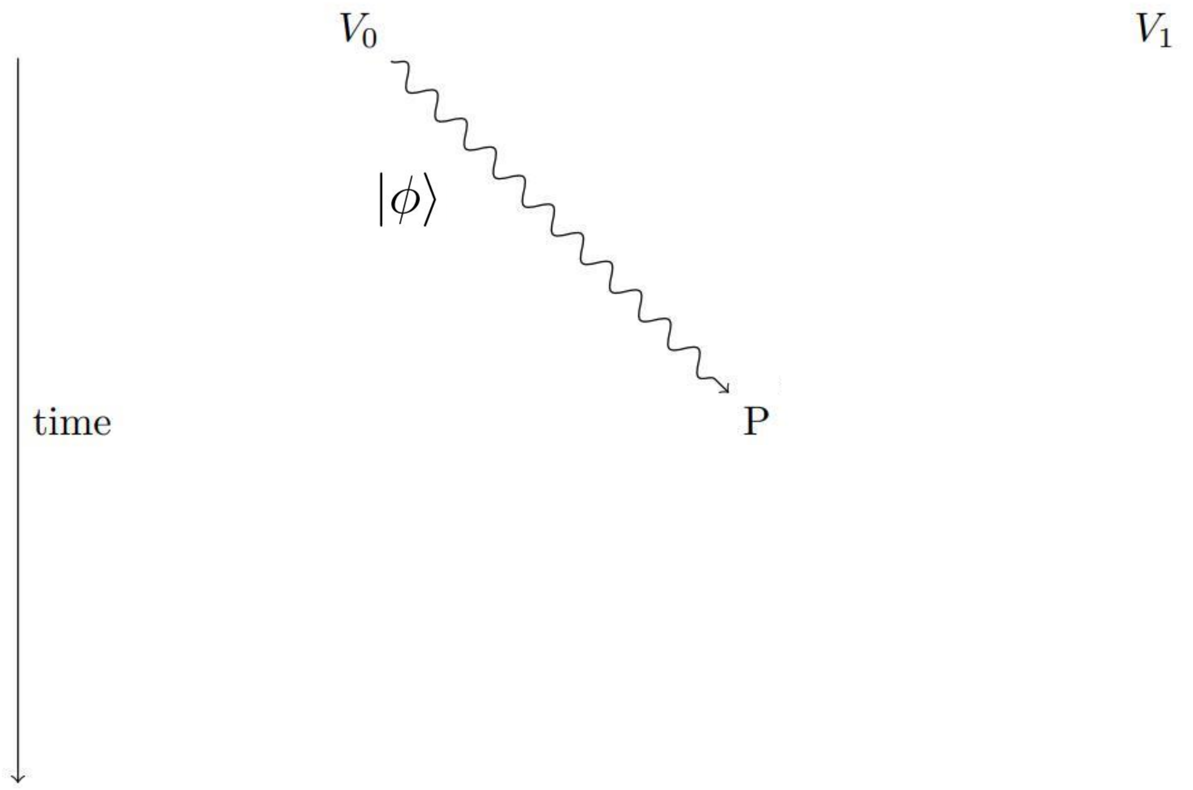

QPV_{BB84}

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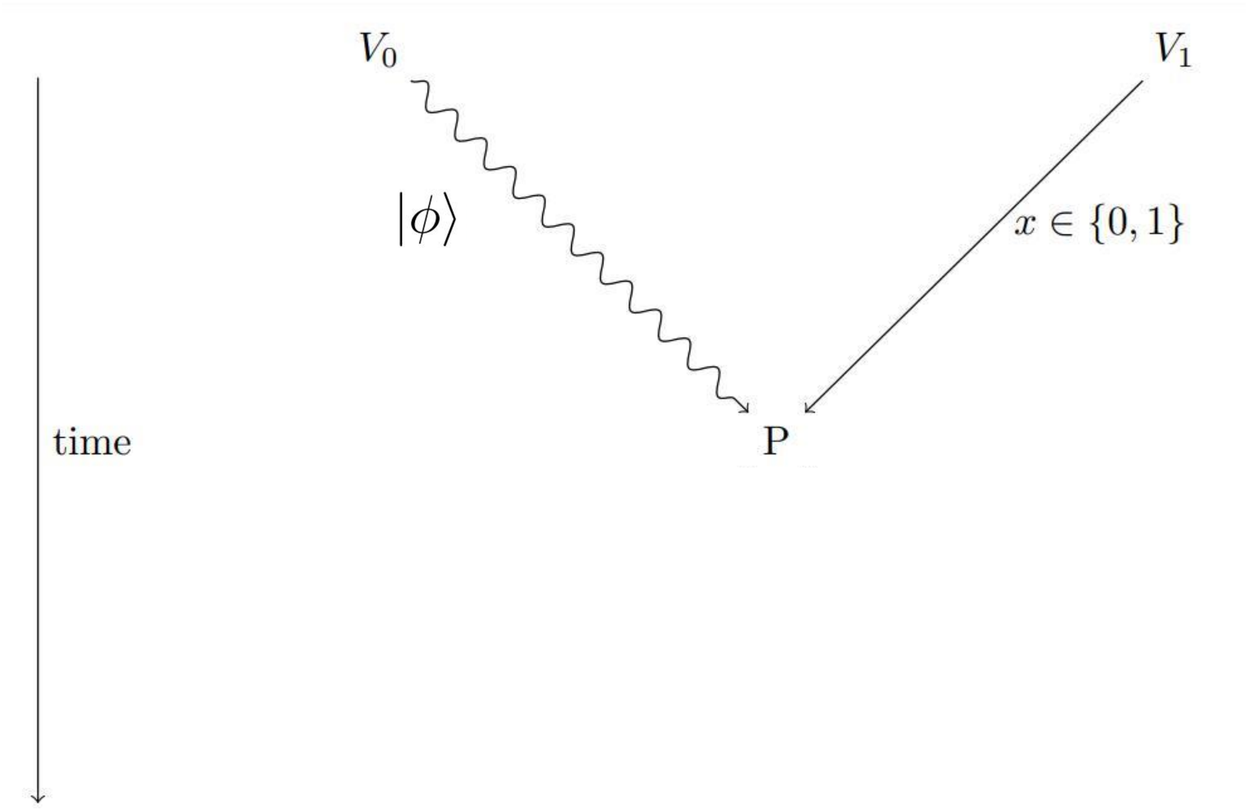

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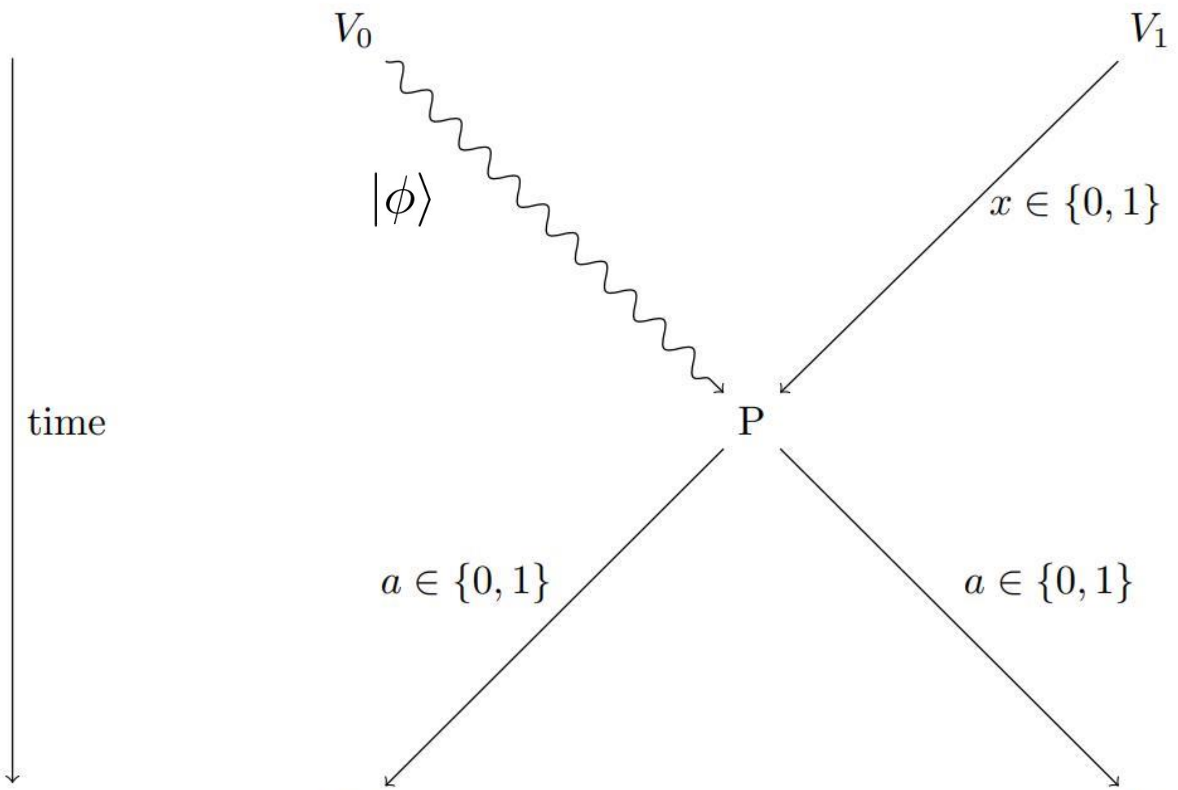

QPV_{BB84}

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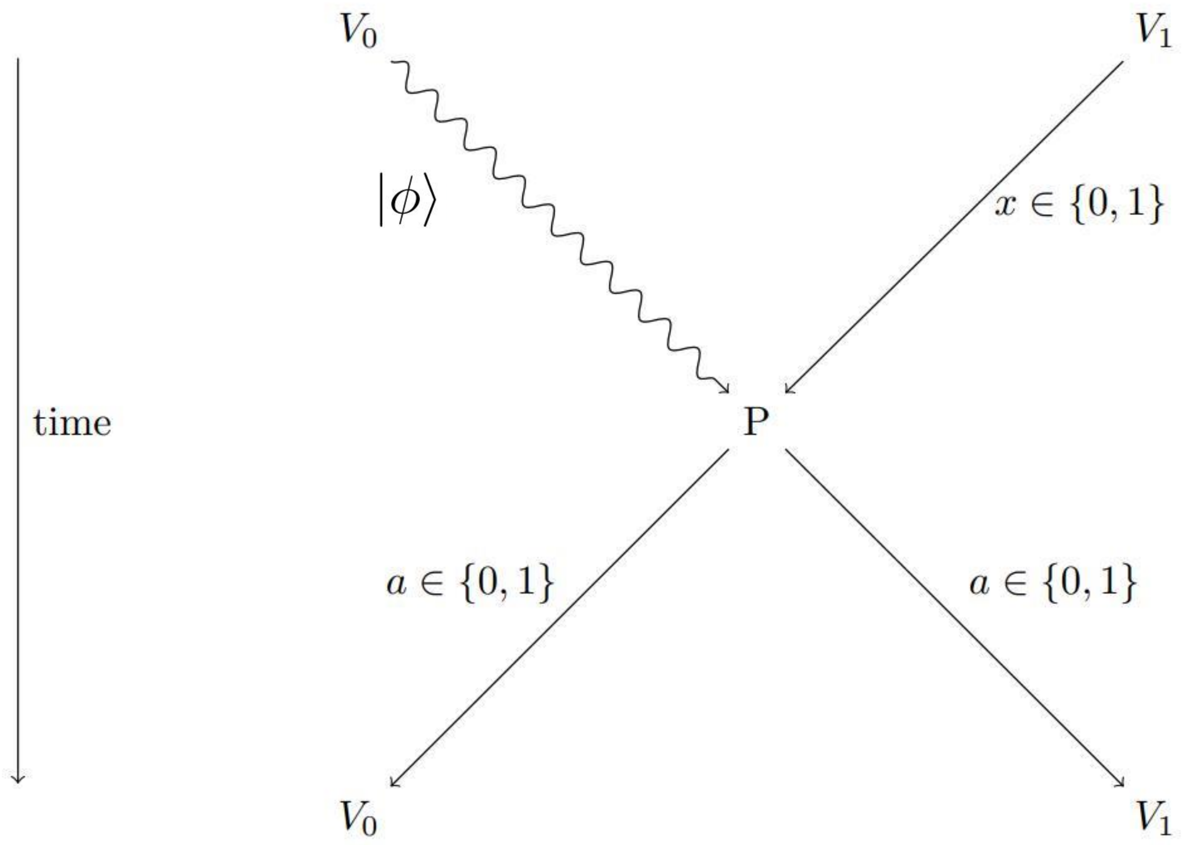
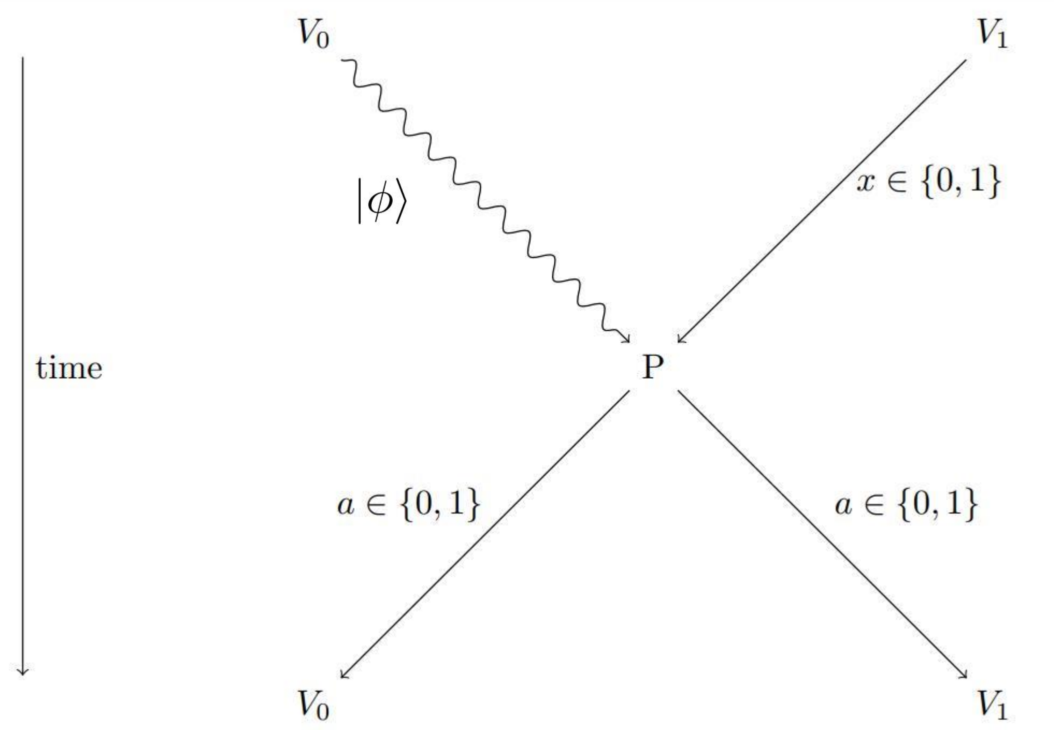


Attacks

Attack pre-sharing entanglement [KMS11]



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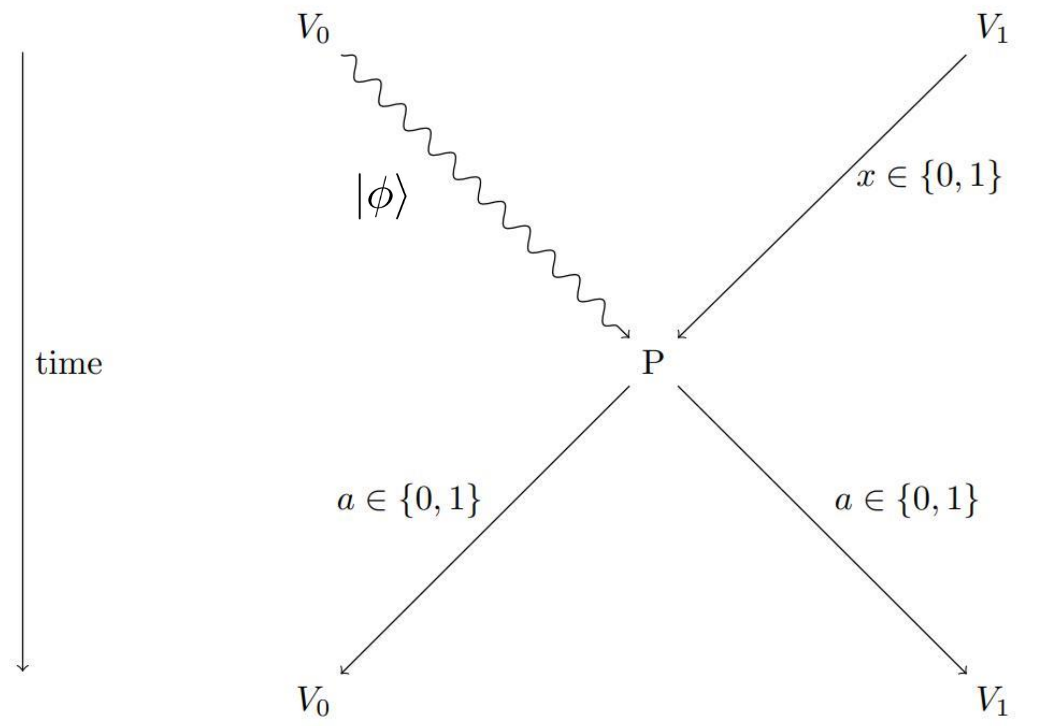

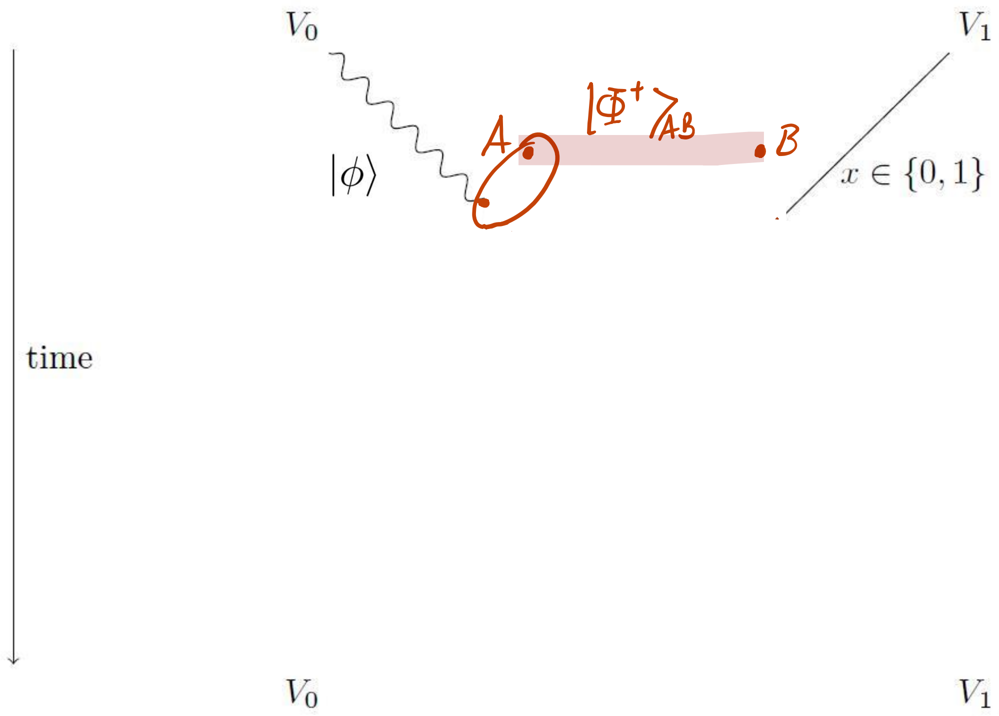

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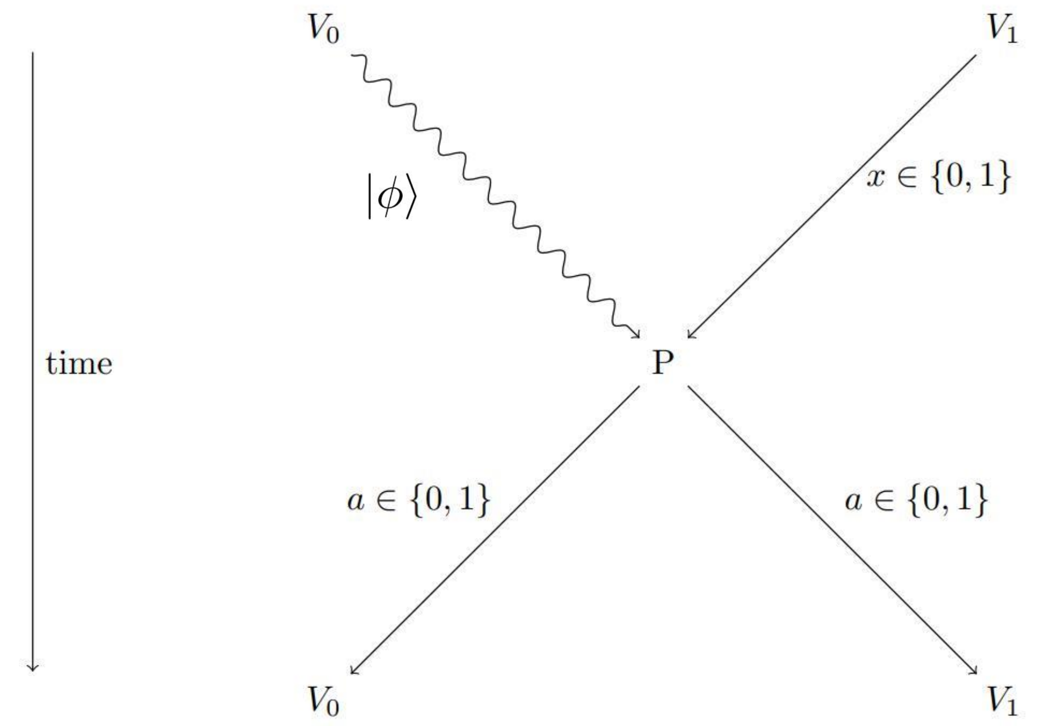

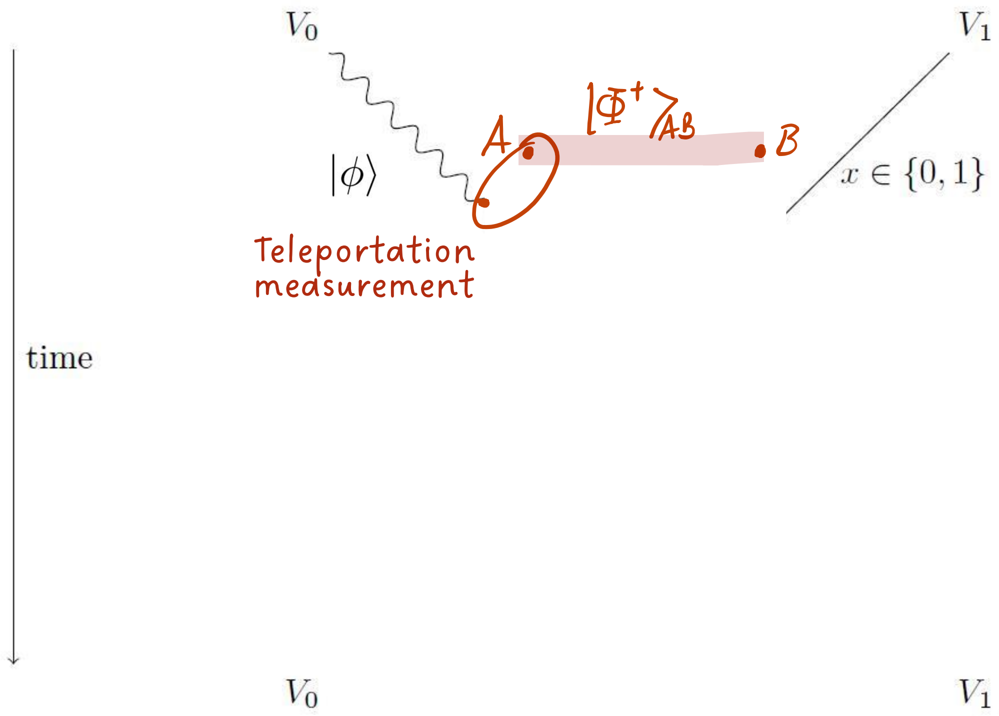

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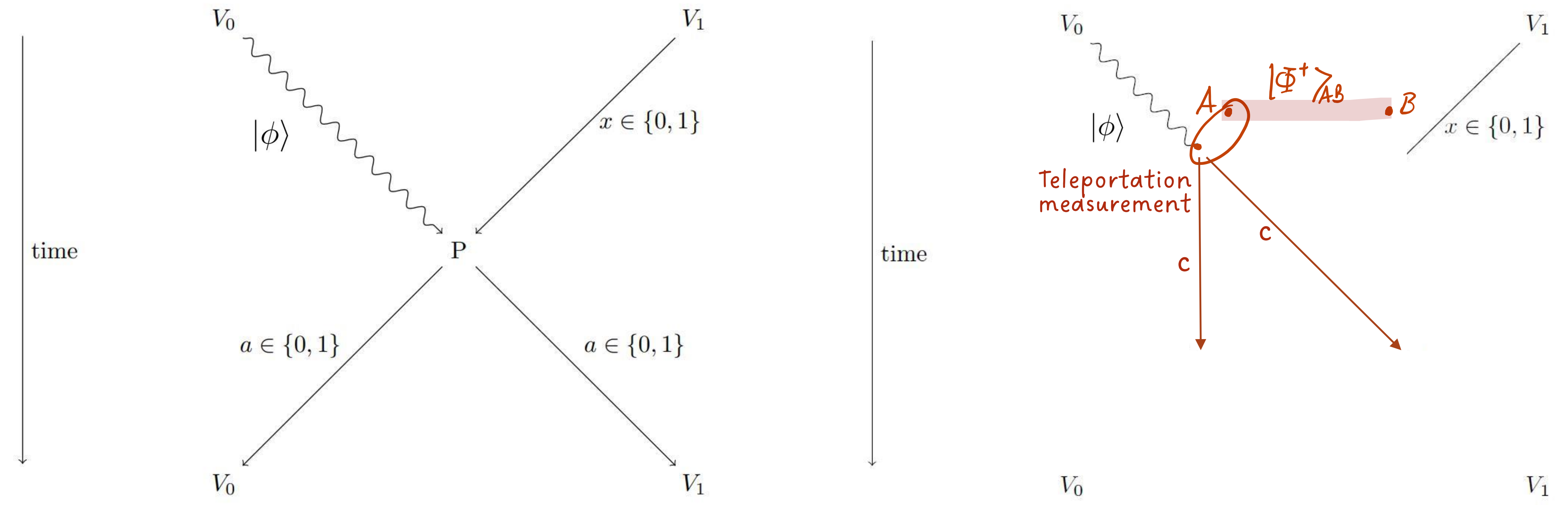

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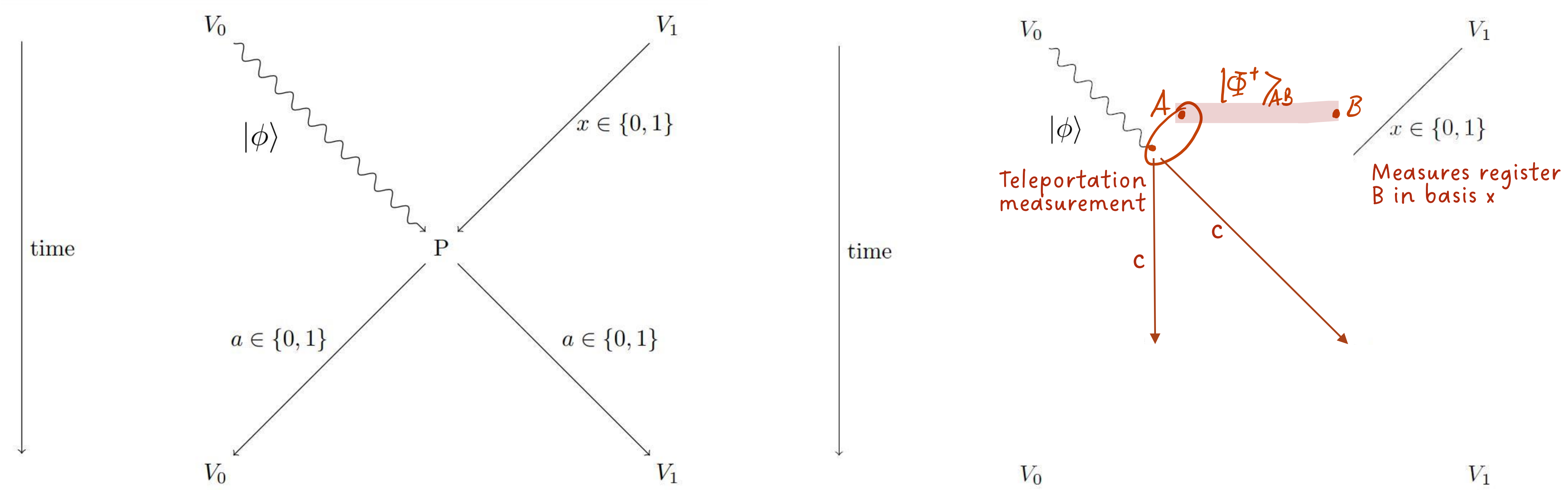

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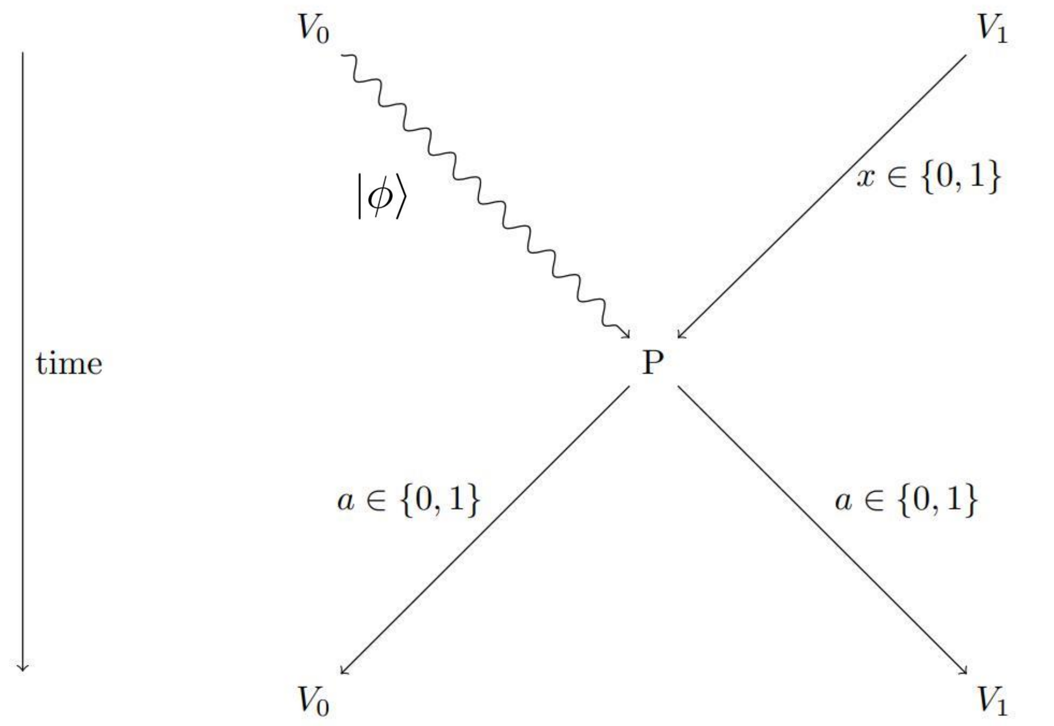

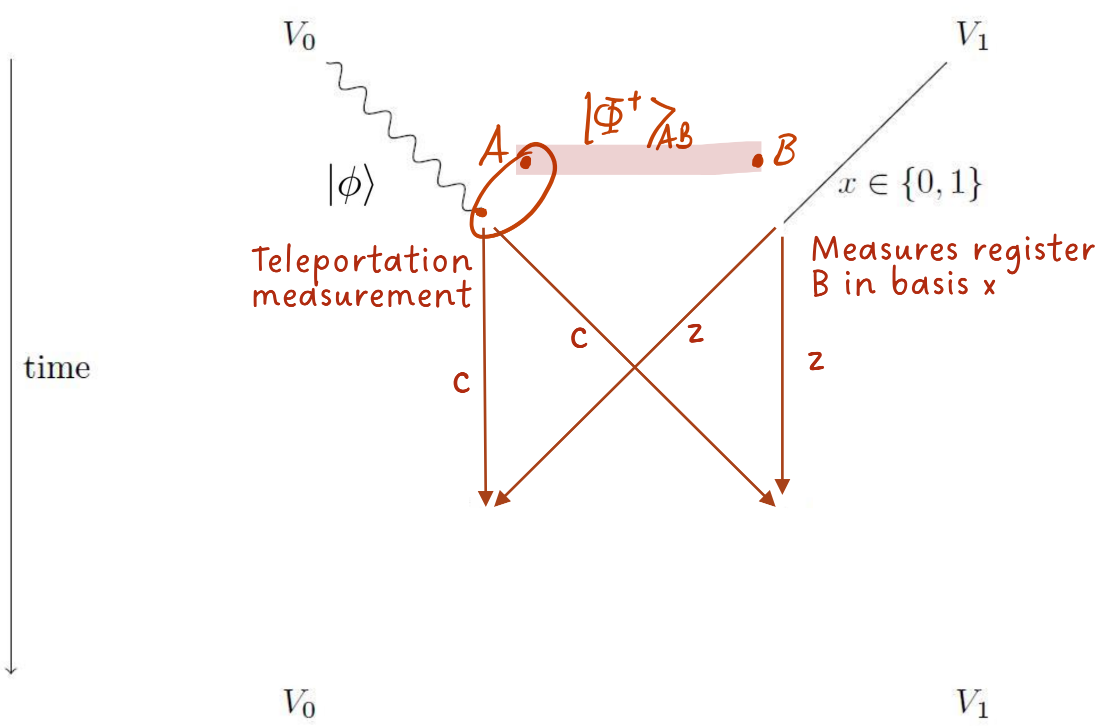

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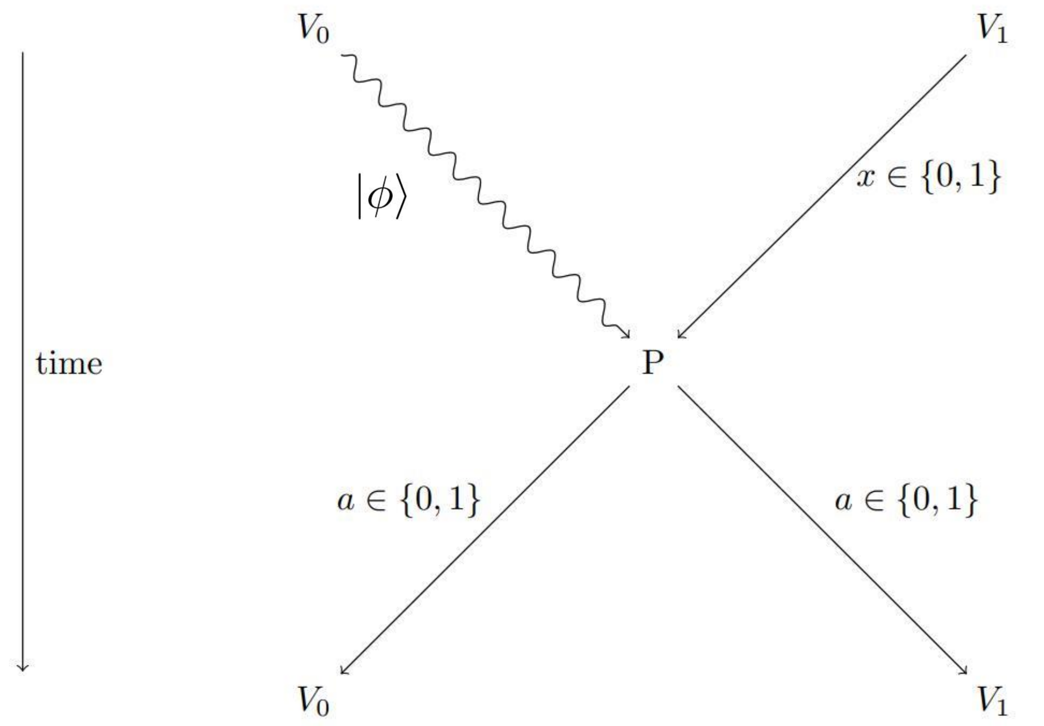

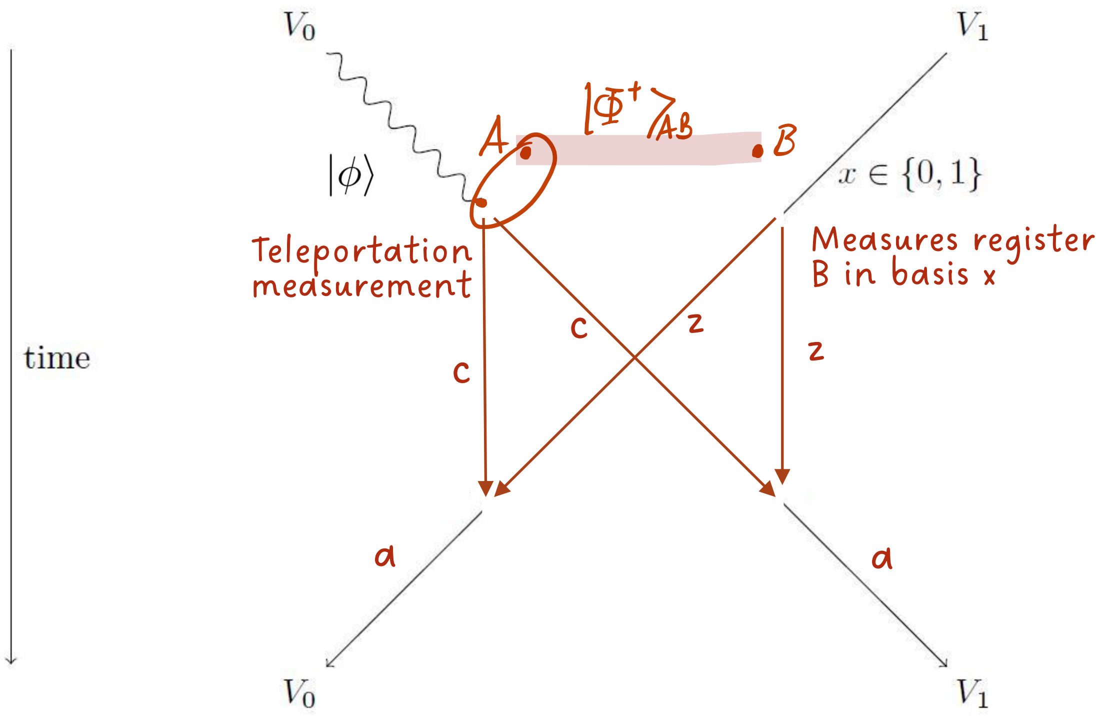

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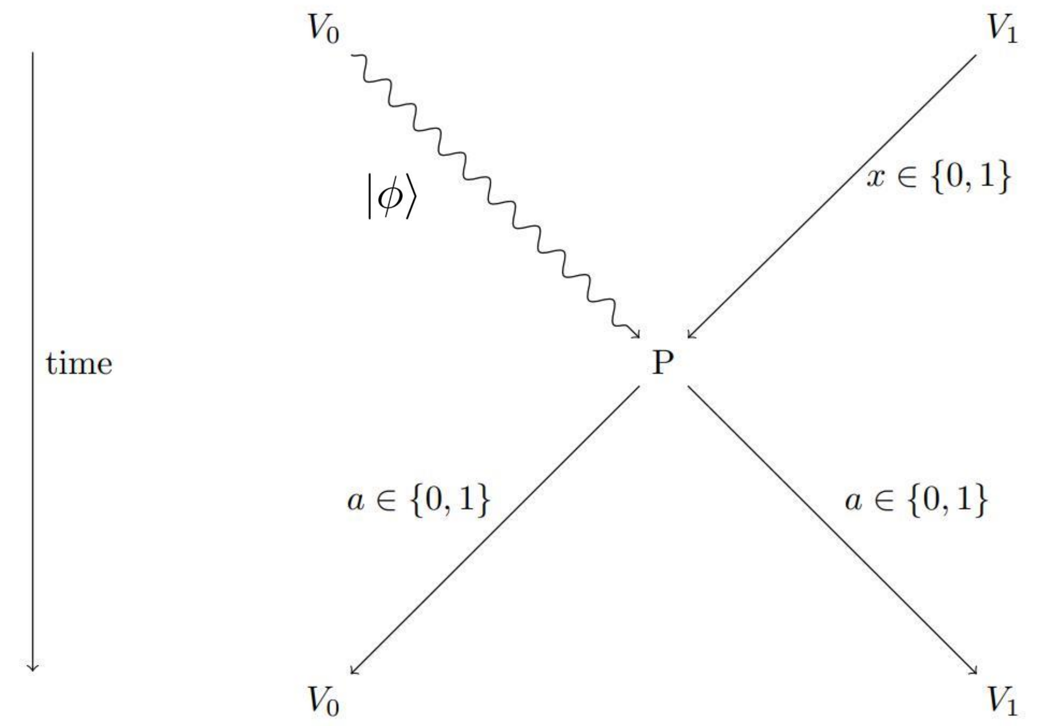
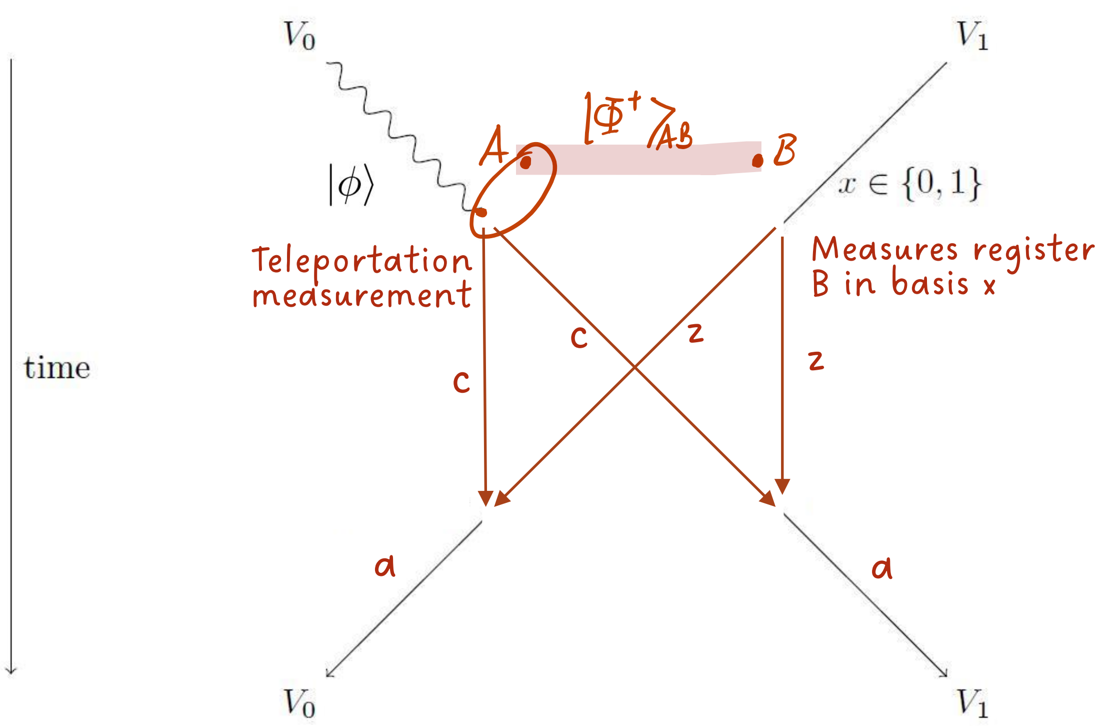
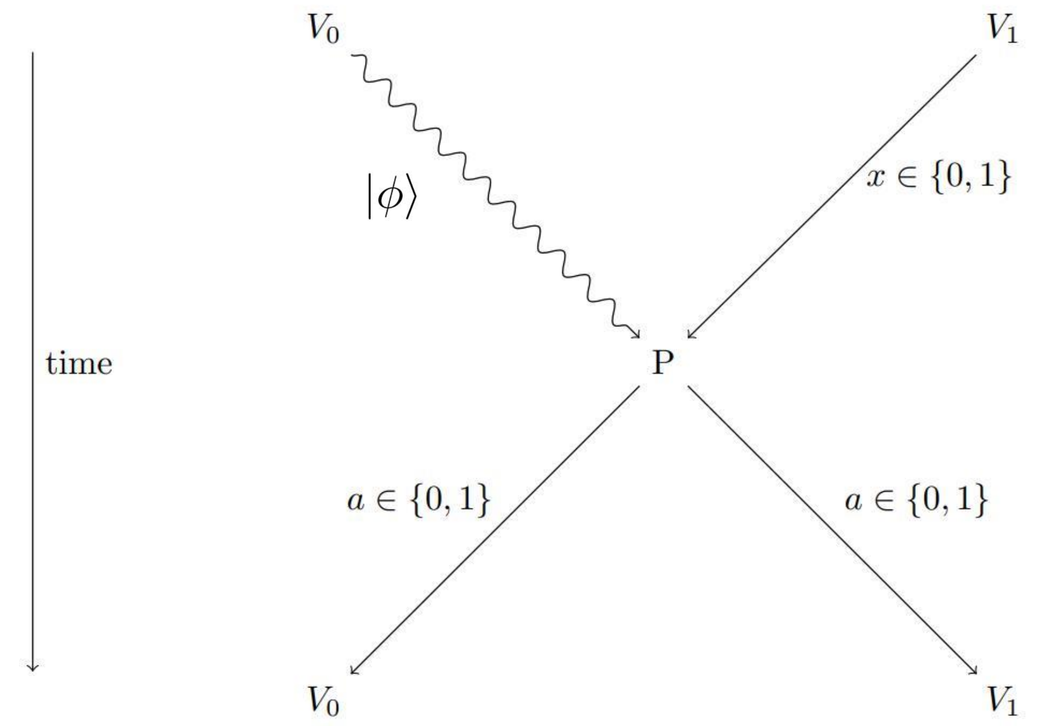
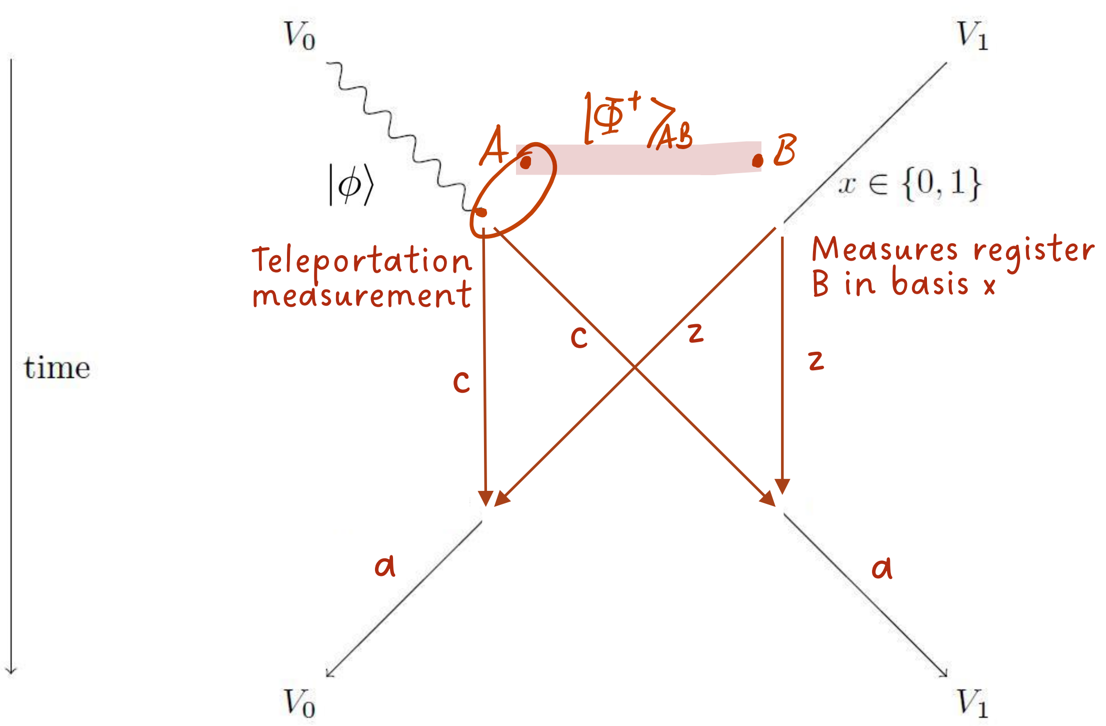




If no pre-shared entanglement [TFKW13]:

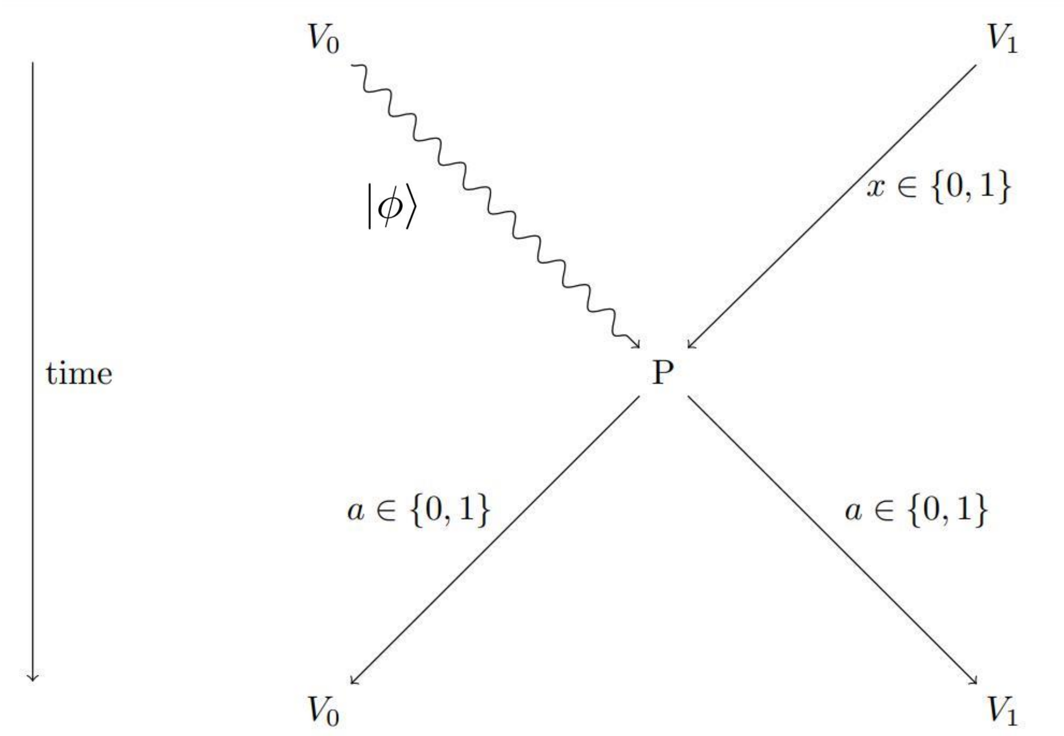
Attack pre-sharing entanglement [KMS11]

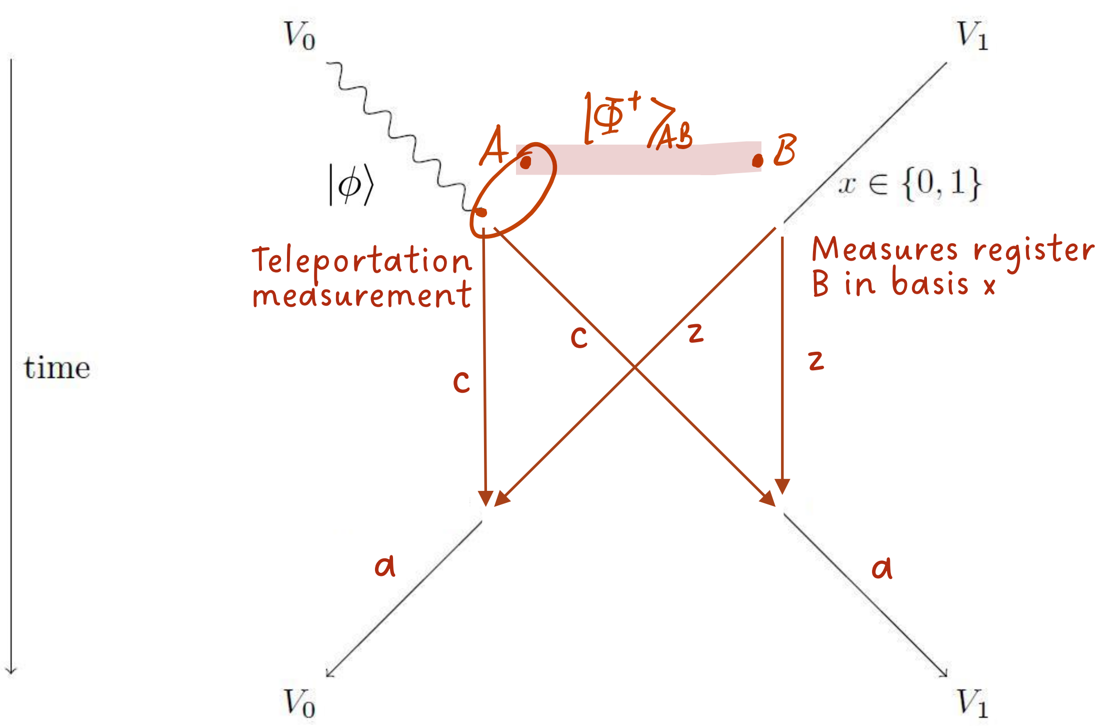




If no pre-shared entanglement [TFKW13]: $\mathbb{P}_{attack} \leq \cos^2\left(\frac{\pi}{8}\right)$

Attack pre-sharing entanglement [KMS11]





If no pre-shared entanglement [TFKW13]: $\mathbb{P}_{attack} \leq \cos^2\left(\frac{\pi}{8}\right) \approx 0.85$

All quantum position verification protocols can be attacked... [BCFGGOS11]

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…the best know general attack requires exponential amount of pre-shared entanglement.

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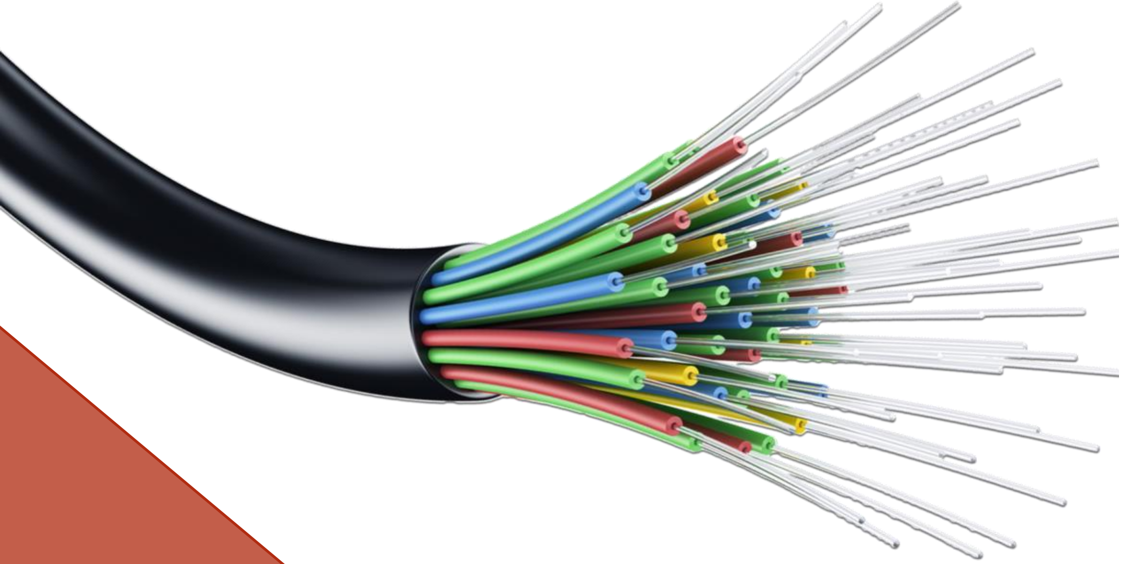
…the best know general attack requires exponential amount of pre-shared entanglement.

Goal: easy protocol which is very difficult to attack.

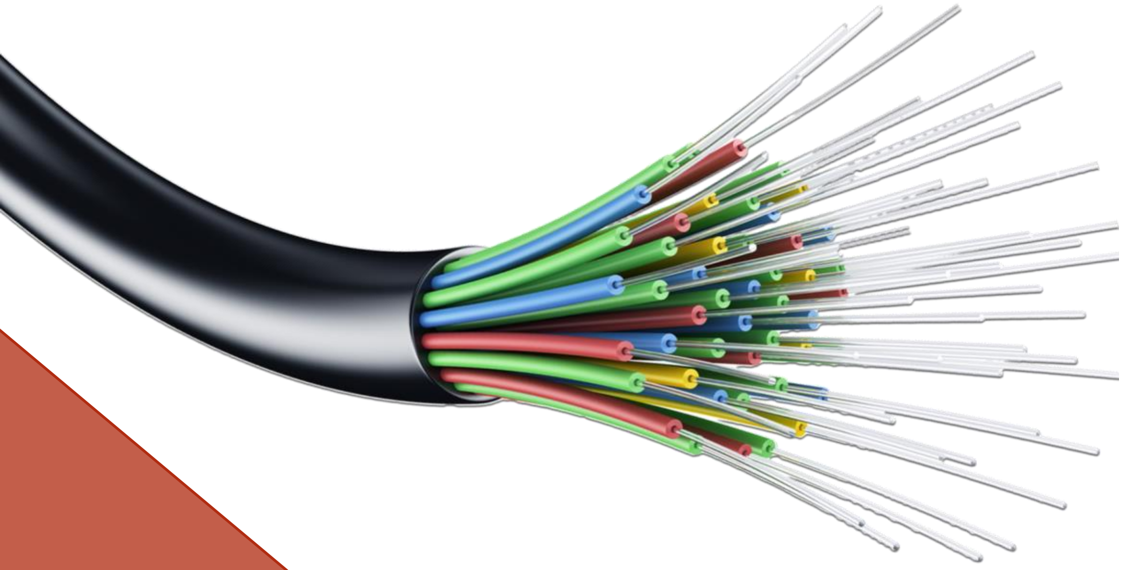
Experimental implementation encounters problems

Experimental implementation encounters problems



Experimental implementation encounters problems

Photon loss



Experimental implementation encounters problems

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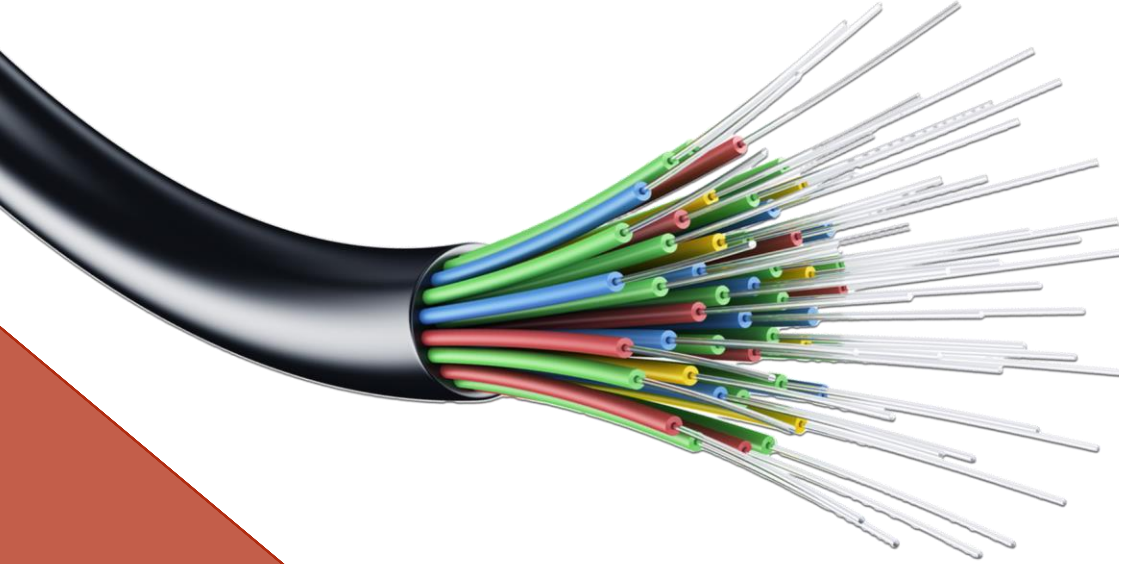

Slow quantum info: ~2/3c

Experimental implementation encounters problems

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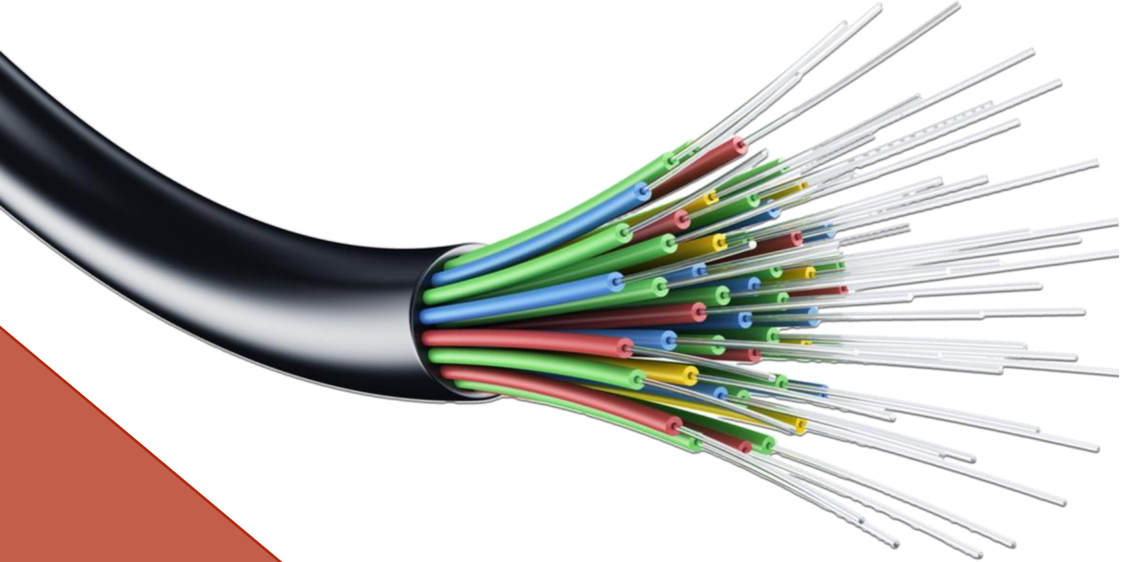
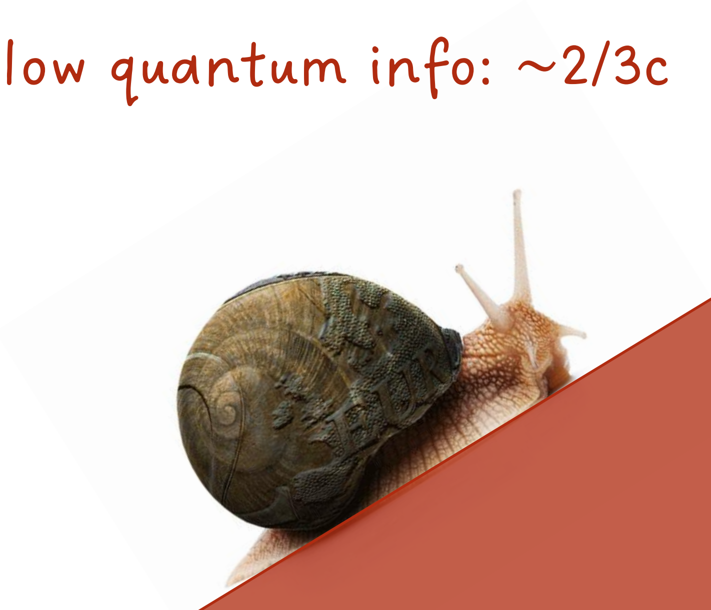
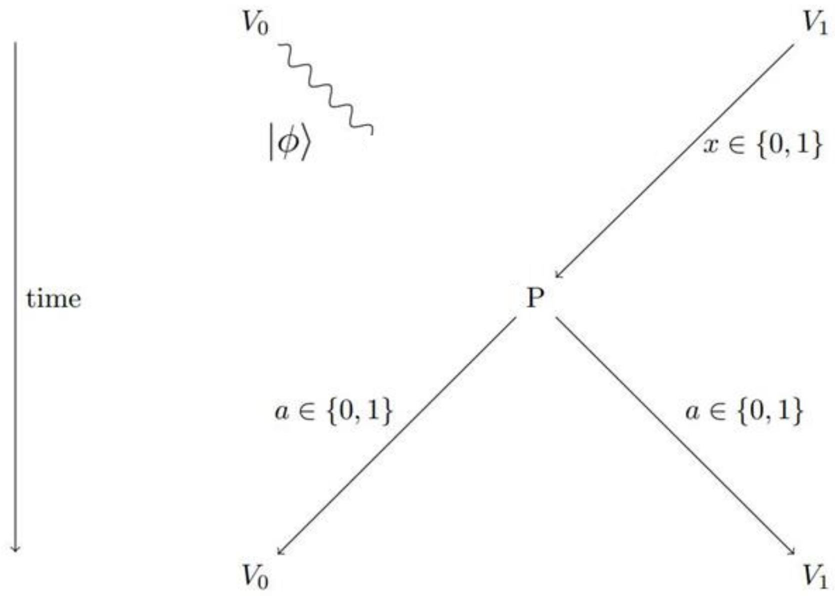


Slow quantum info: ~2/3c



Taking advantage of photon loss



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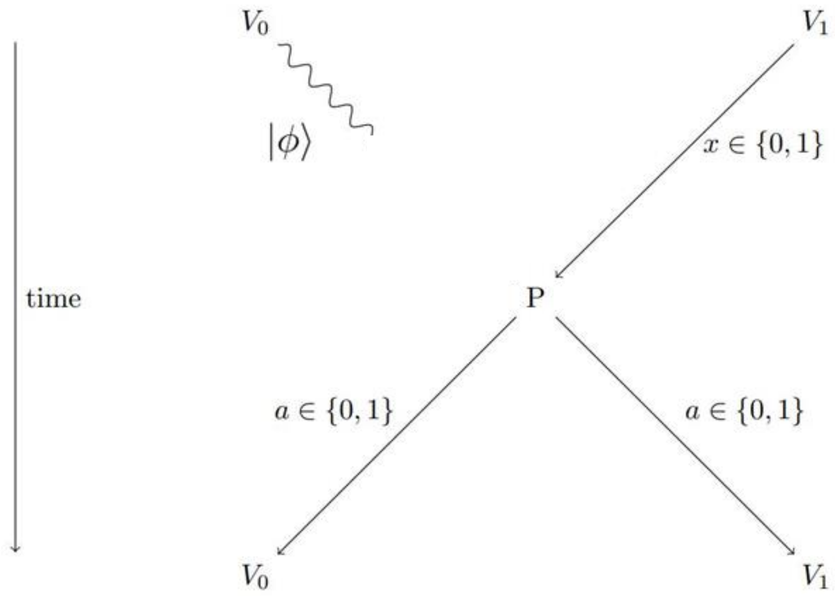

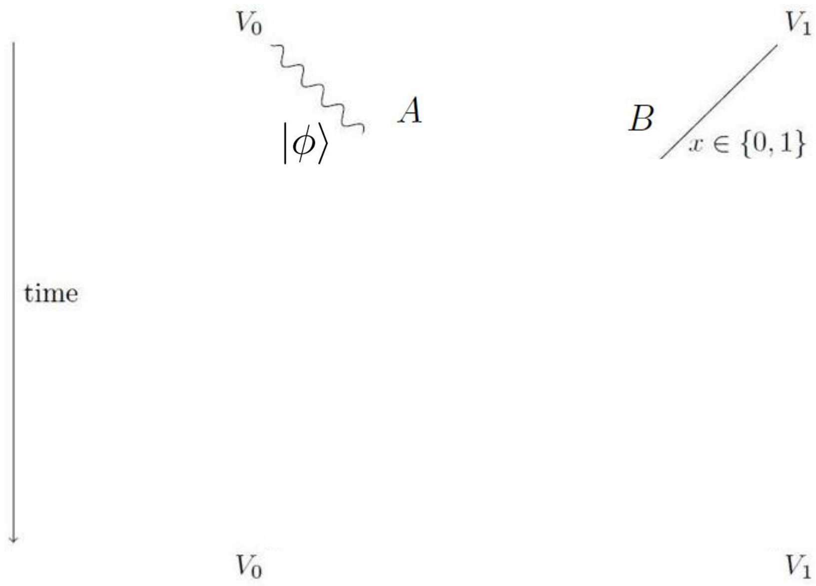

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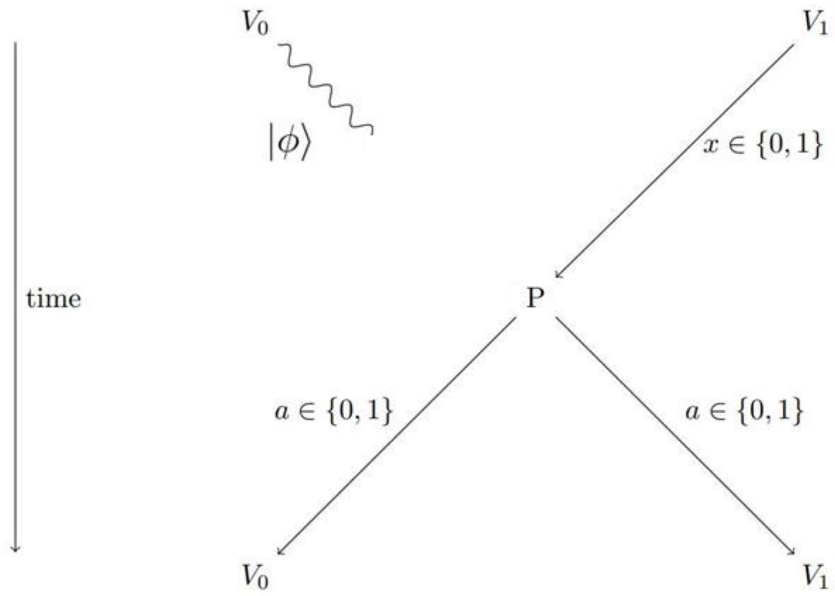

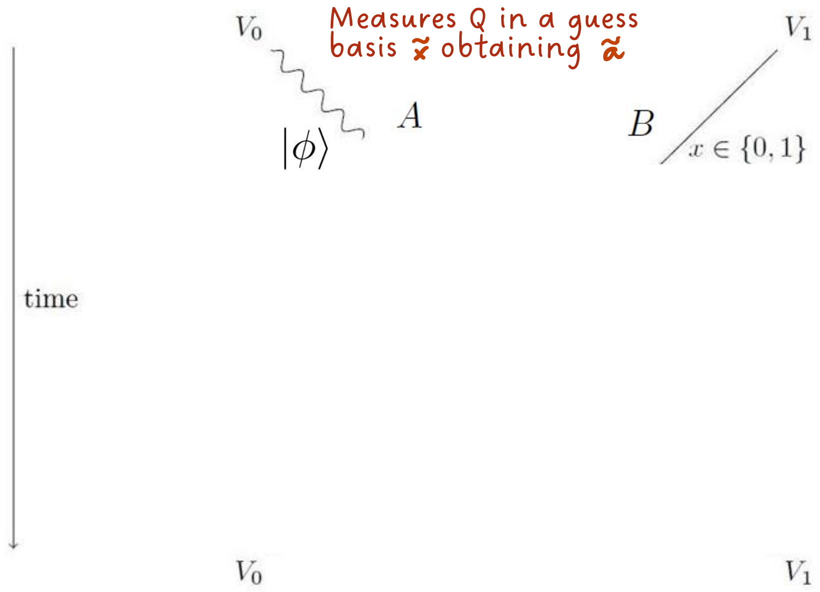

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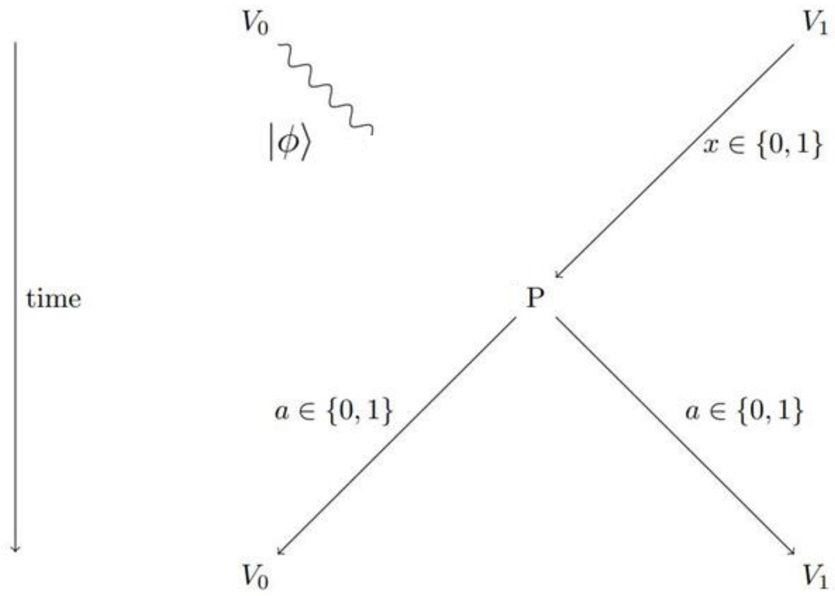

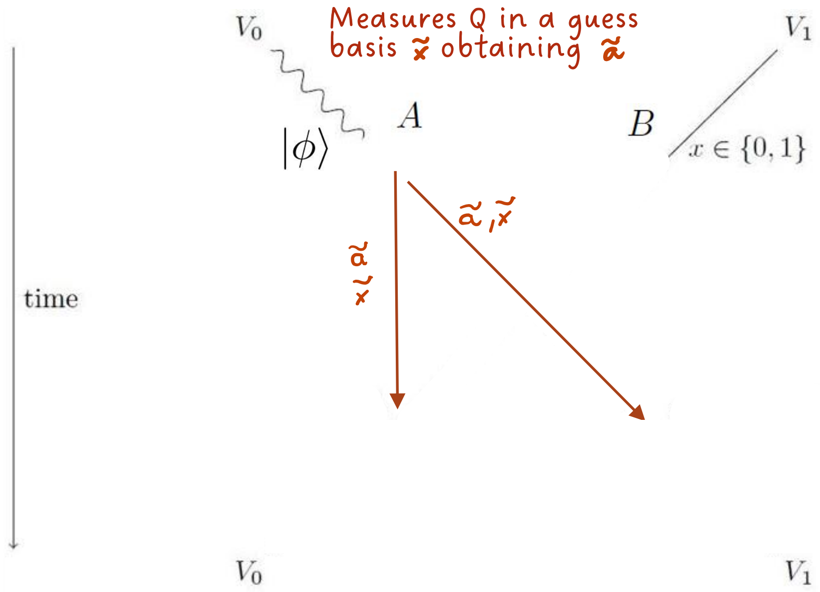

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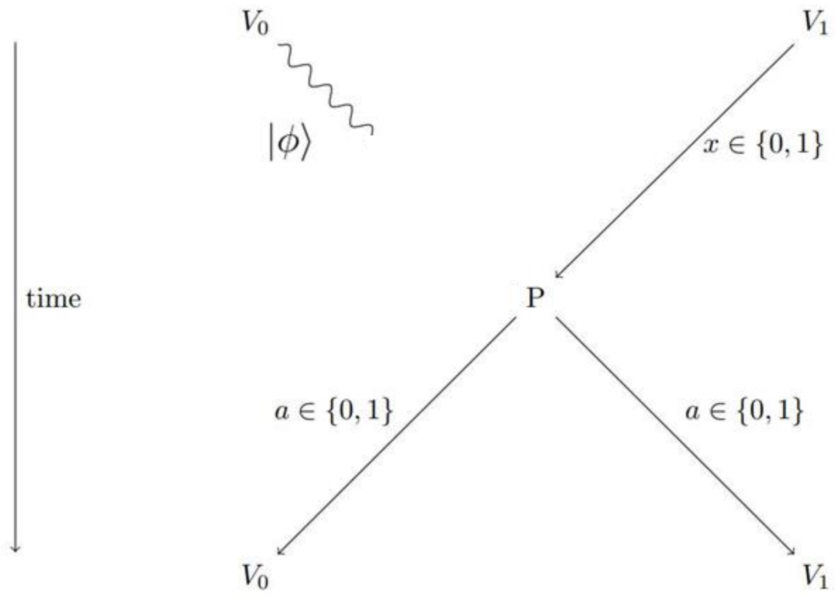

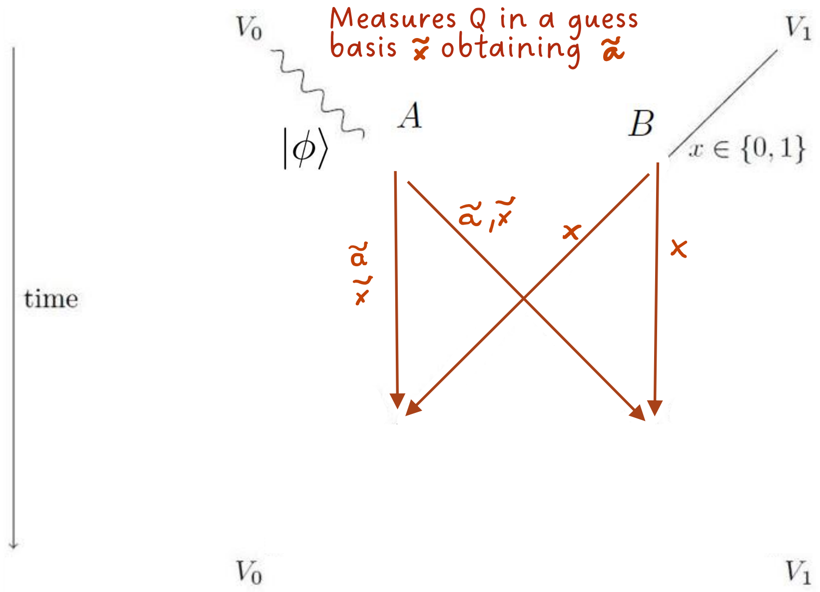

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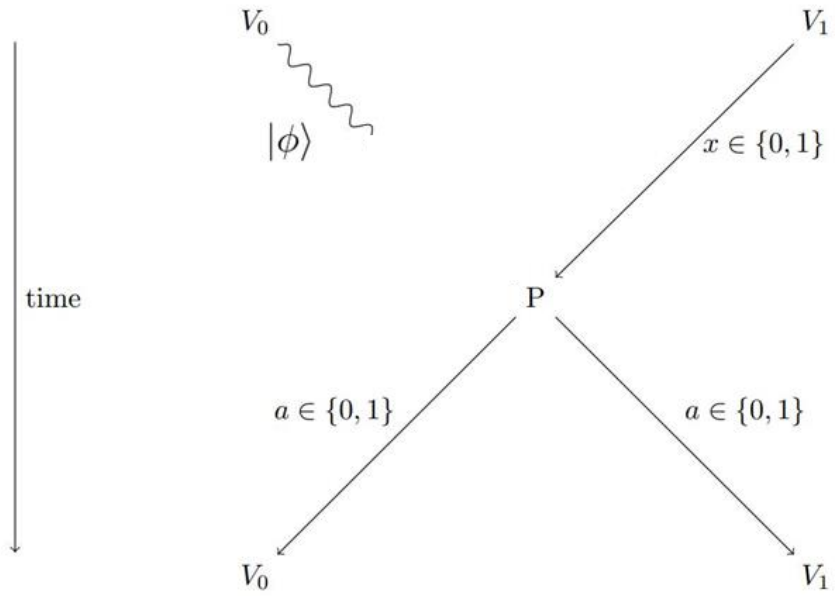
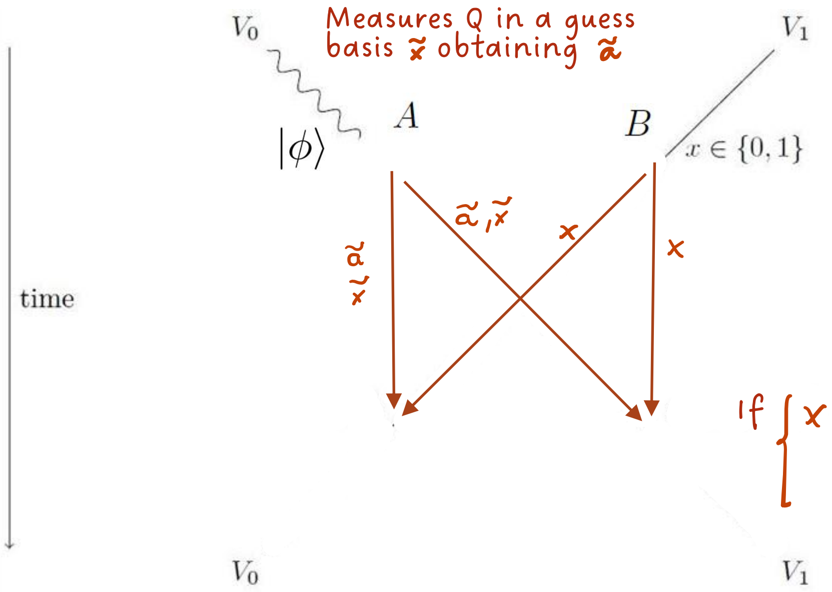
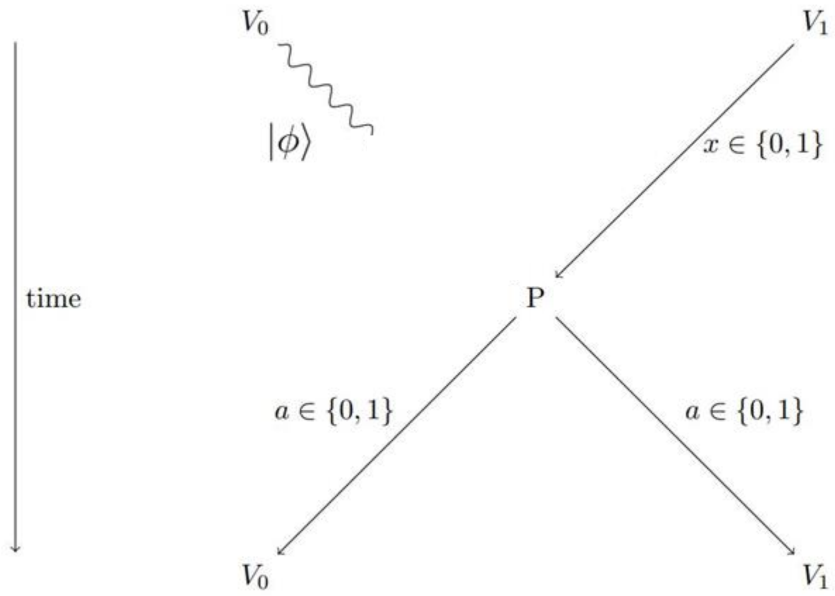
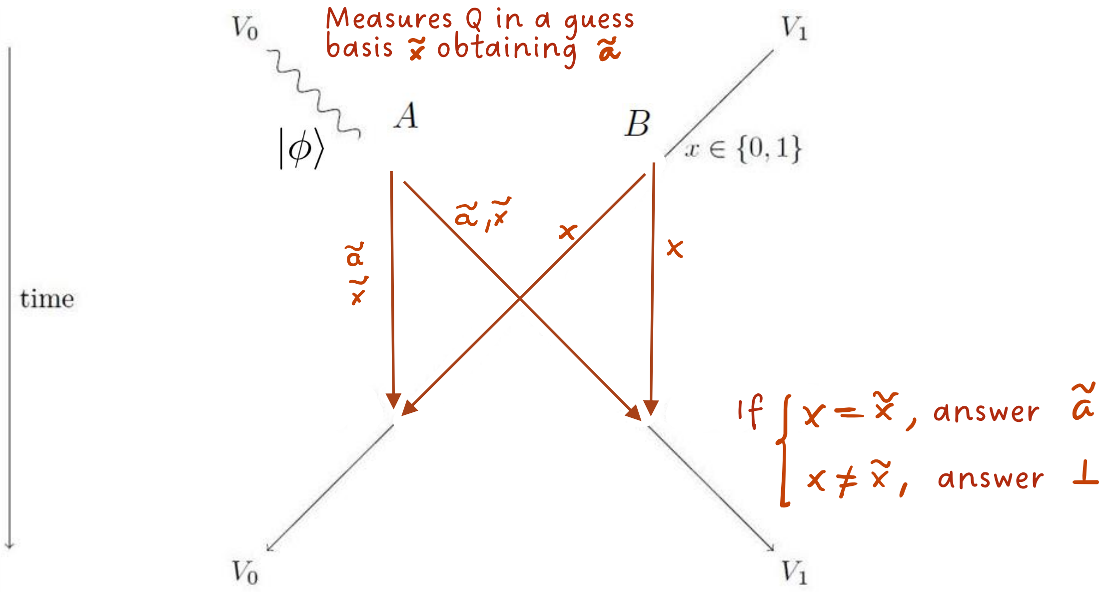




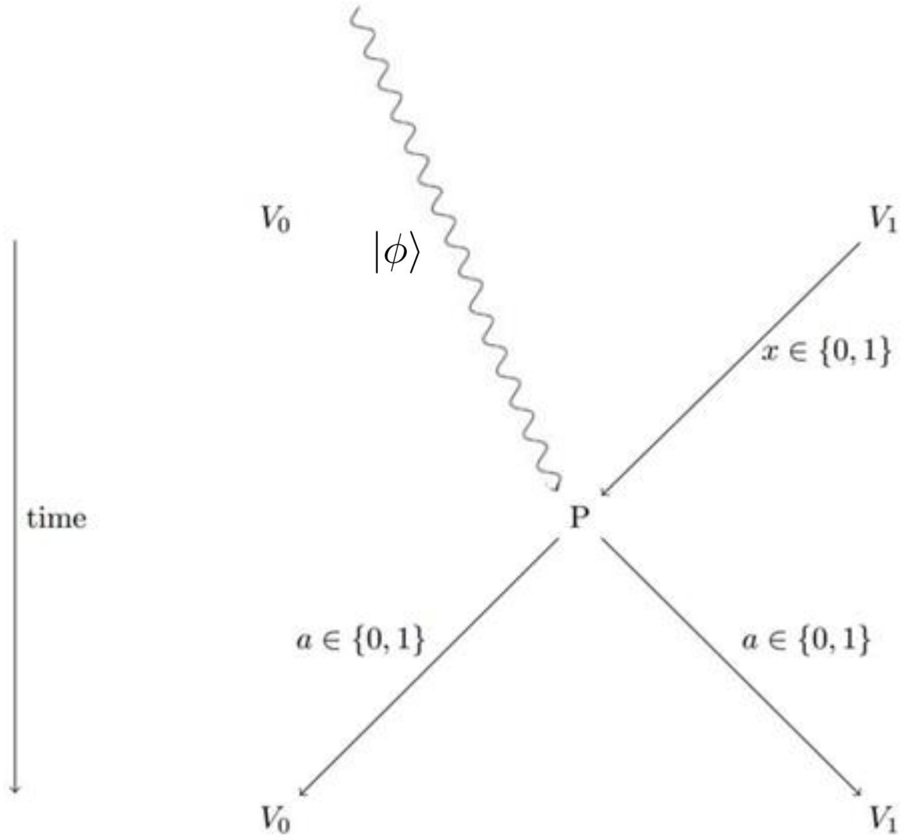
If $\begin{cases} x = \tilde{x}, \text{ answer } \tilde{a} \end{cases}$

Taking advantage of photon loss





Taking advantage of slow quantum information



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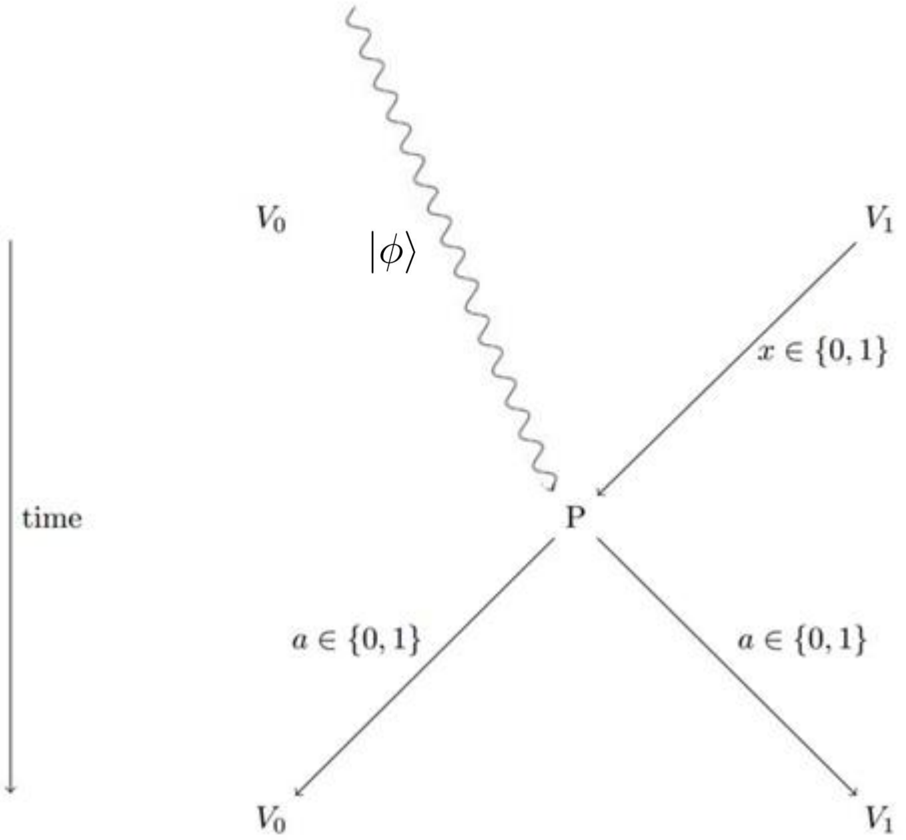

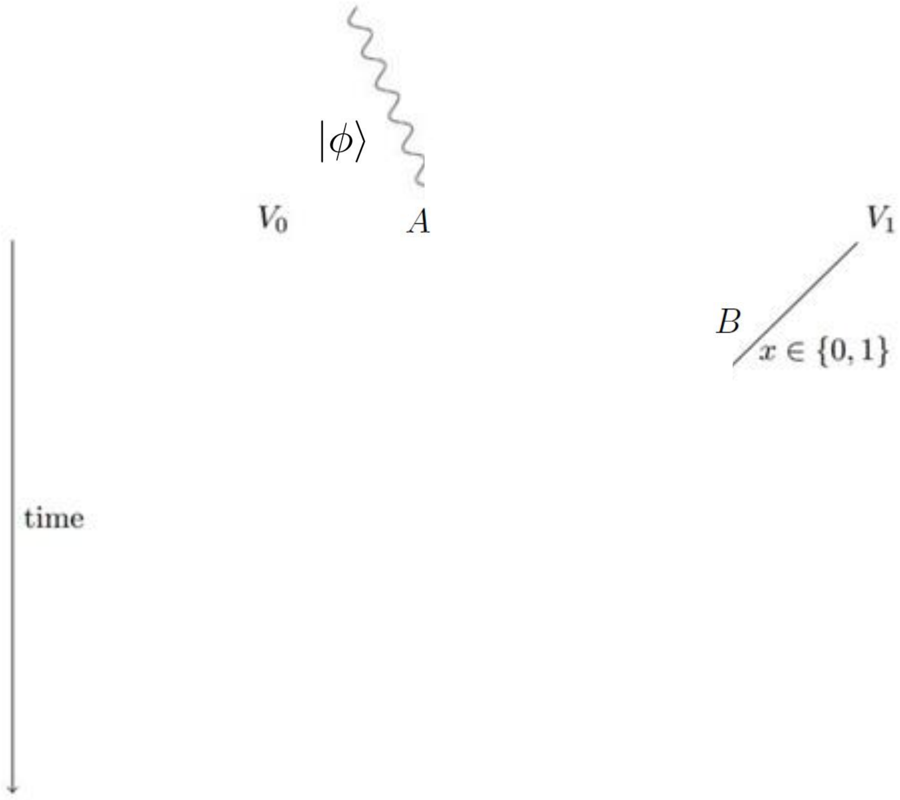

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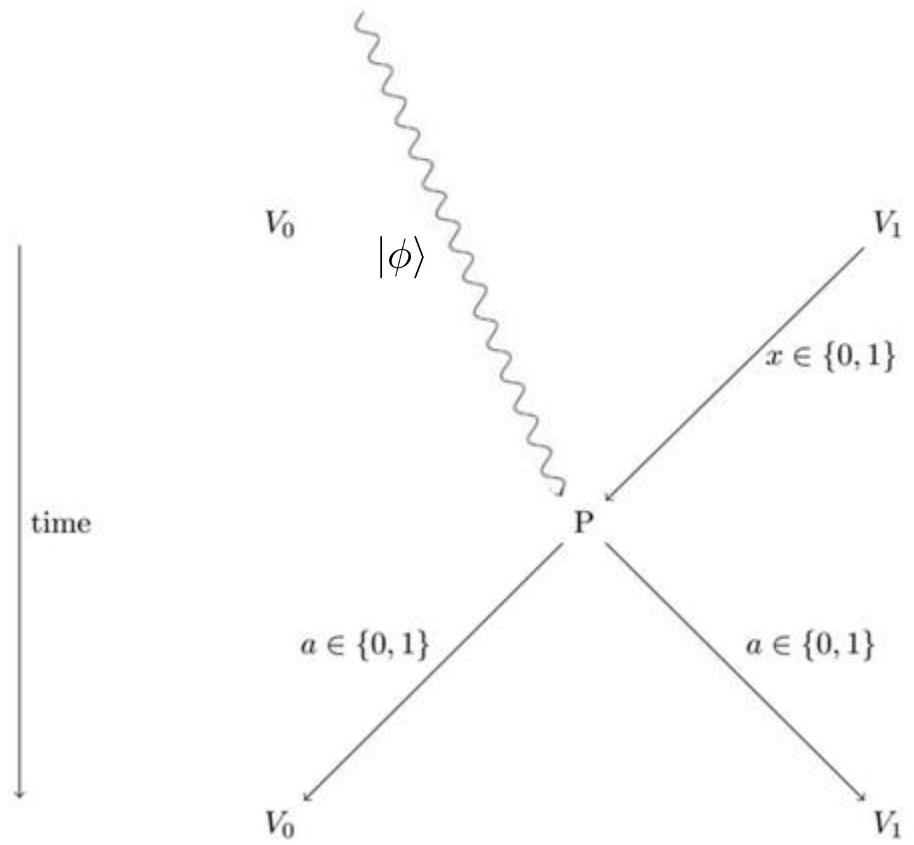

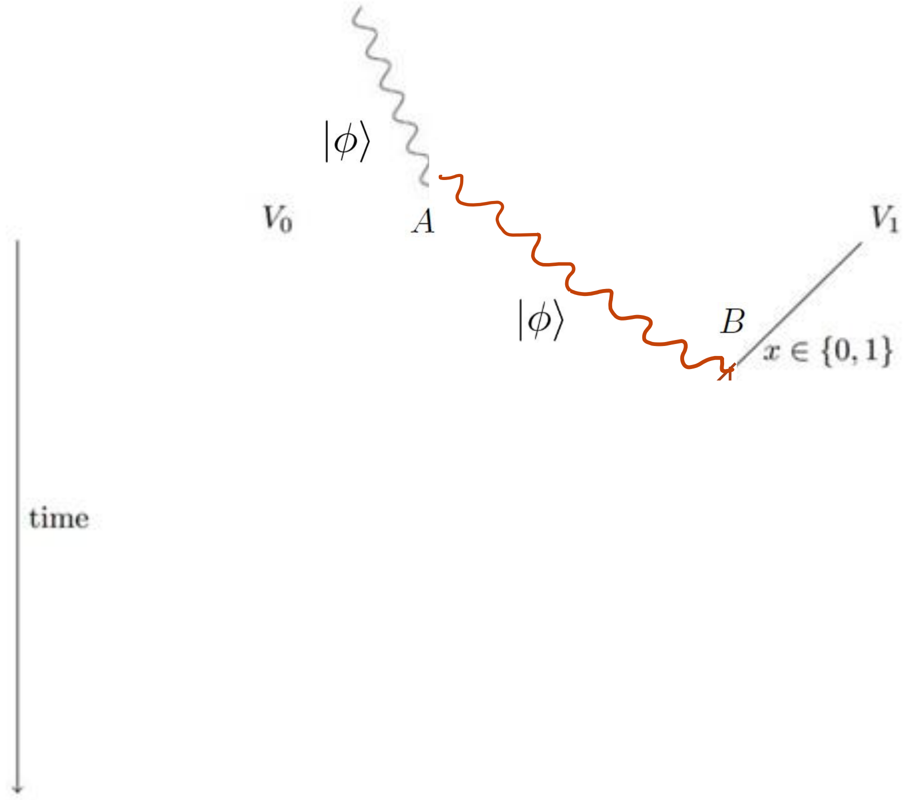

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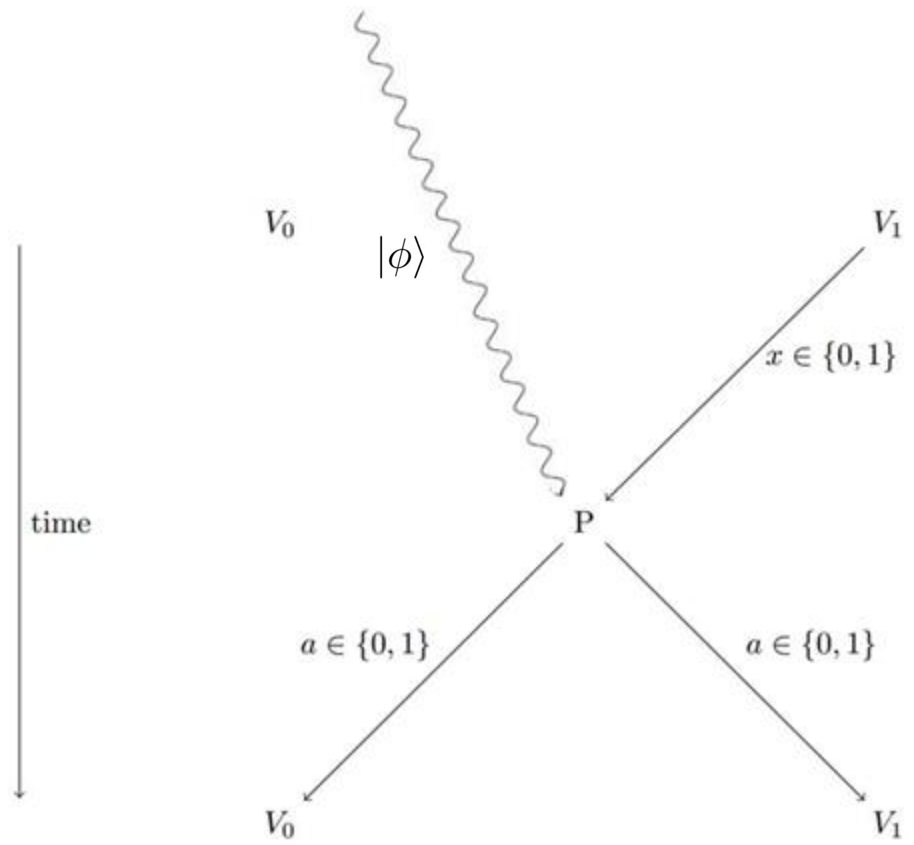

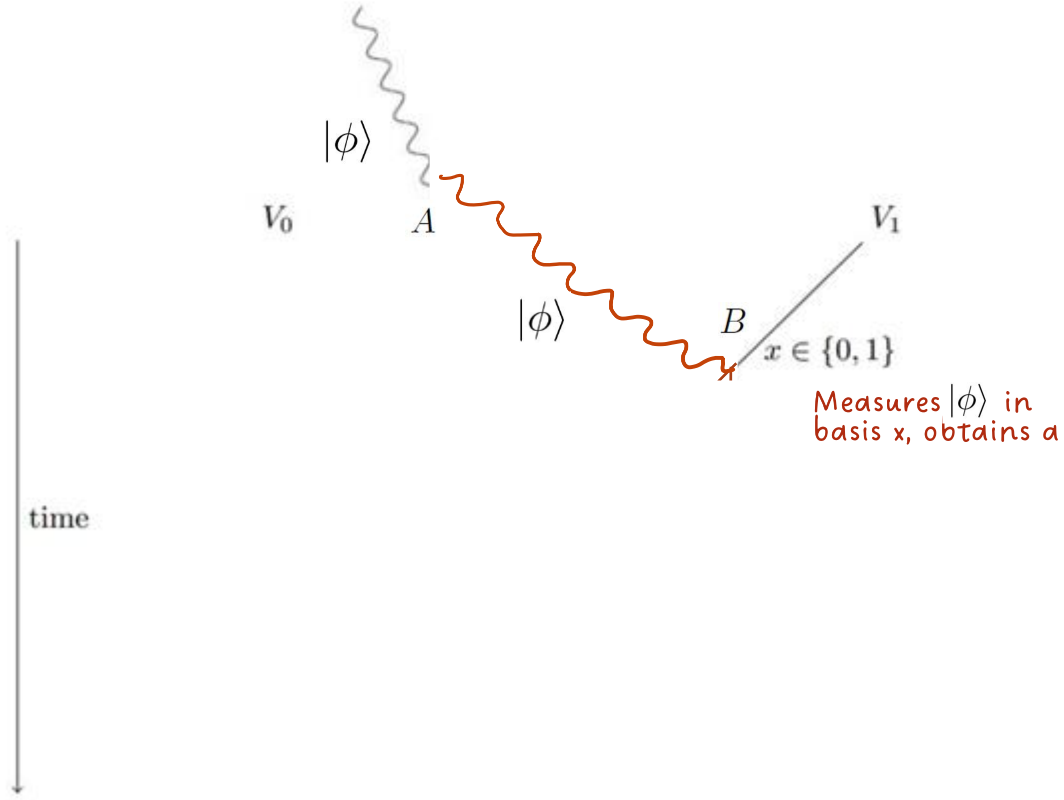

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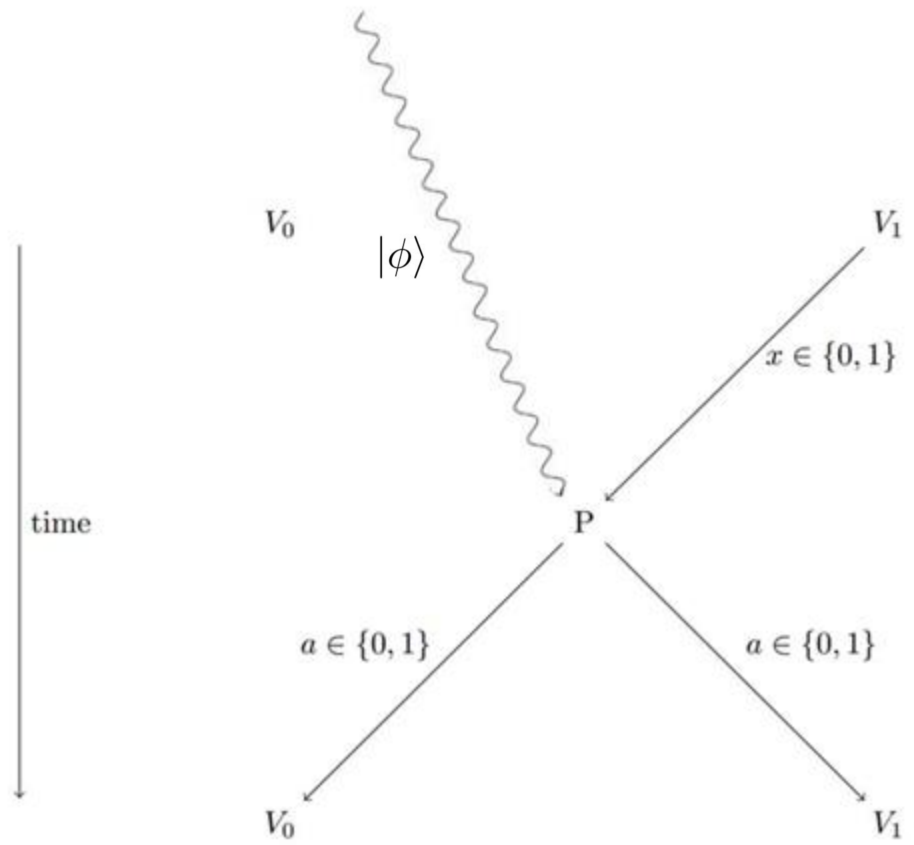

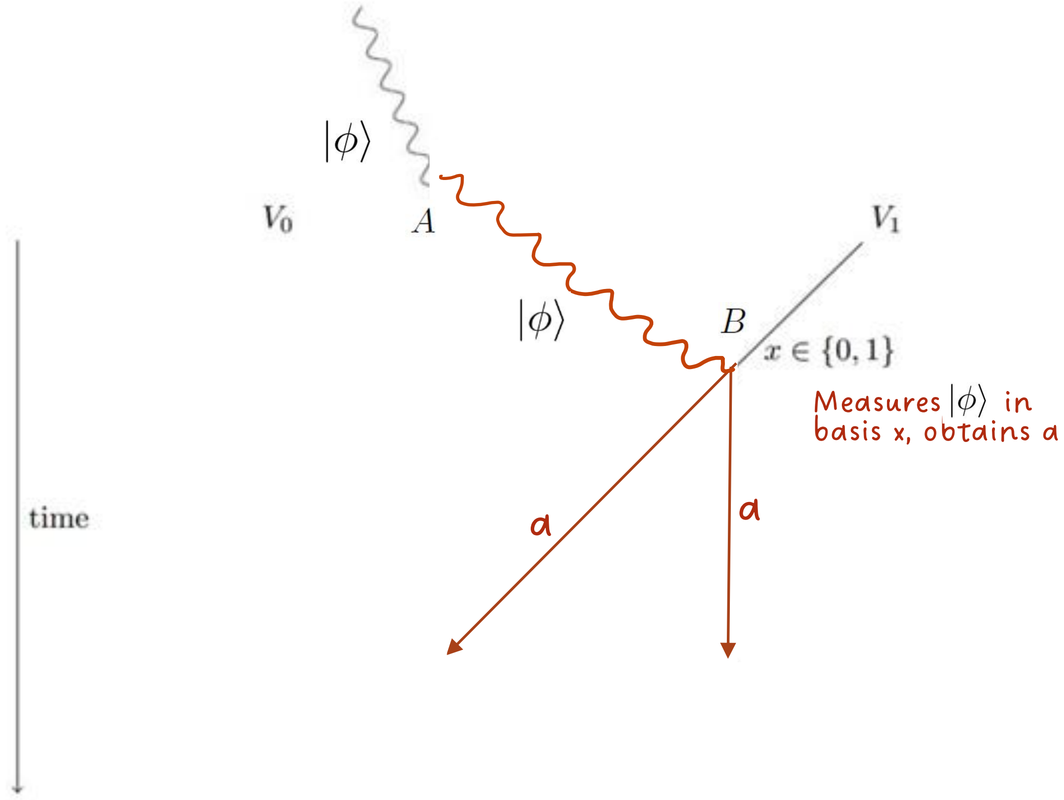

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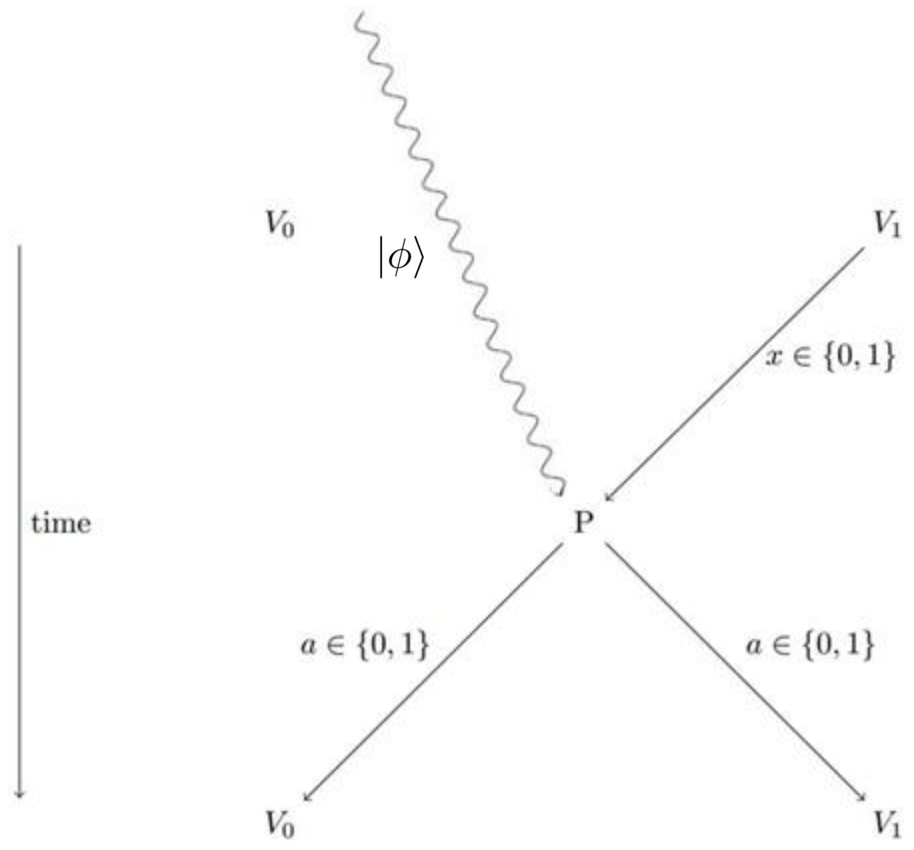

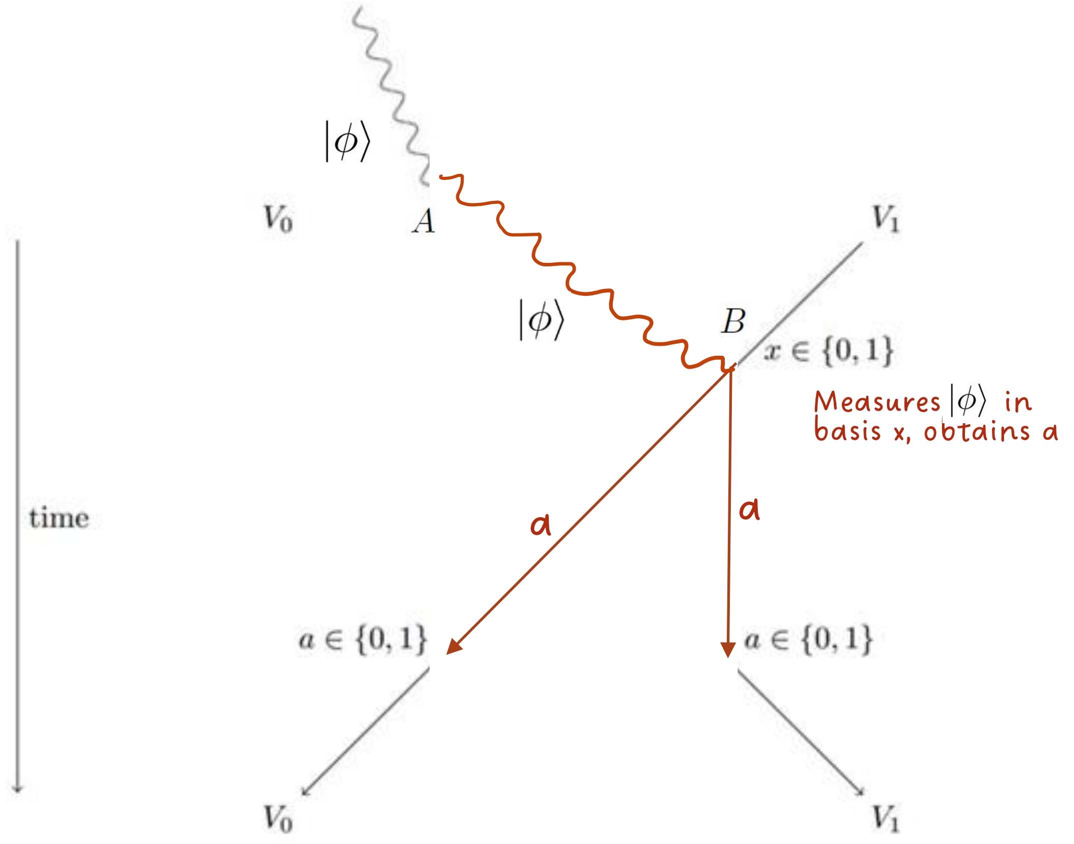


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Photon loss
Slow quantum info
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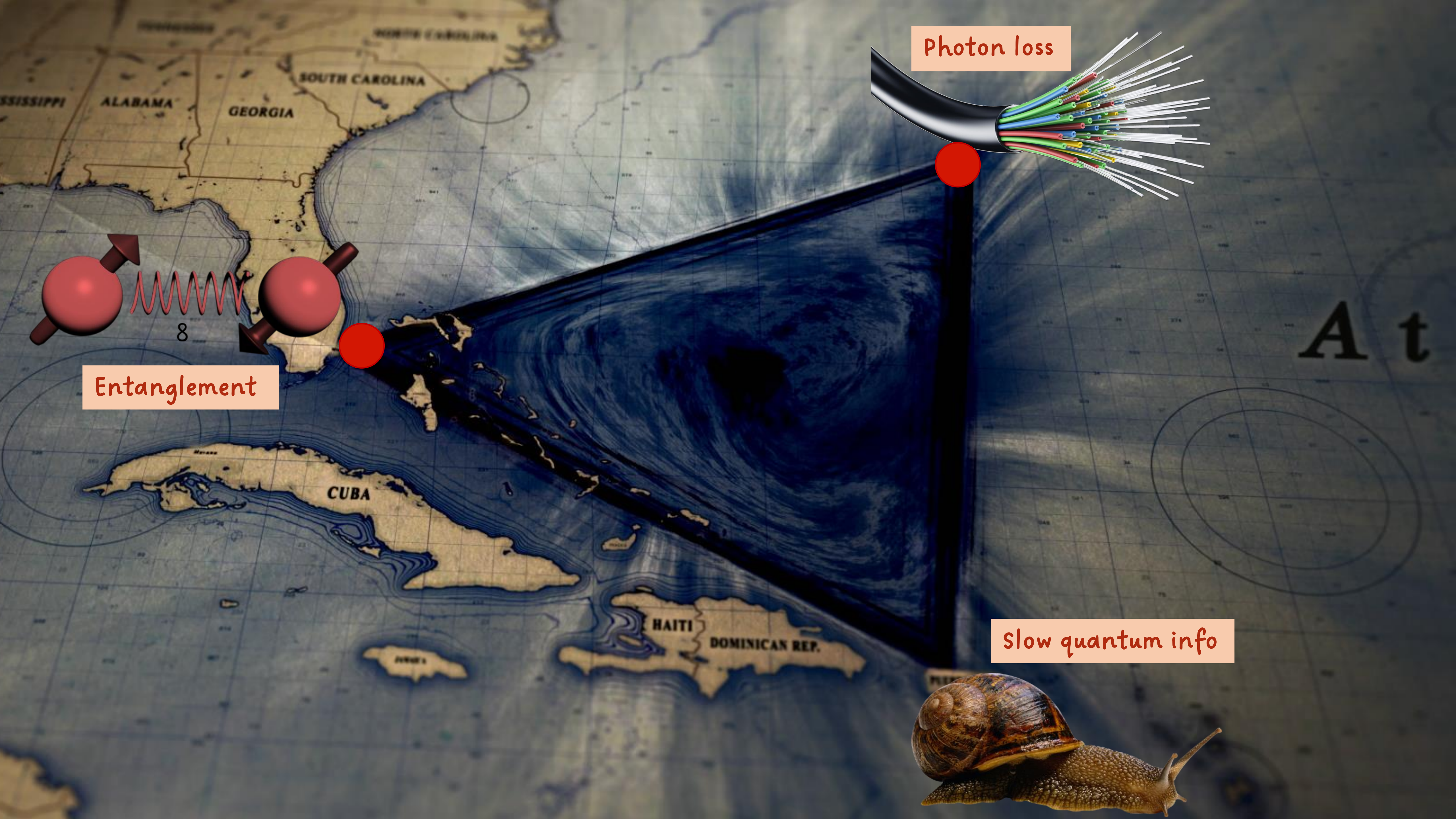
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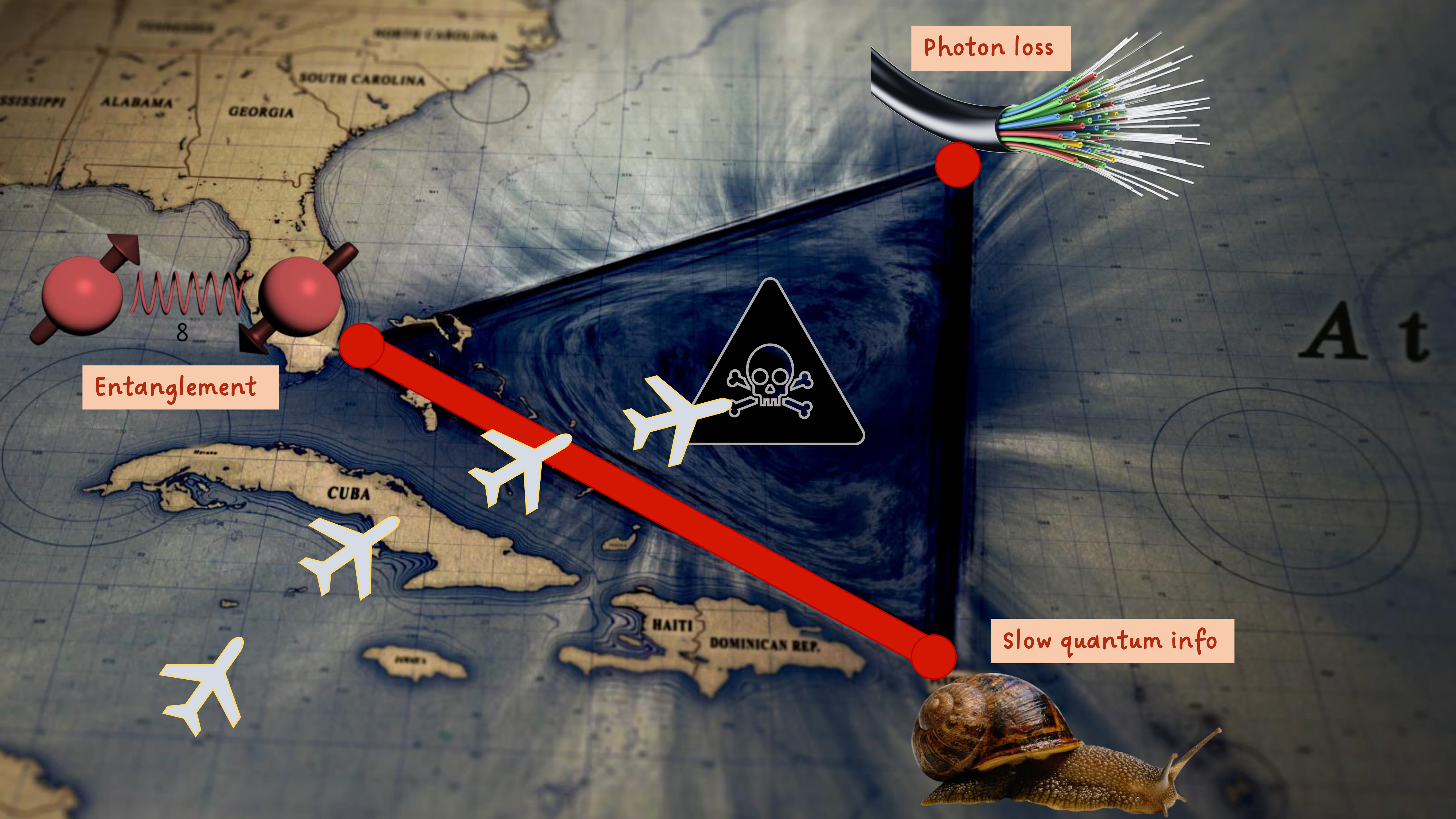
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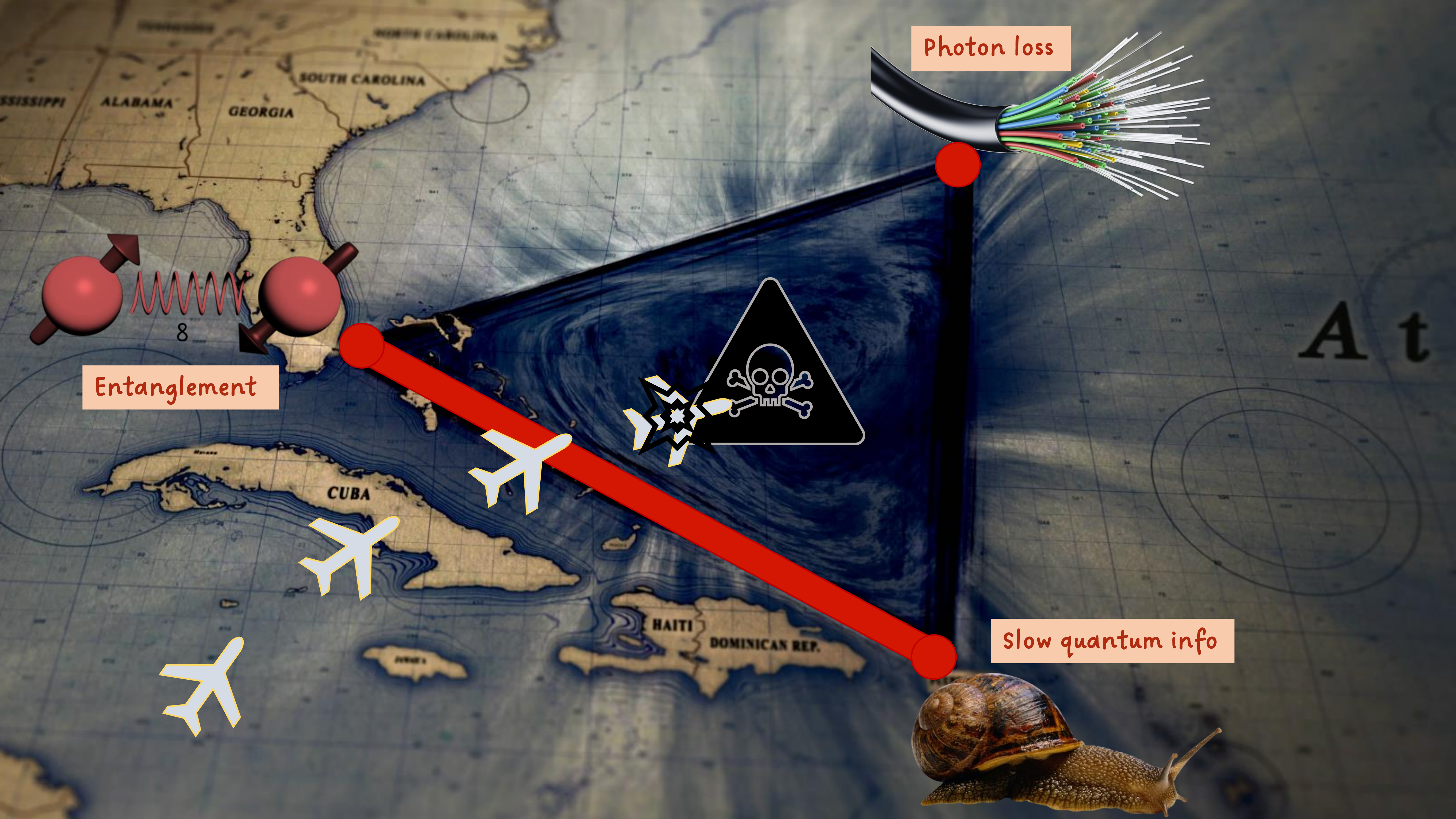
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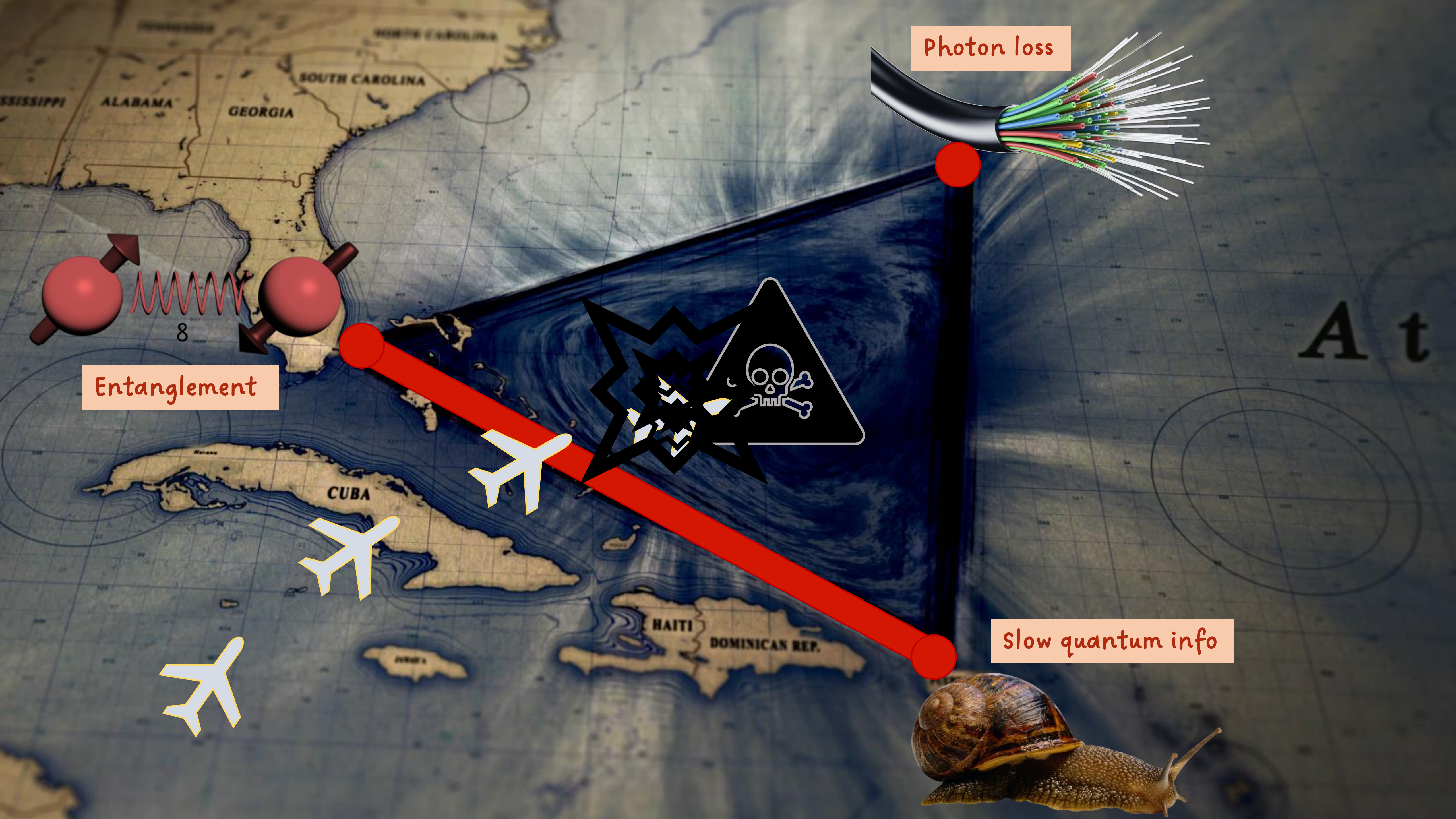

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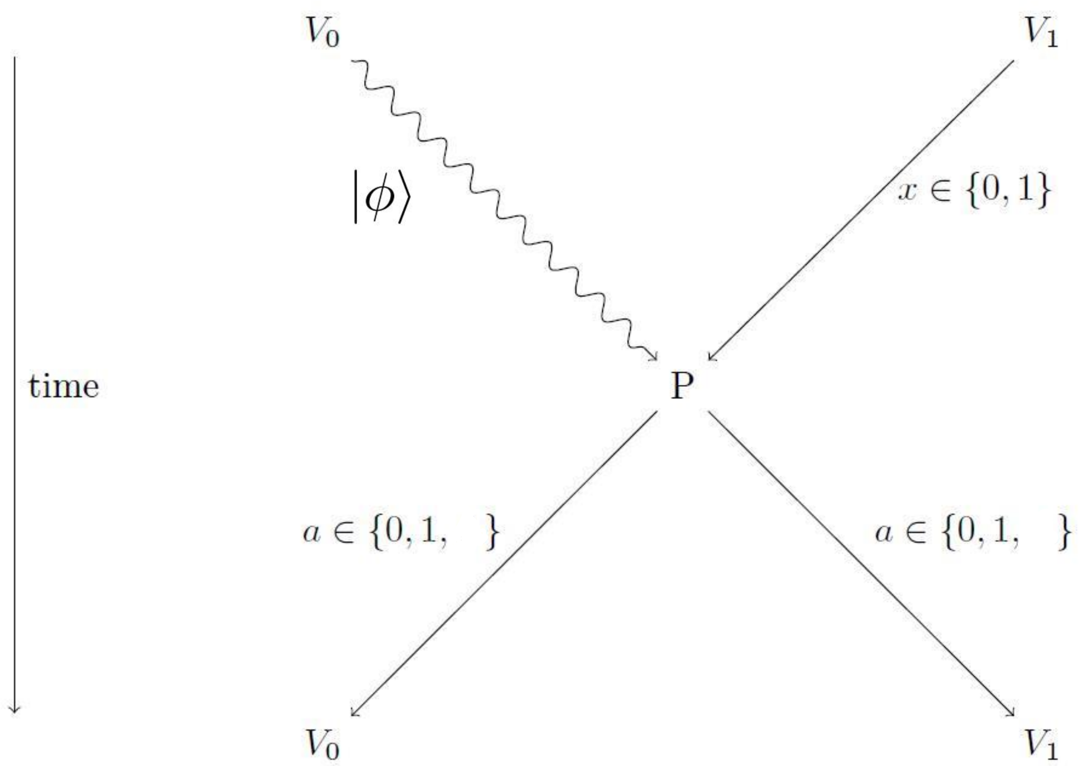
Step 1. Let's analyze the loss

$\mathrm{QPV}^{\eta}_{\mathrm{BB84}}$



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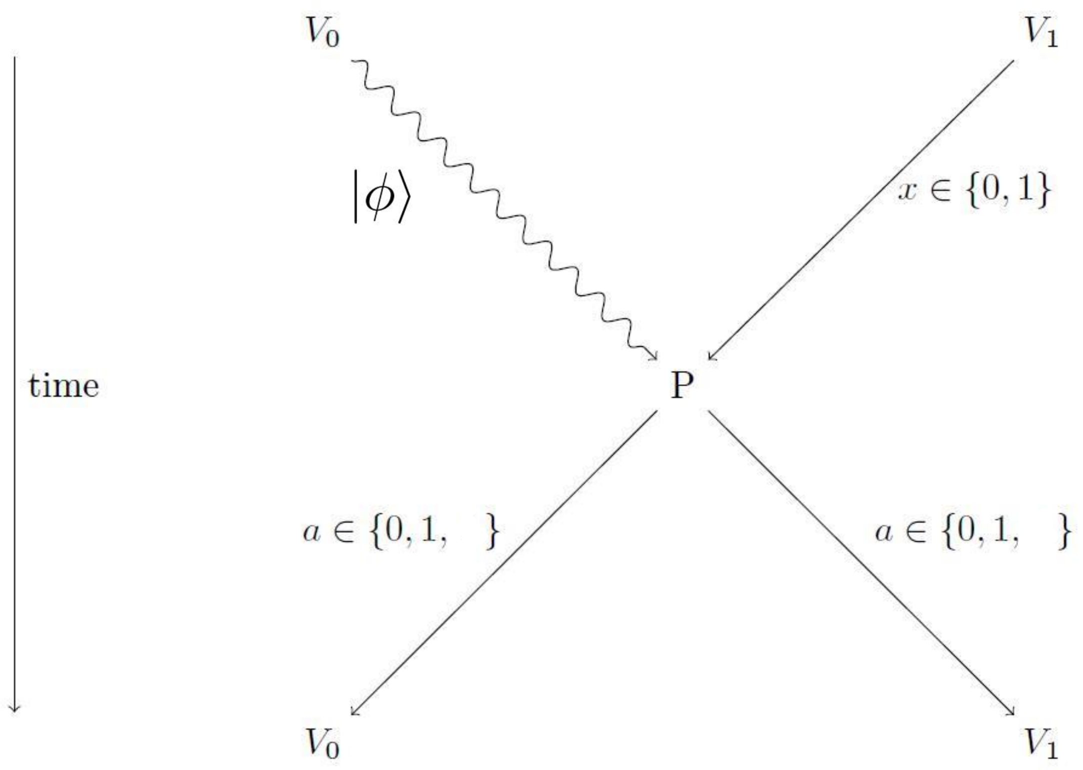

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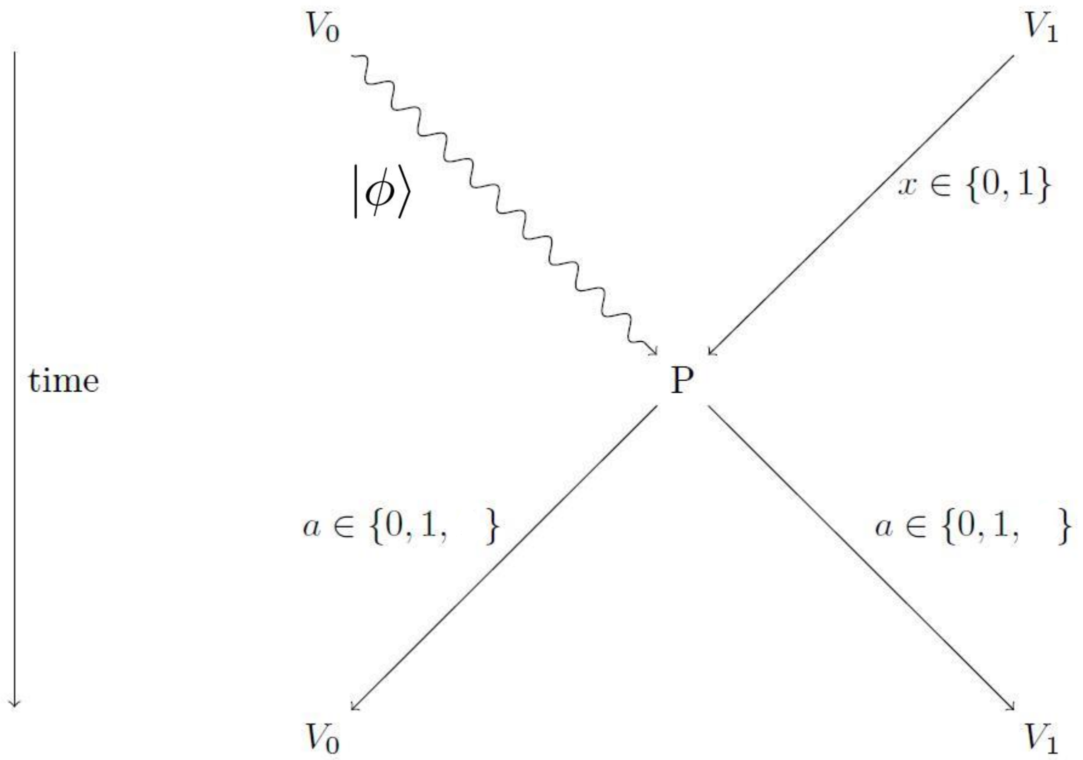

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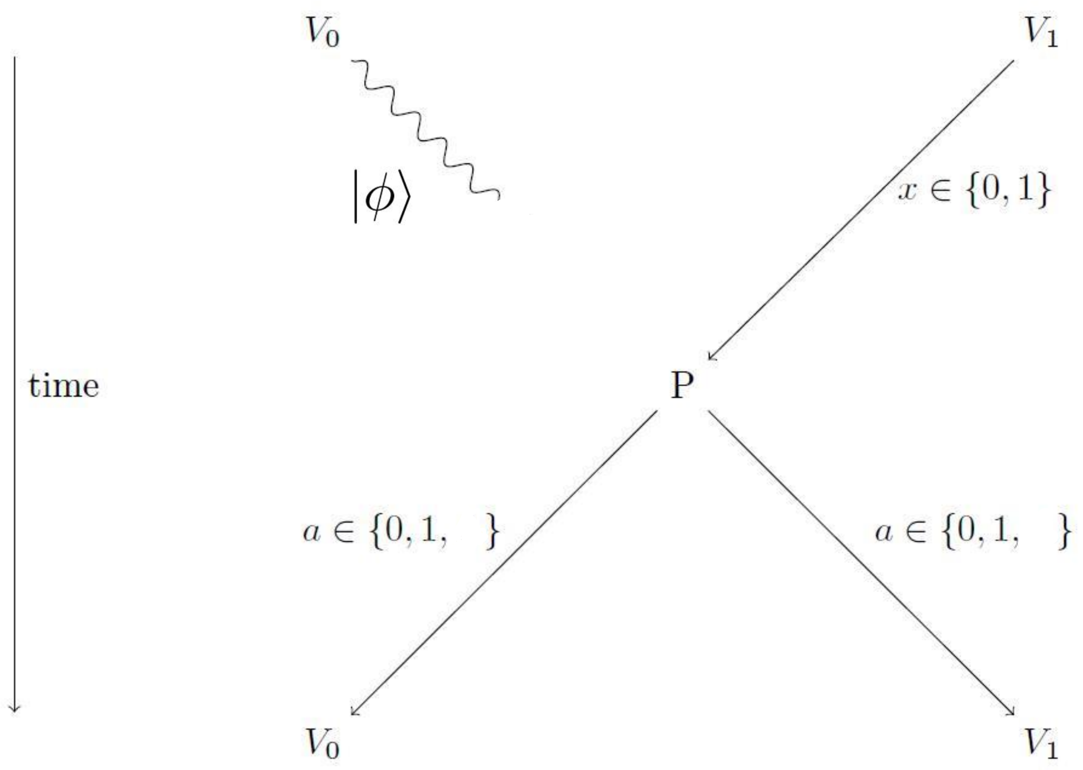

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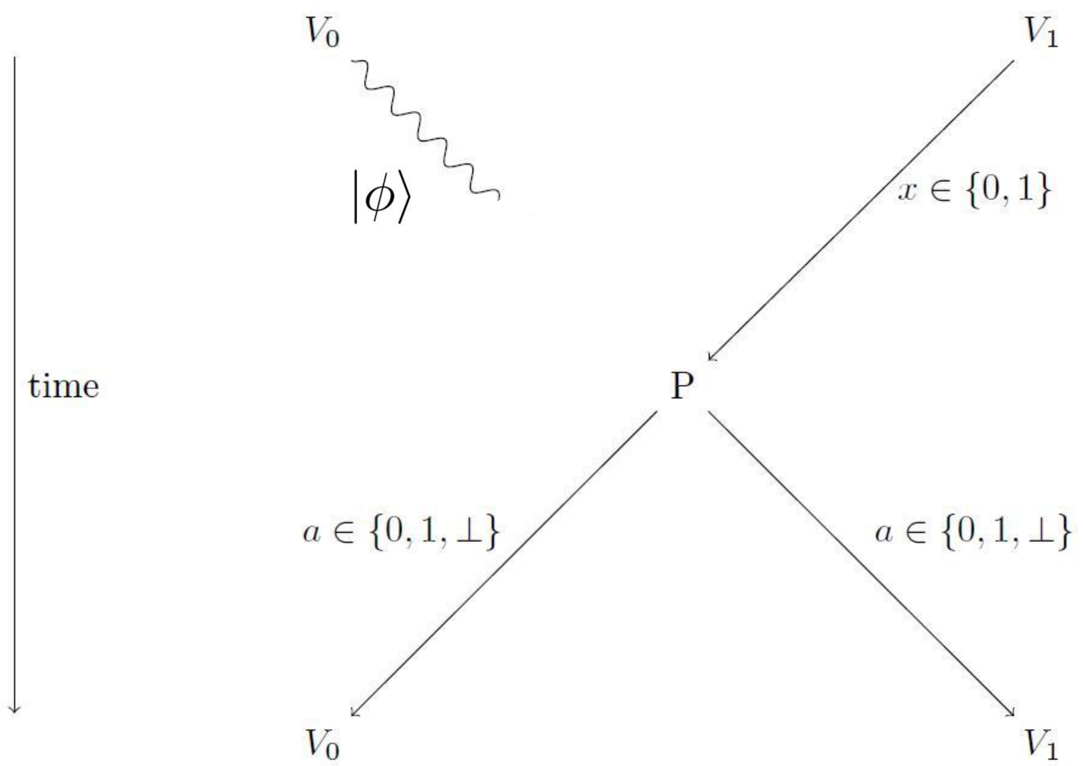

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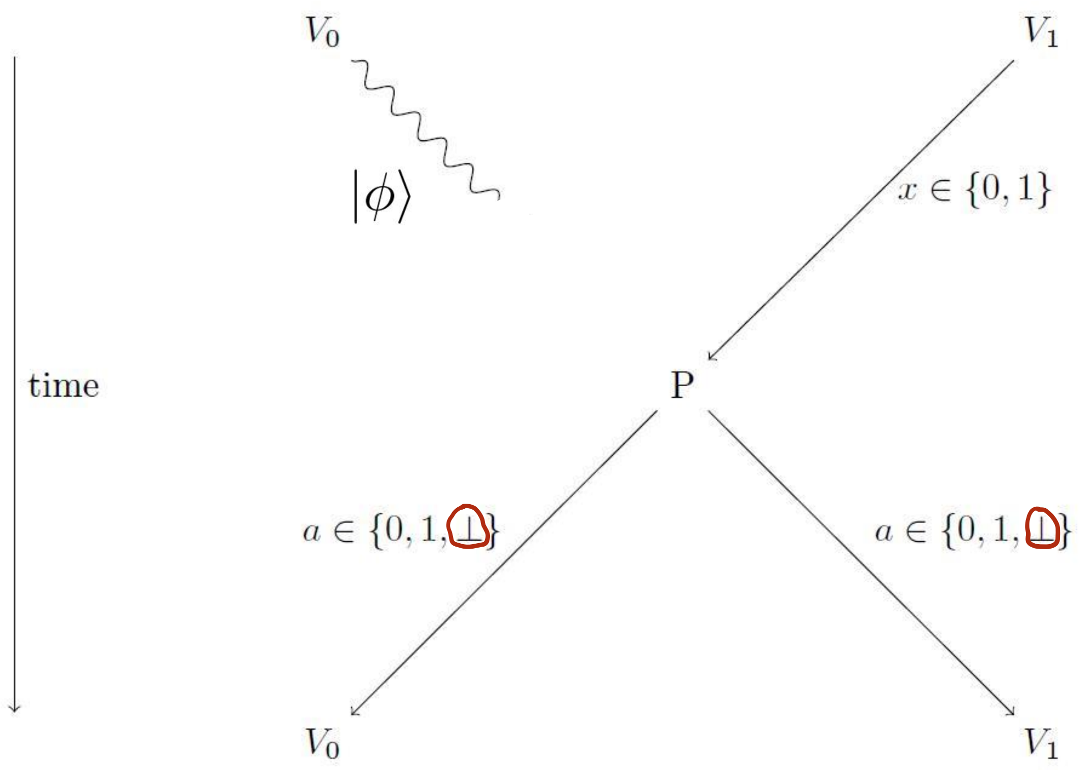

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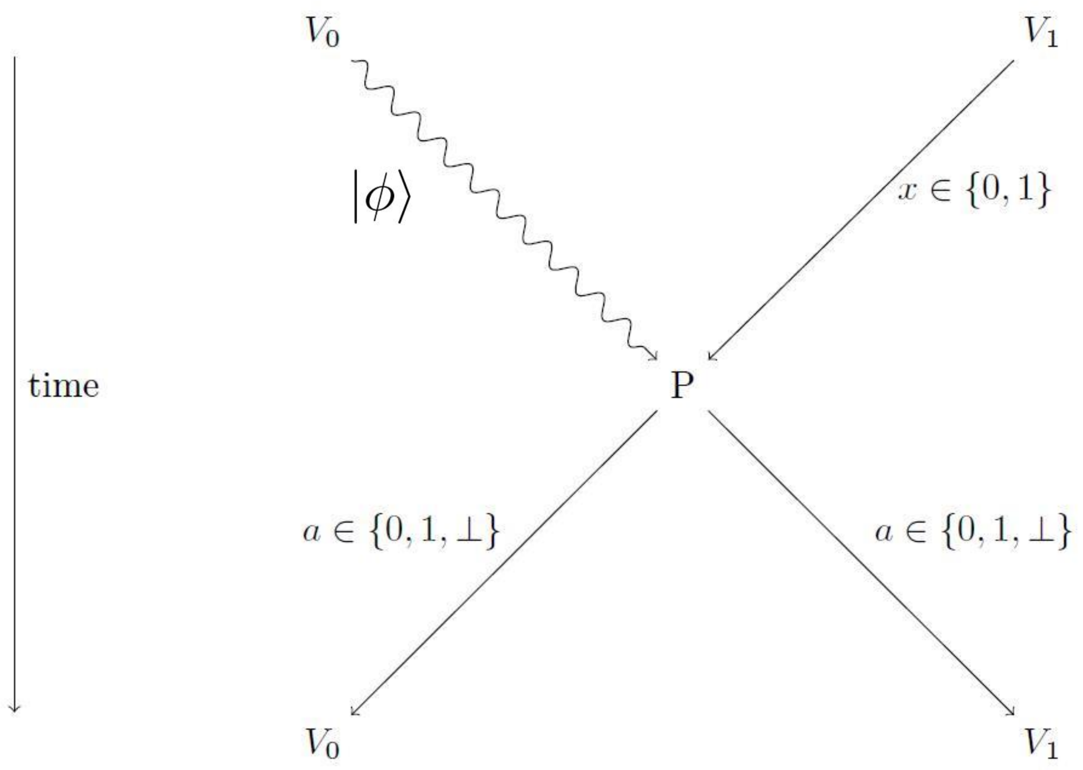

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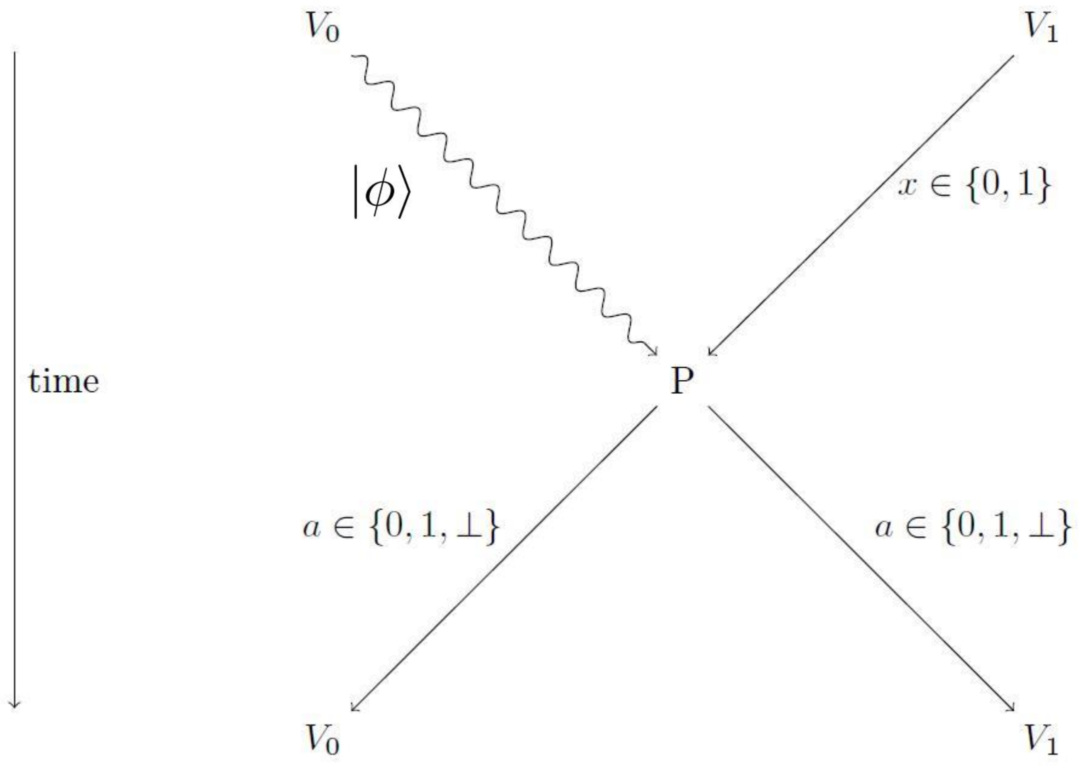
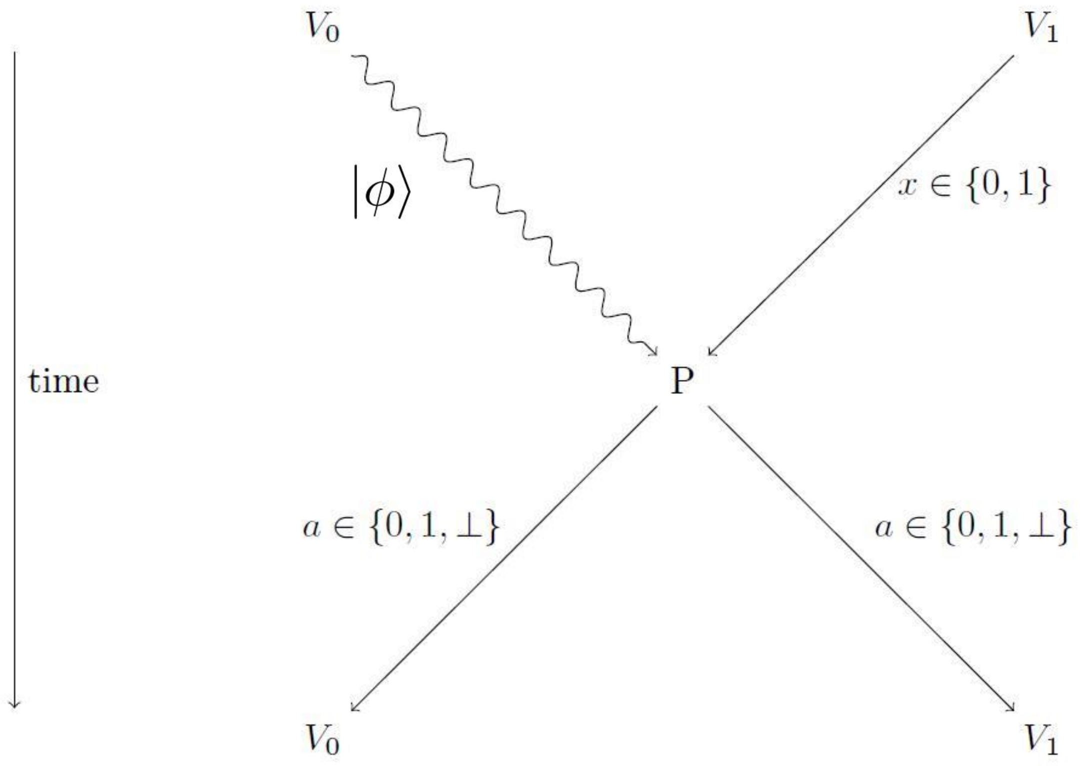


Given an error p_{err},
the prover is going to be correct w.p.

$$\mathrm{QPV}_{\mathrm{BB84}}^{\eta}$$



Given an error p_{err},
the prover is going to be correct w.p.

$$p_{\mathrm{C}} = \eta(1 - p_{err})$$

Security:
unentangled attackers

$$\mathrm{QPV}^{\eta}_{\mathrm{BB84}}$$

Security:
unentangled attackers

$$\mathrm{QPV}^{\eta}_{\mathrm{BB84}}$$

Goal: to upper bound attackers' prob of answering correctly q_{C}

$\mathrm{QPV}^{\eta}_{\mathrm{BB84}}$

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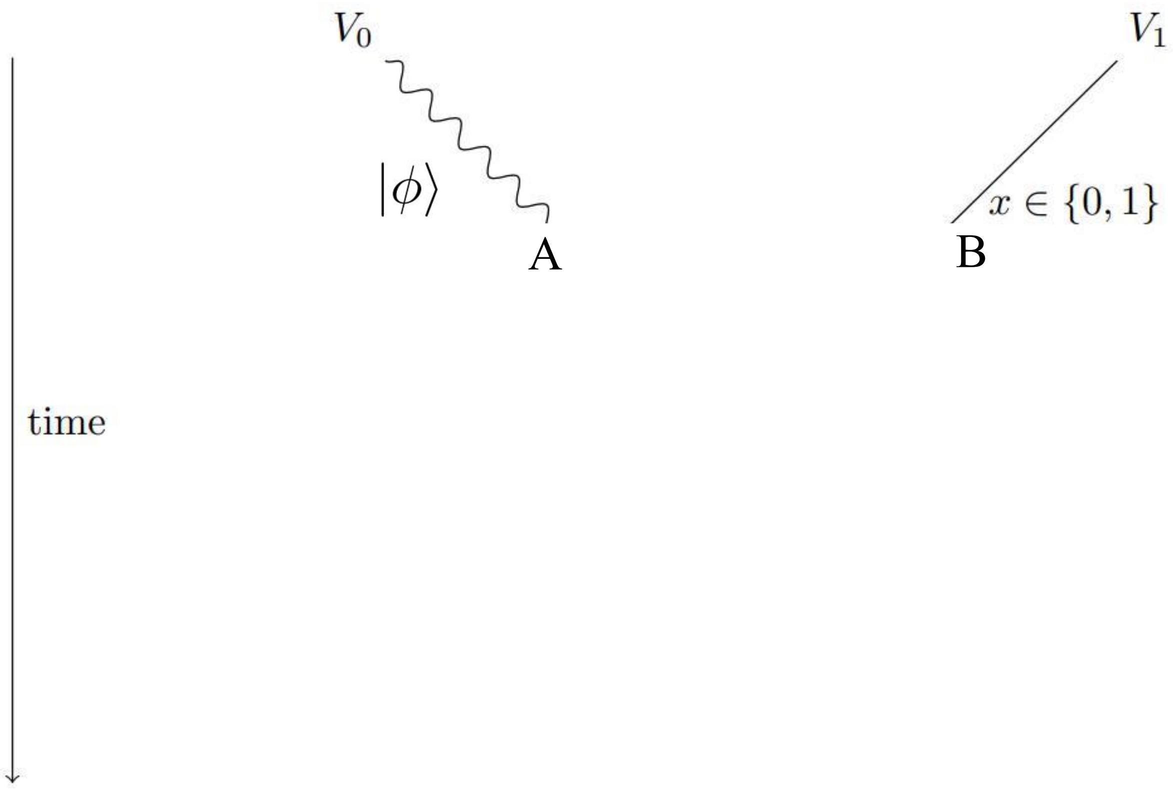
Goal: to upper bound attackers' prob of answering correctly q_{C}



Security:
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$\mathrm{QPV}^{\eta}_{\mathrm{BB84}}$

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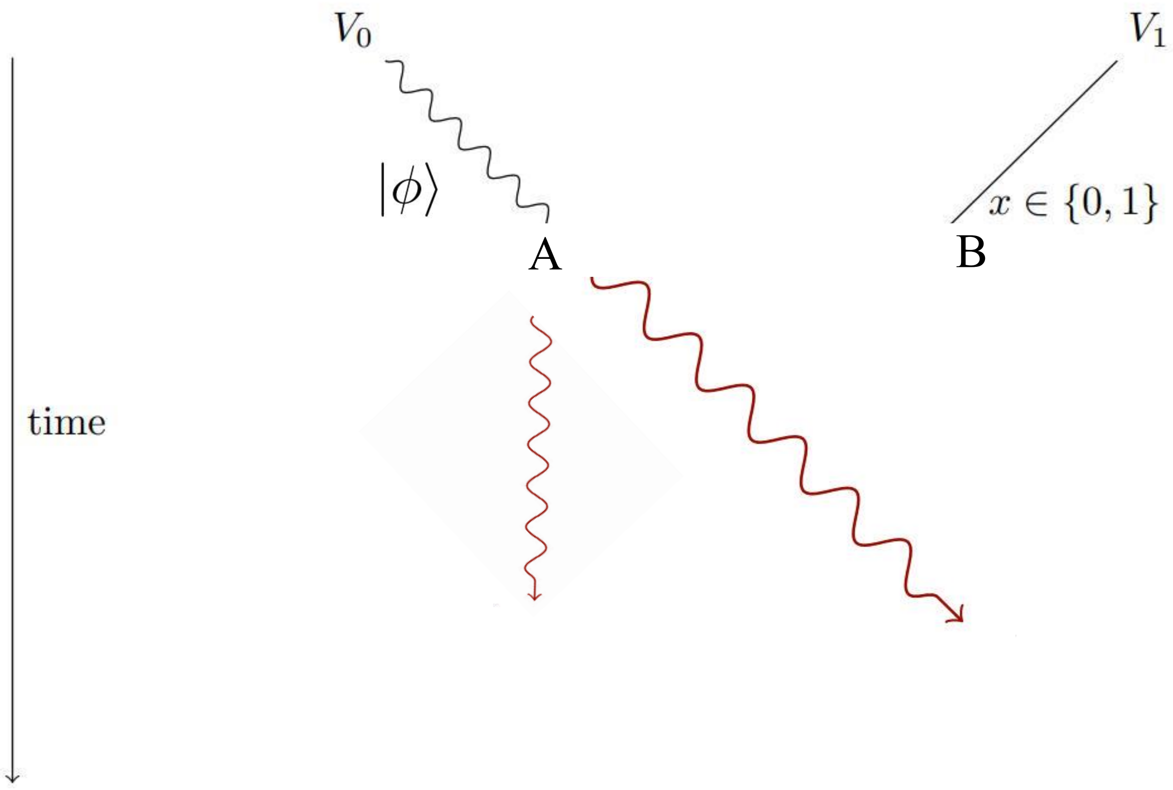

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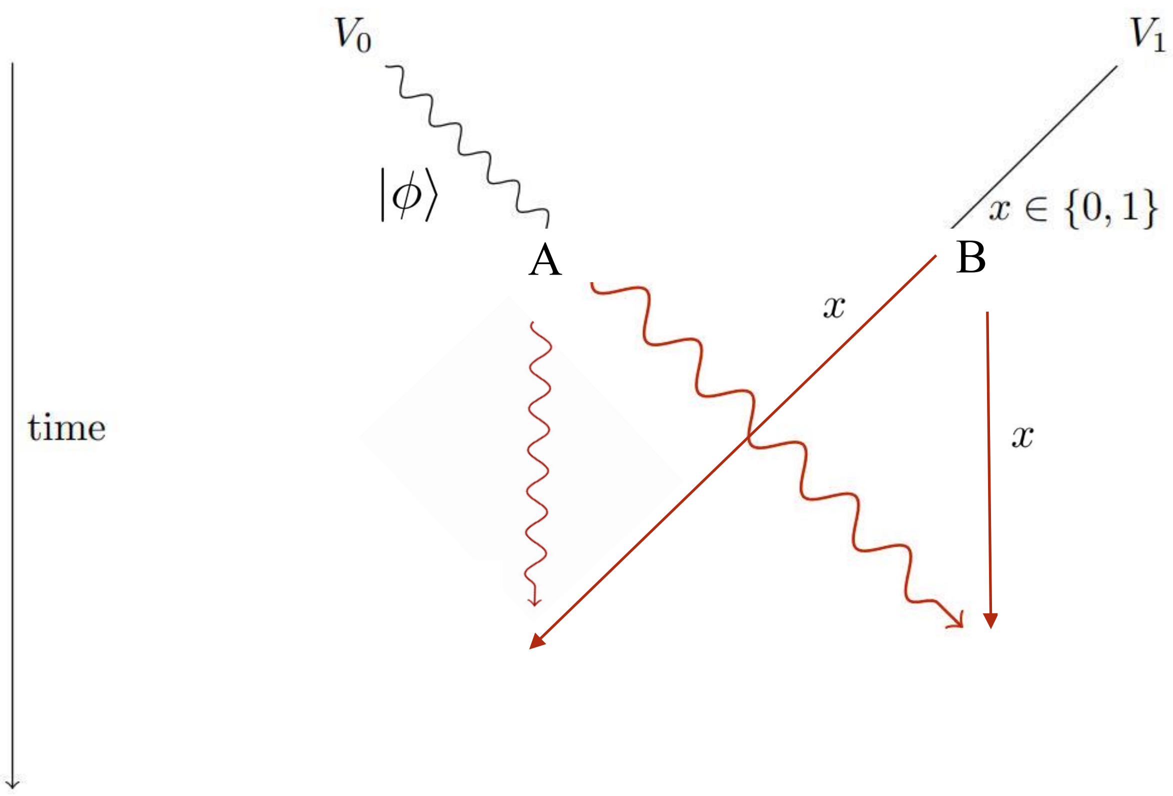

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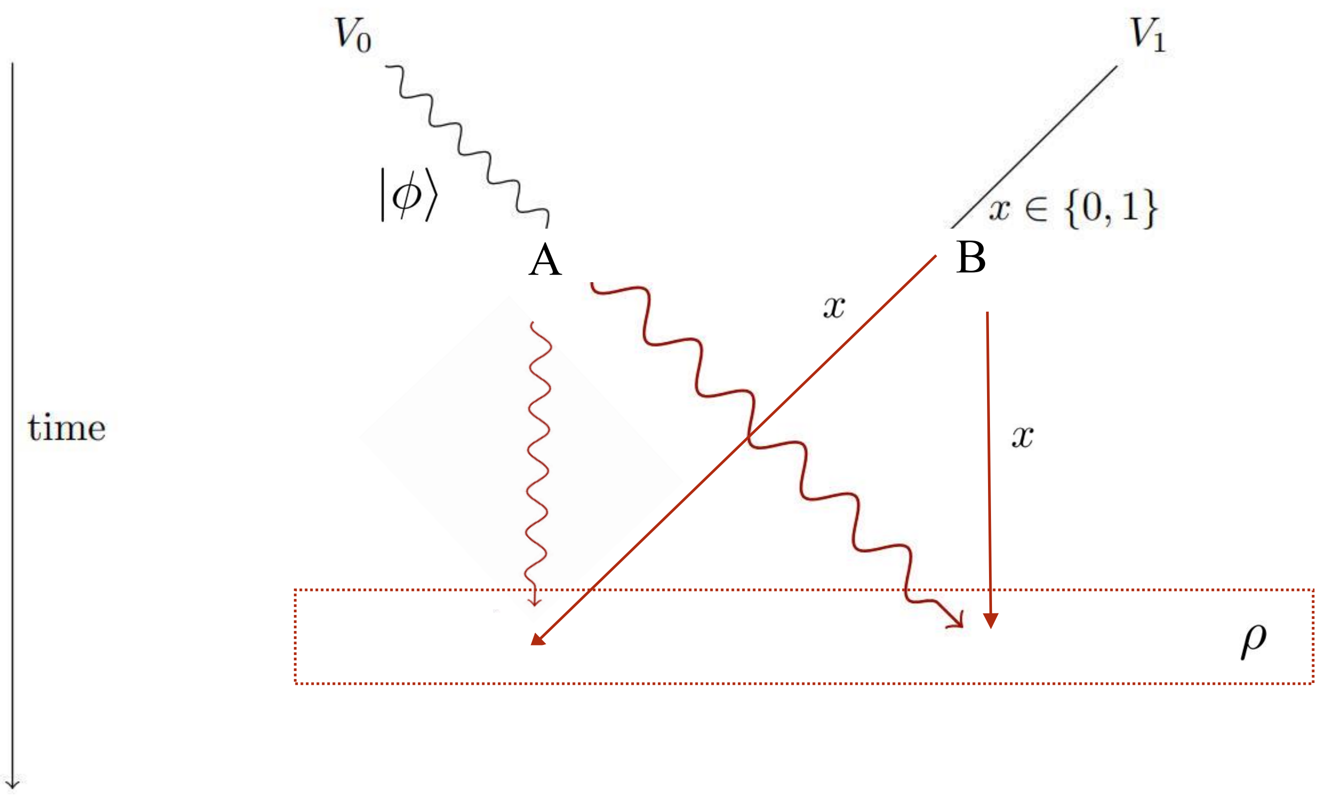

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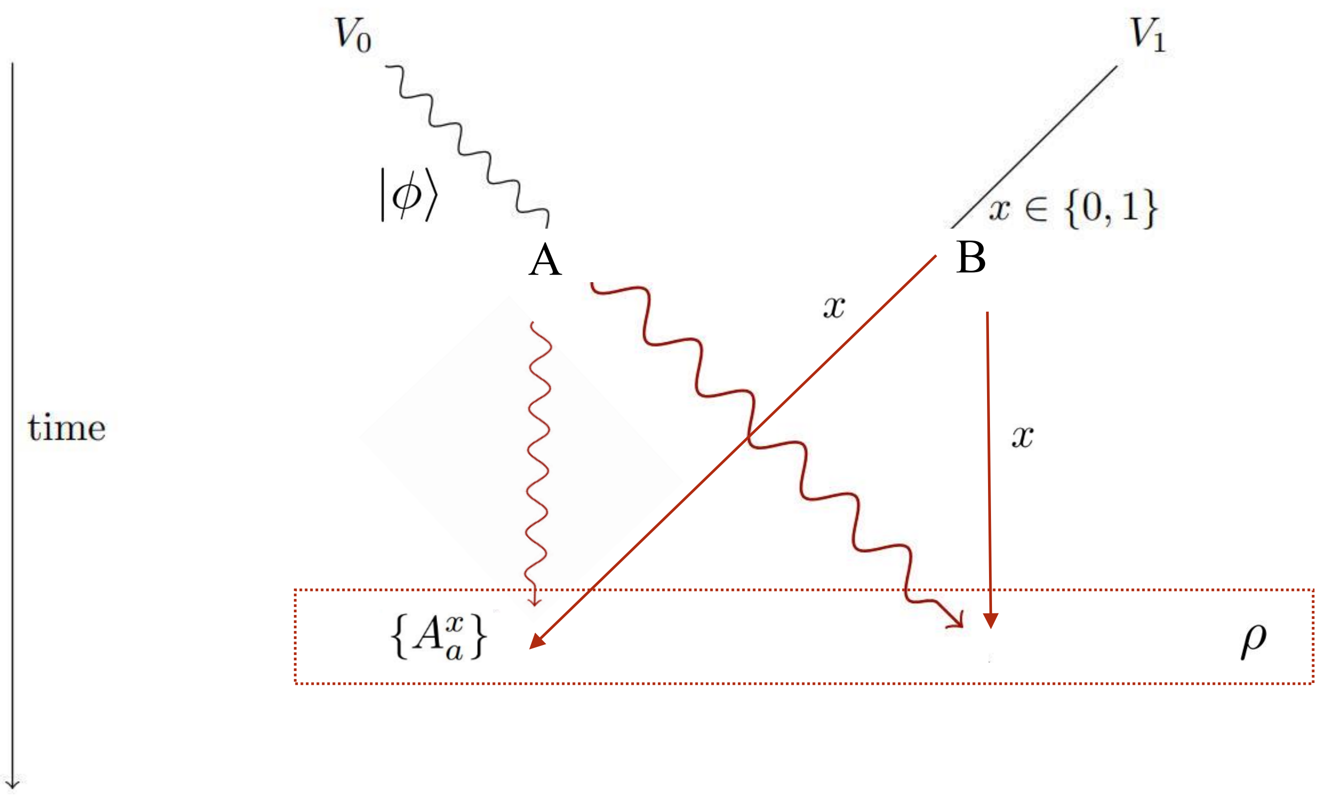

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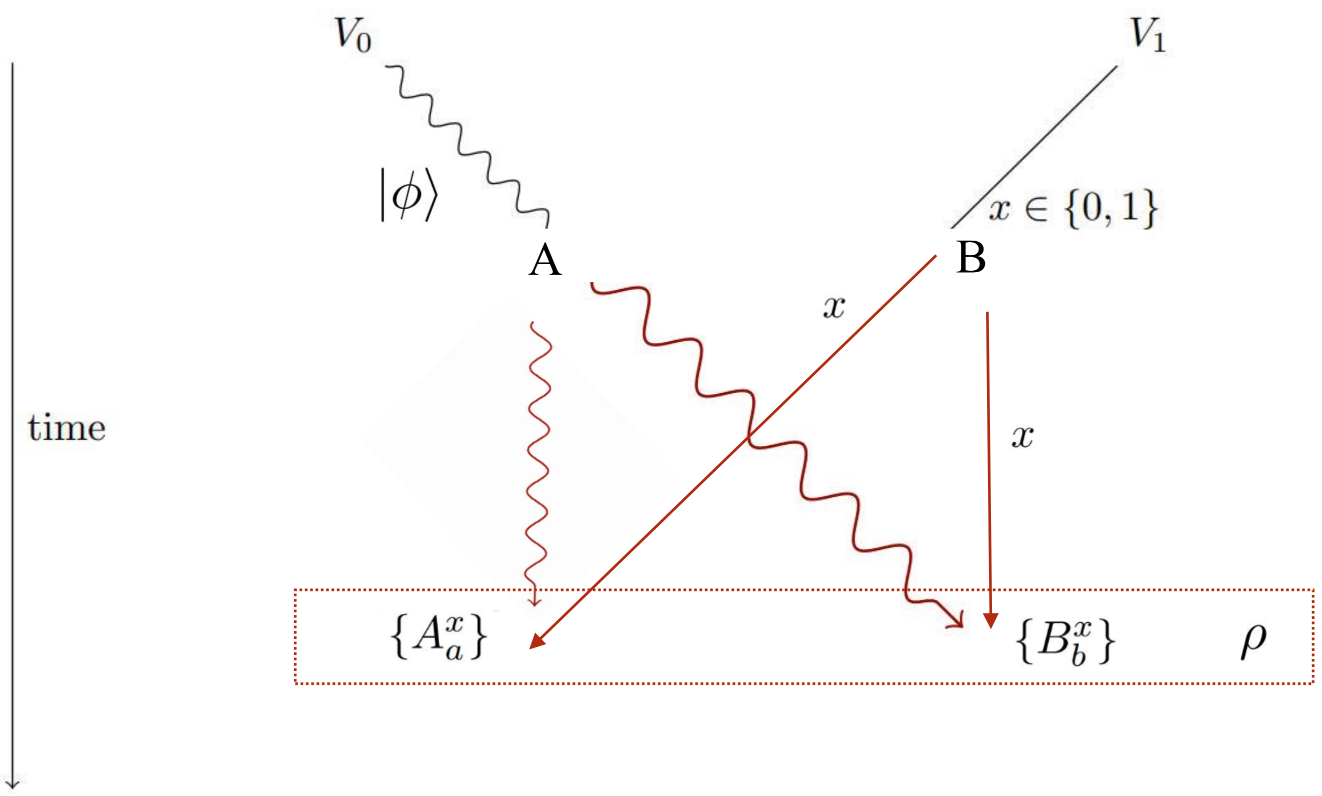

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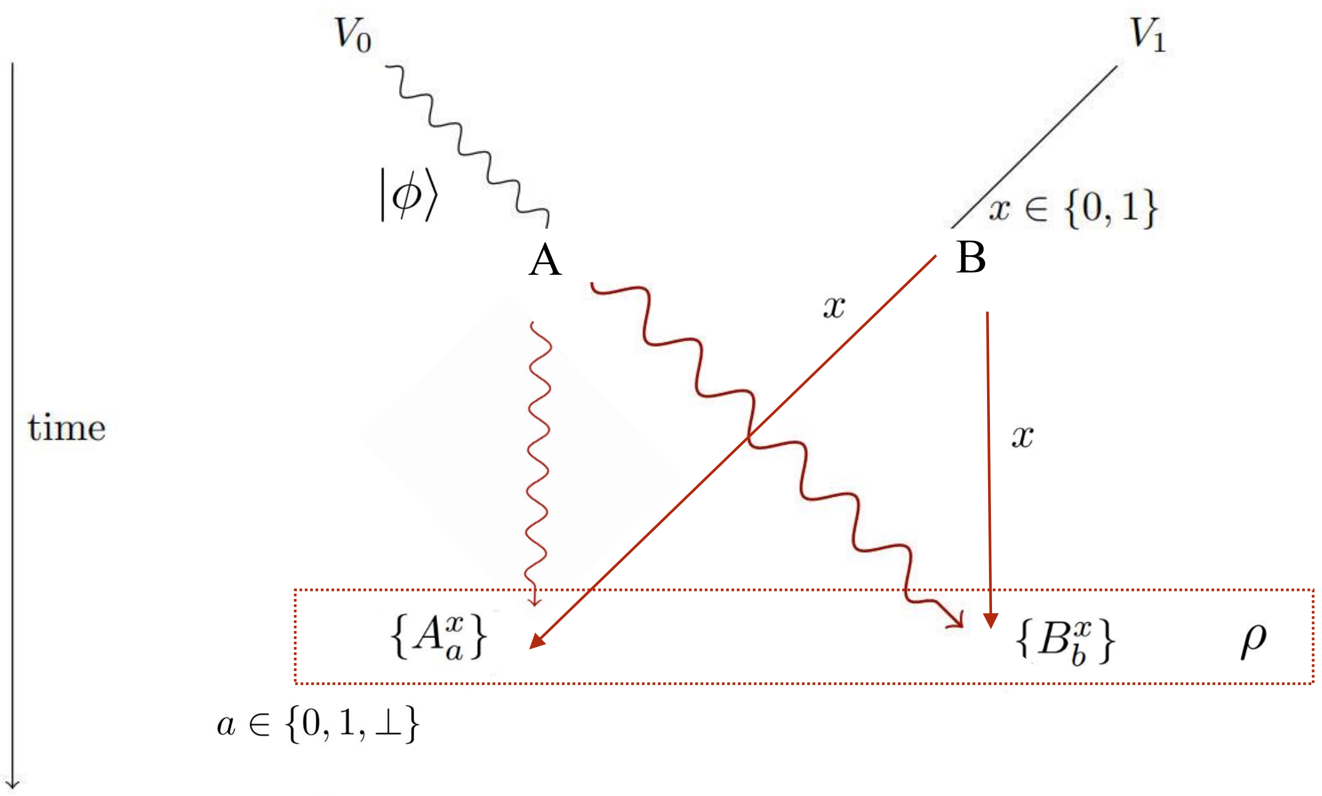

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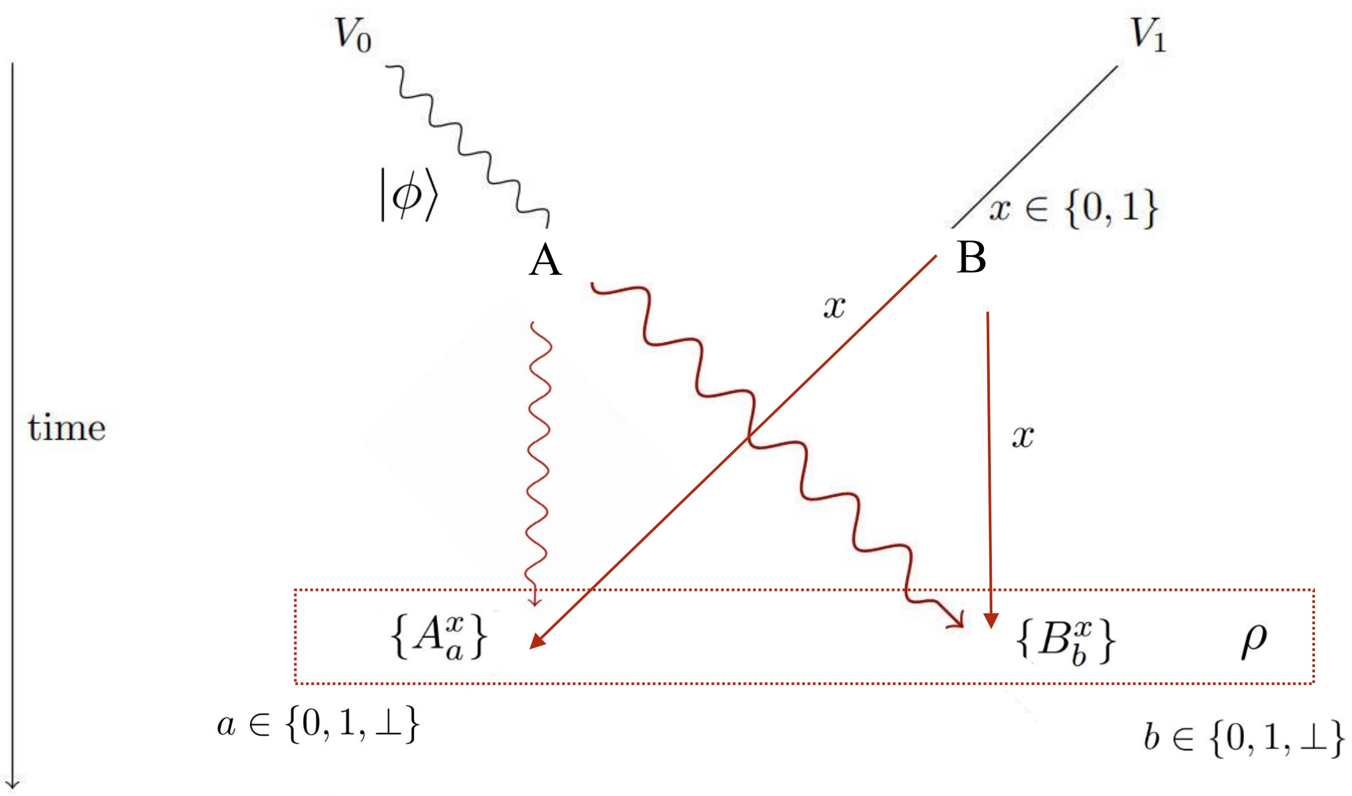

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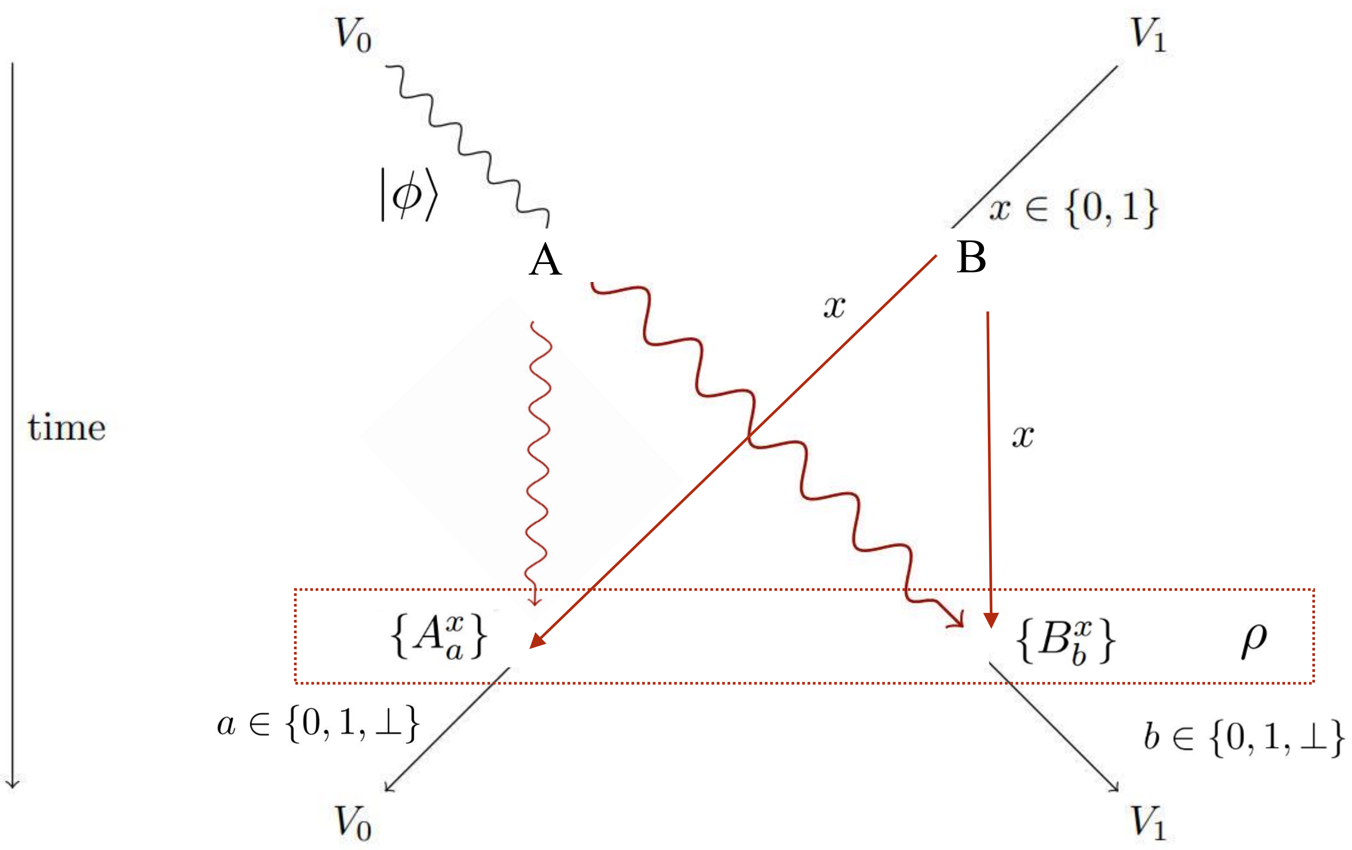

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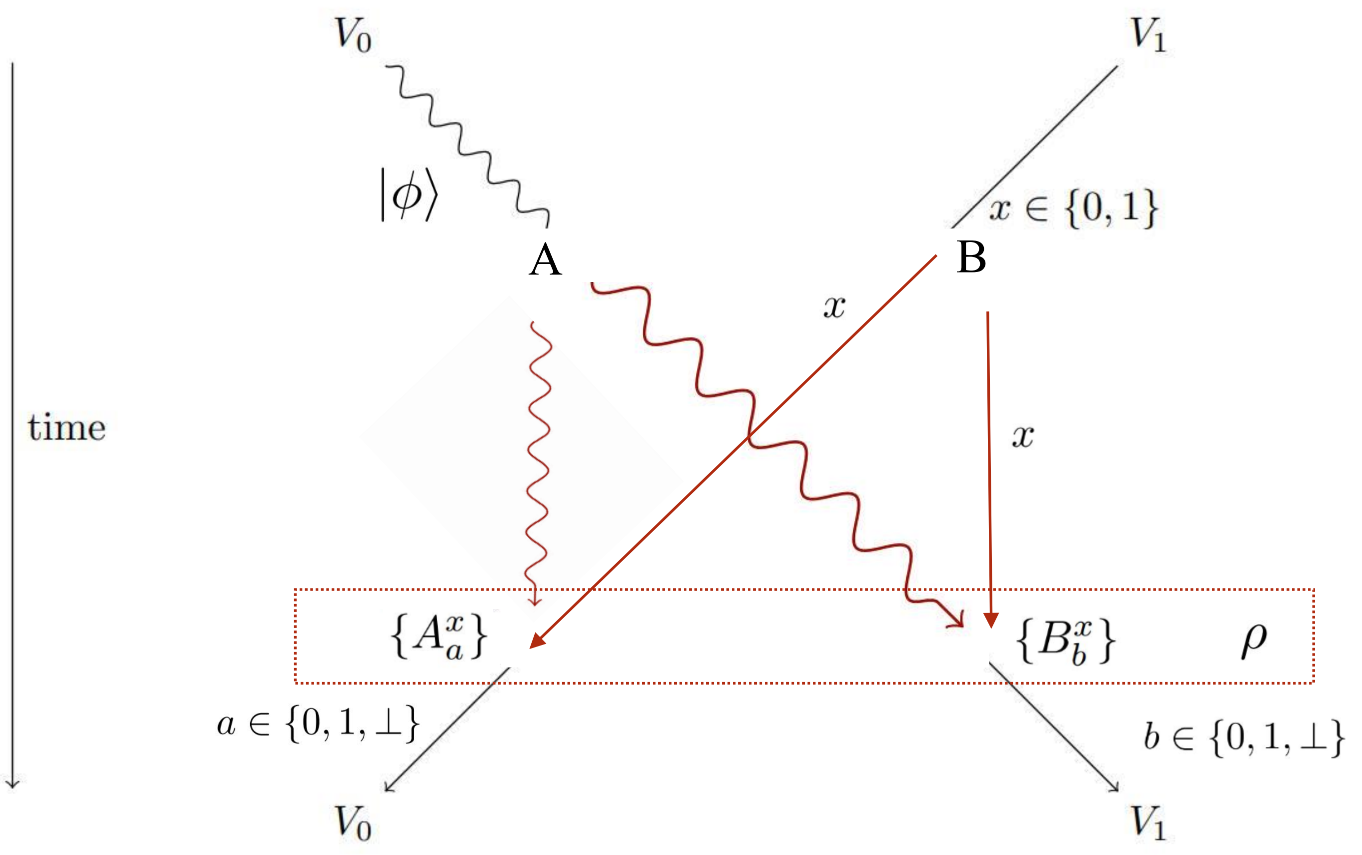
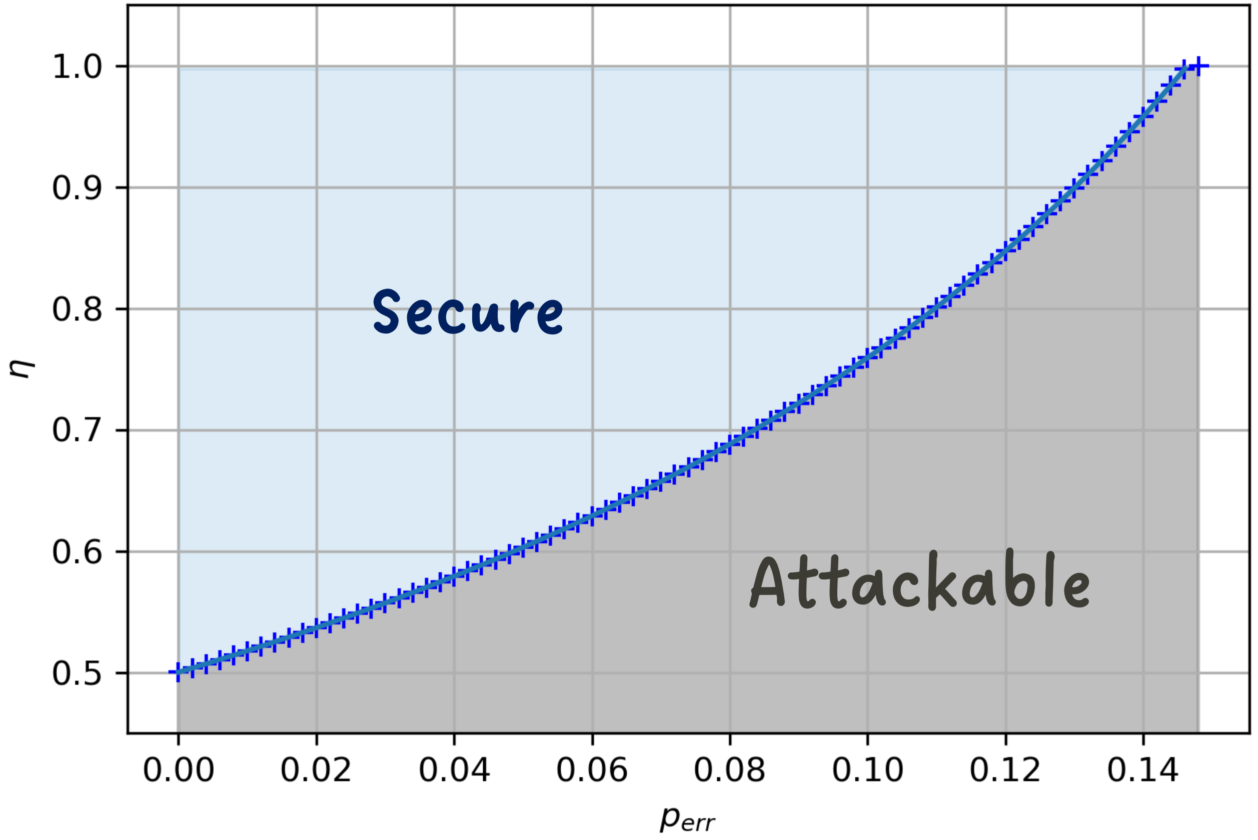


Result $\quad q^*_{\mathrm{C}} = \cos^2\left(\frac{\pi}{8}\right)\eta + \sin^2\left(\frac{\pi}{8}\right)(1-\eta) \qquad \forall\ \eta \in [\frac{1}{2}, 1]$

In experimental parameters, the result translates to



But still insecure if the attackers pre-share one EPR pair

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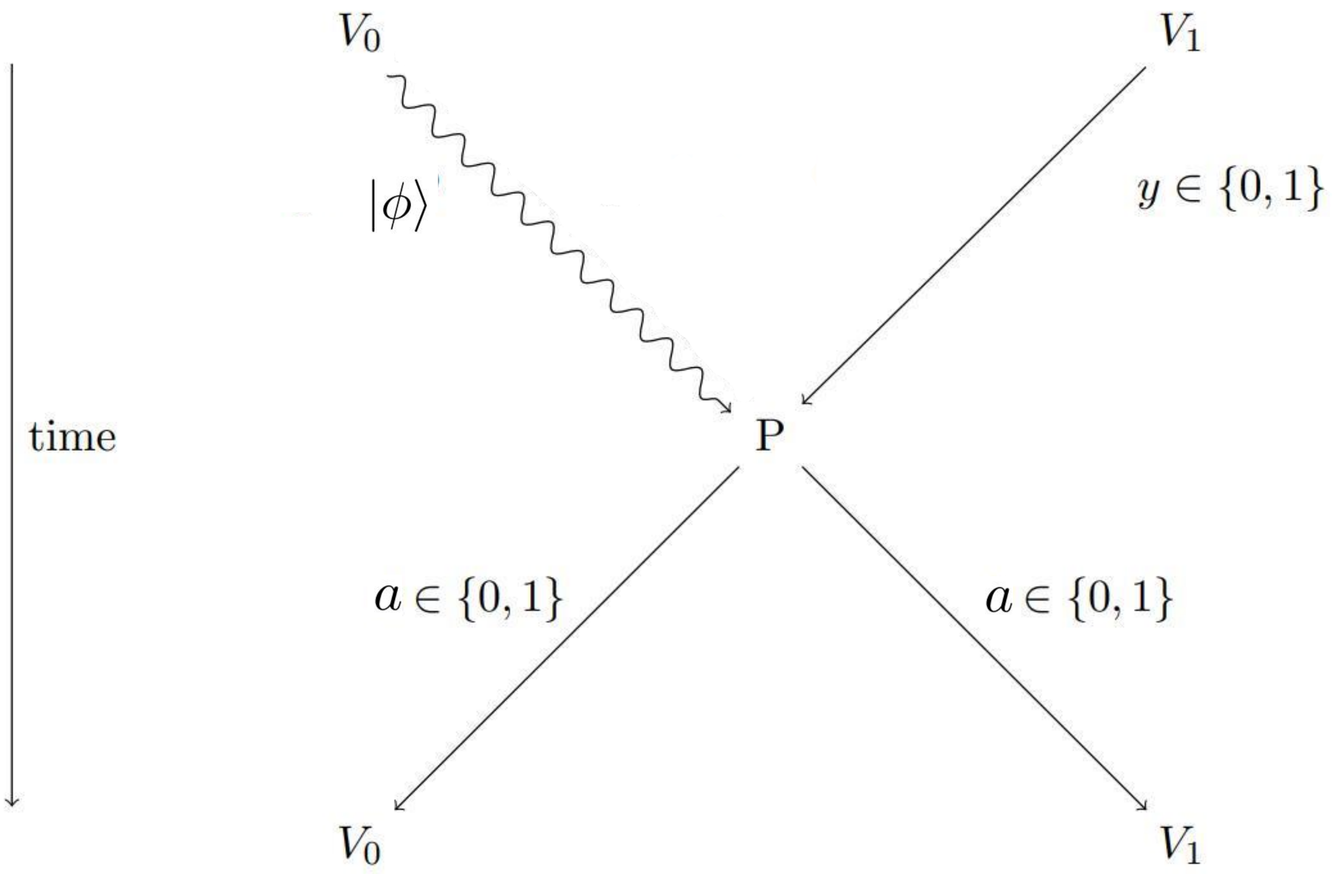
Step 2. Using Step 1 to fix it

$\mathrm{QPV}_{\mathrm{BB84}}^{,f}$



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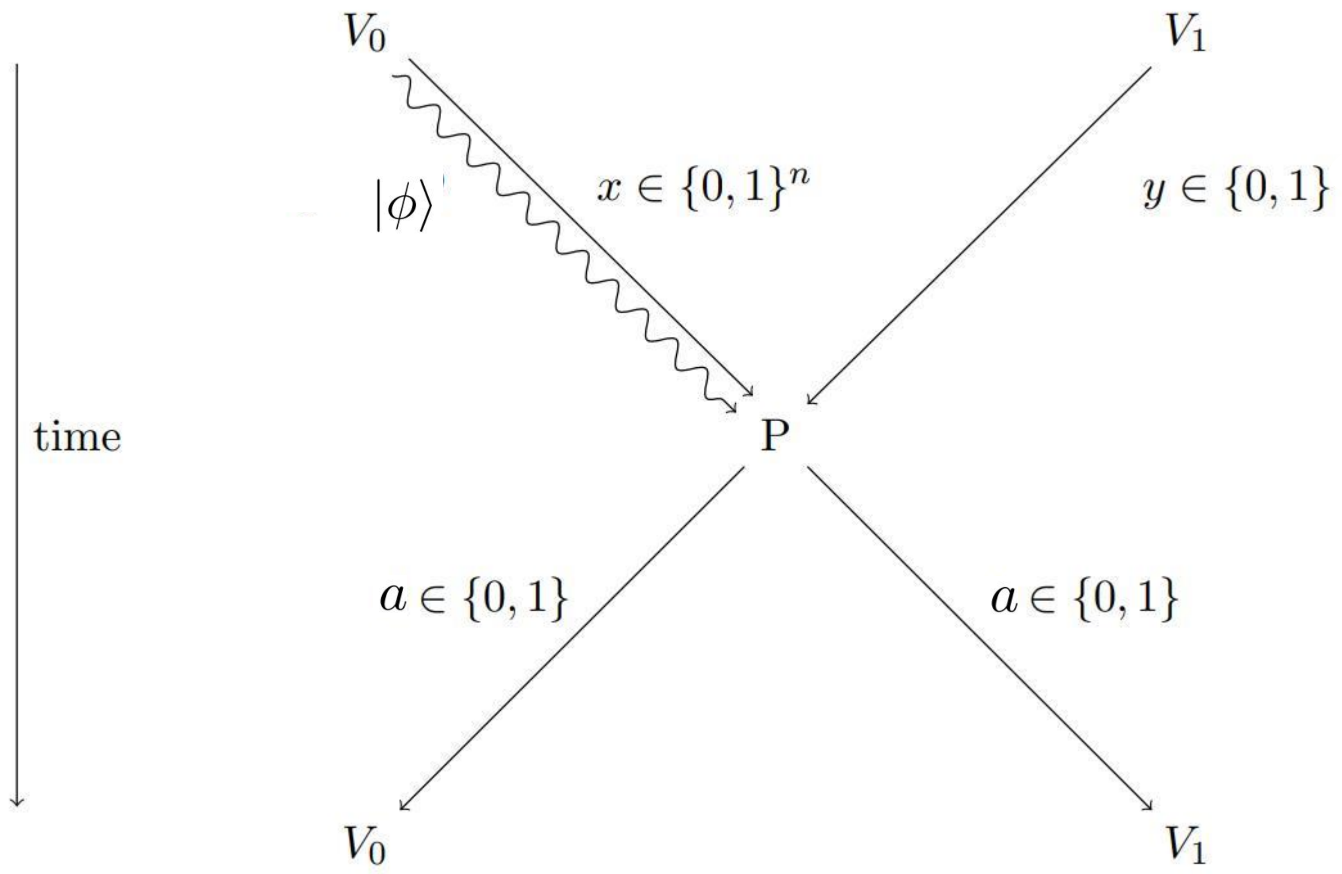

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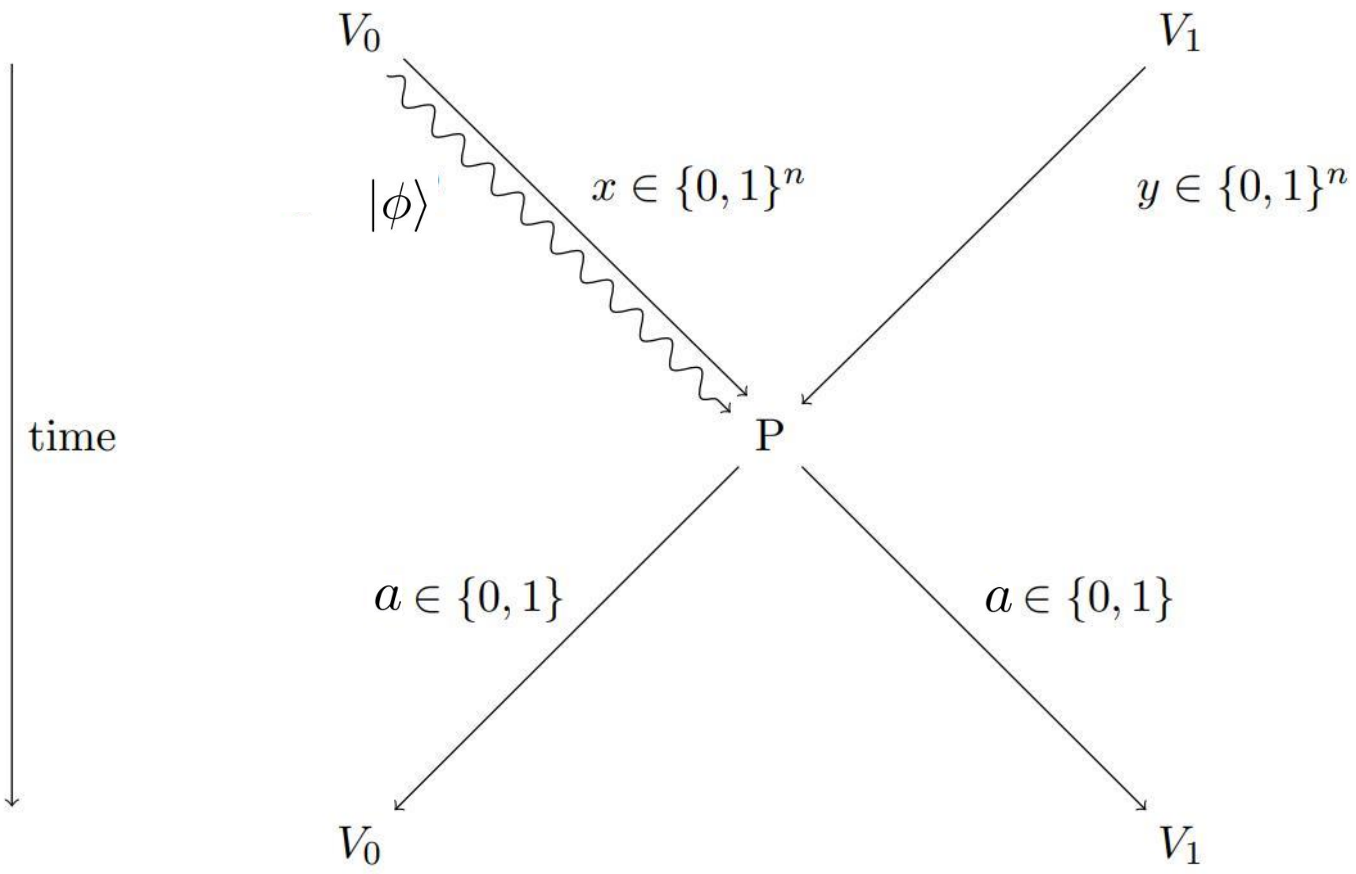

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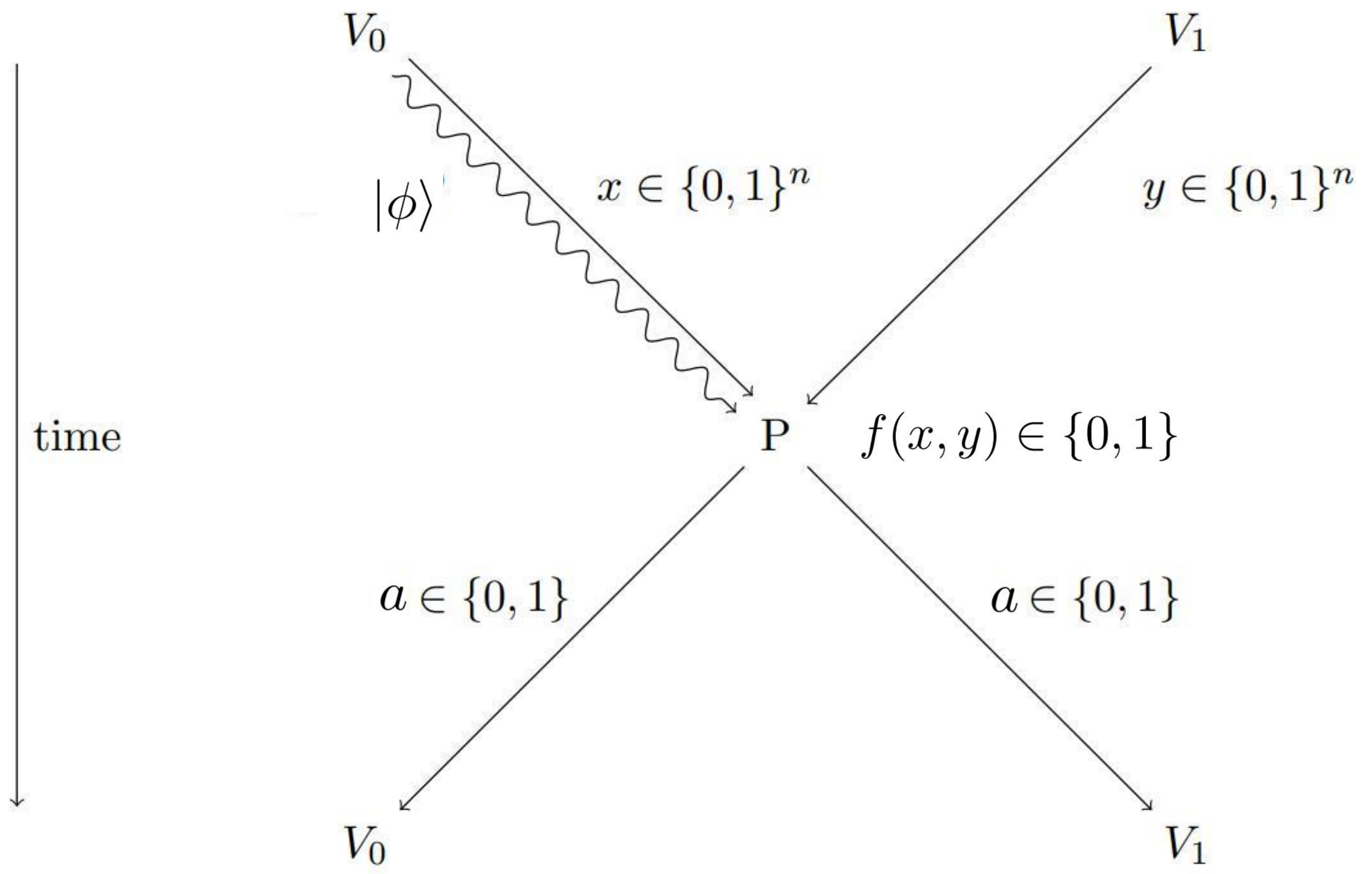

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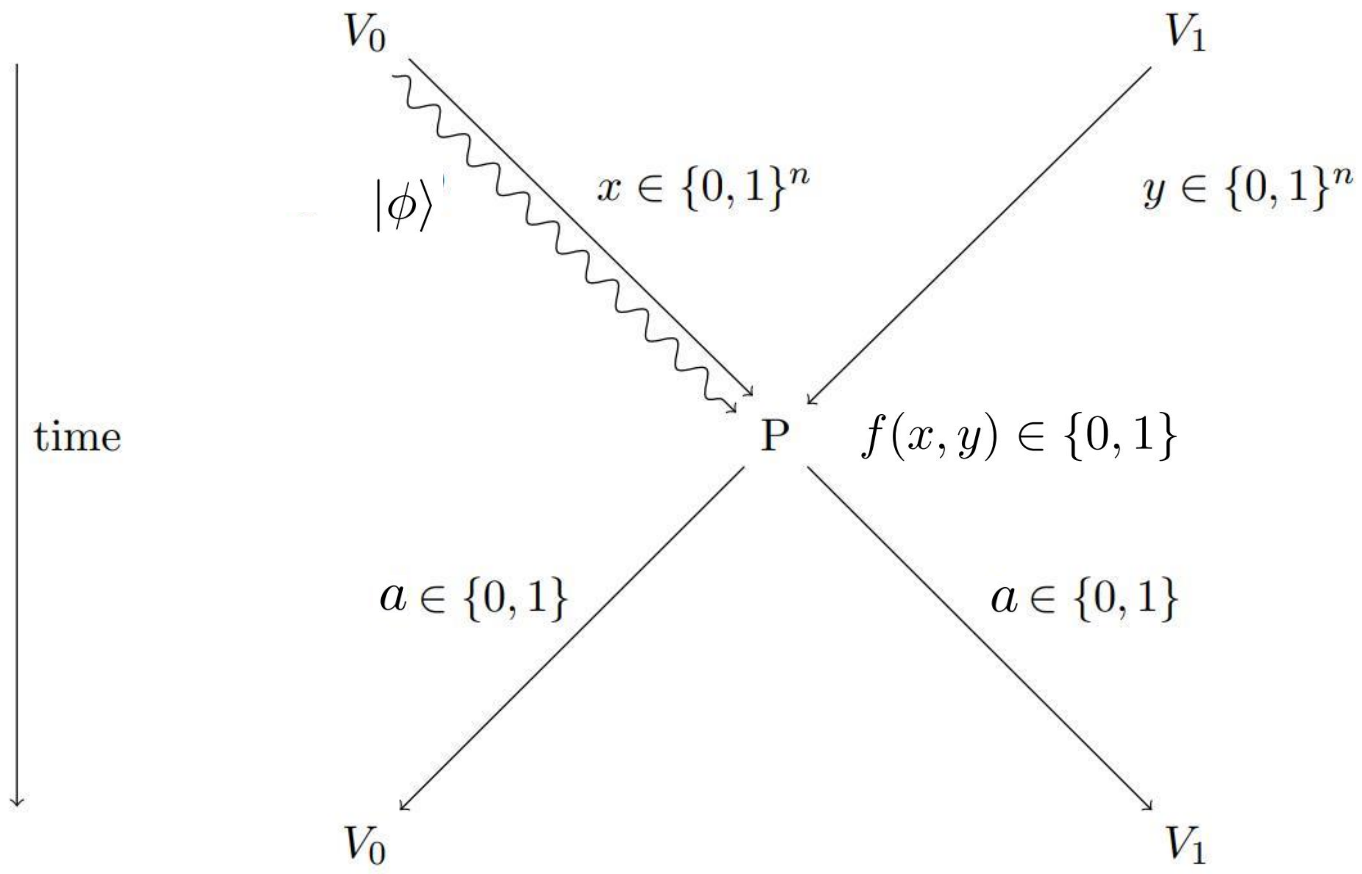
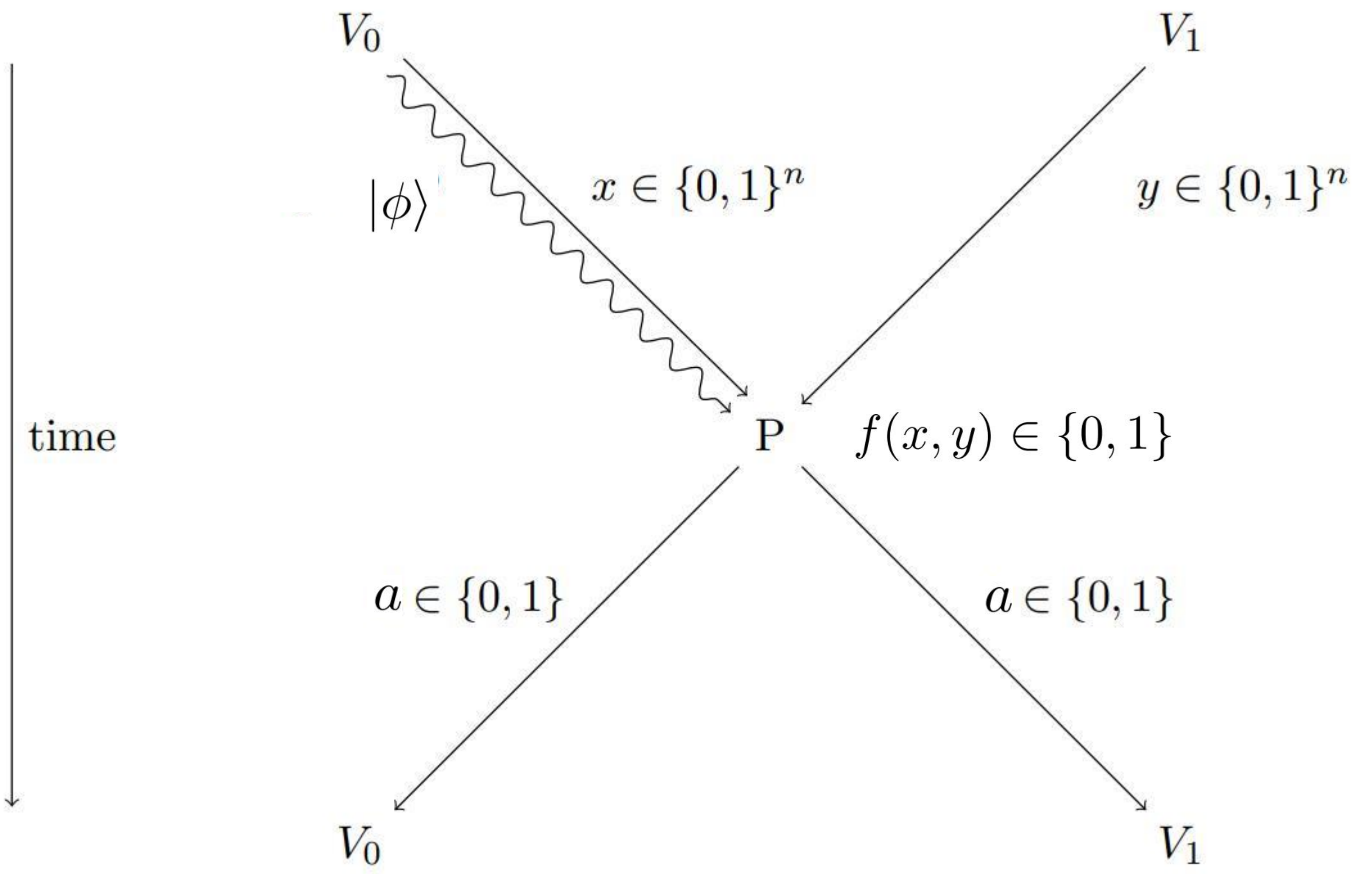


Extension proven secure [BCS22]

$\mathrm{QPV}_{\mathrm{BB84}}^{,f}$



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1. by attackers that pre-share entanglement, and

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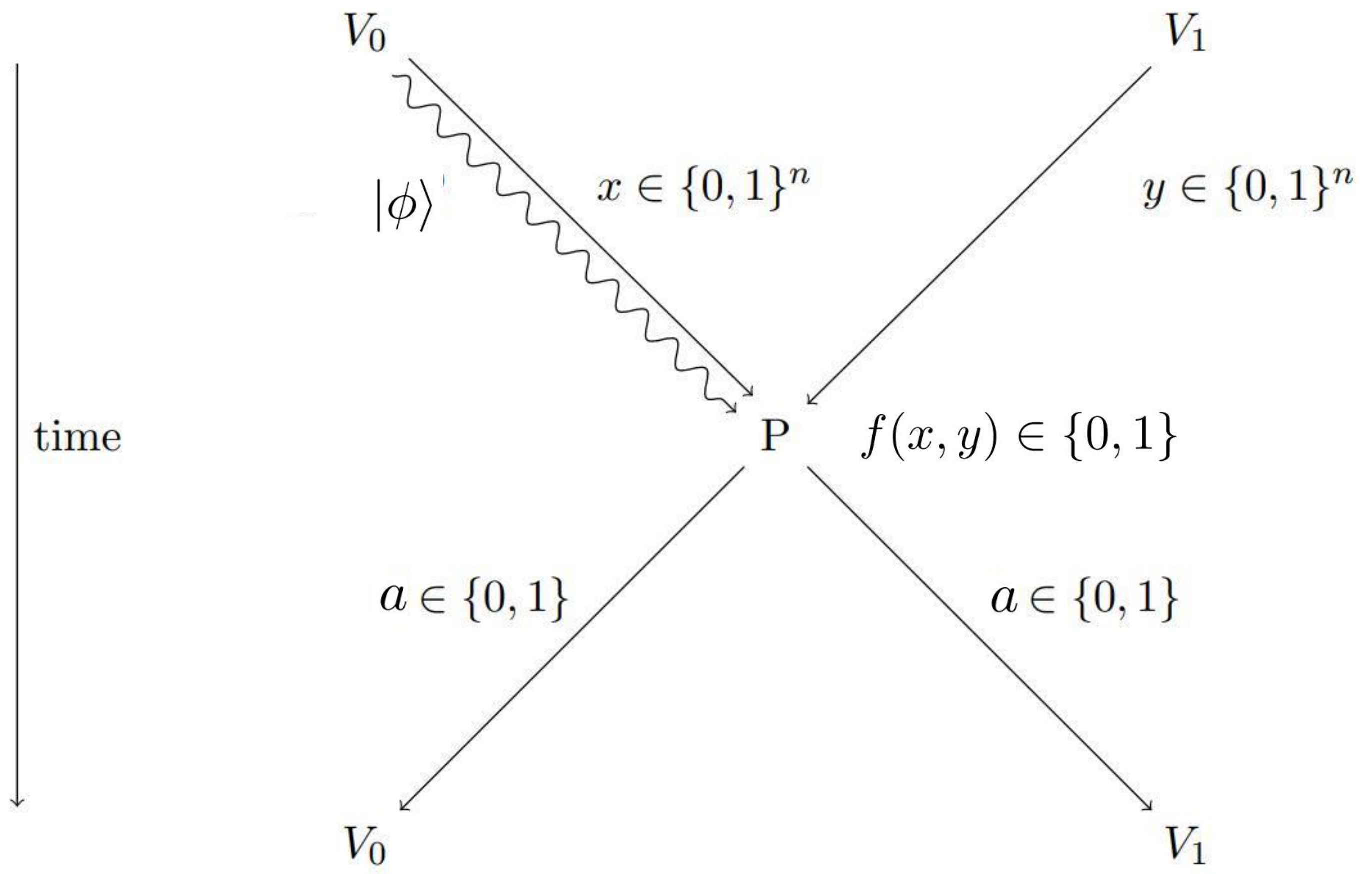
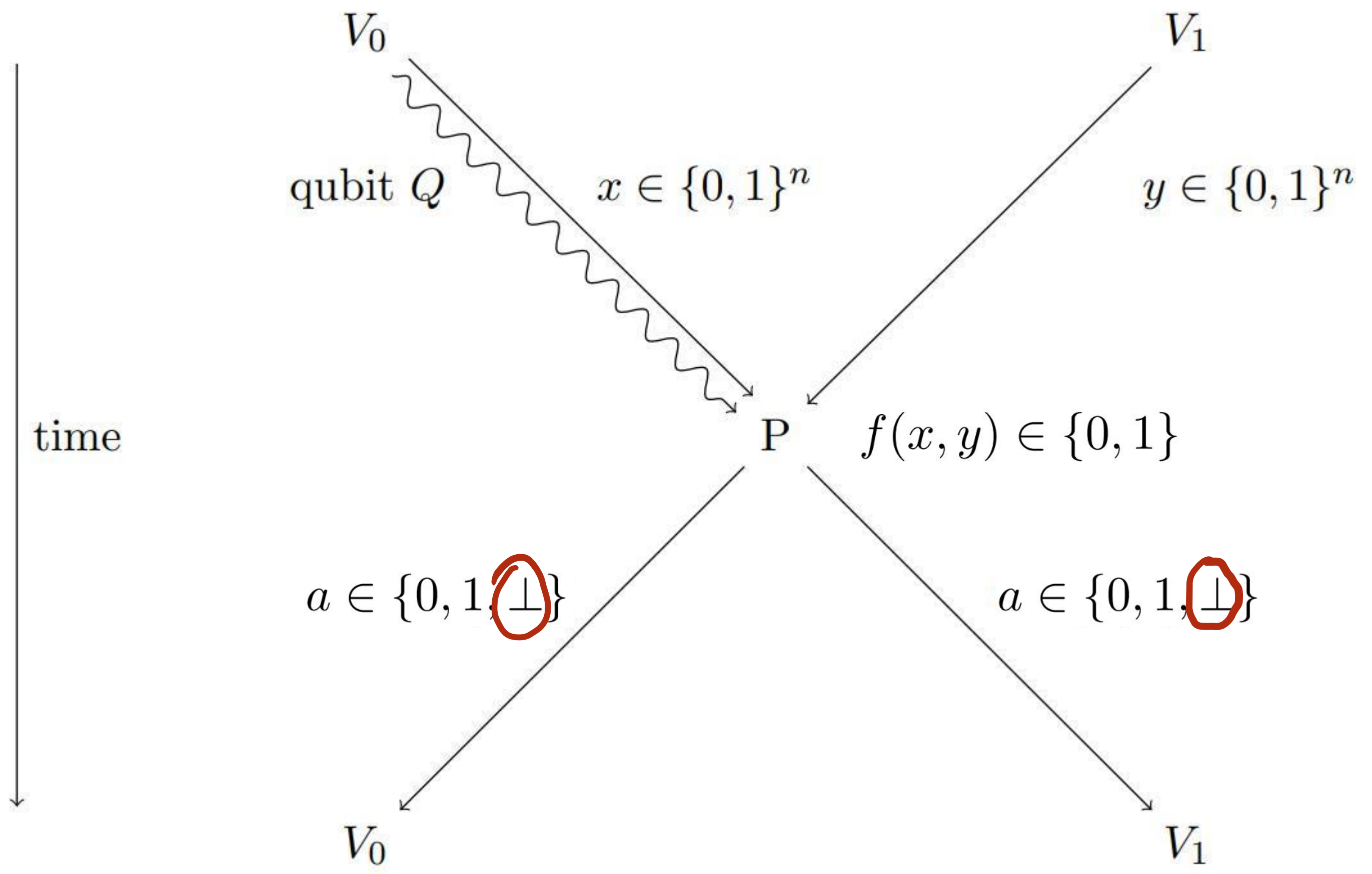


Extension proven secure [BCS22]

1. by attackers that pre-share entanglement, and
2. arbitrary slow quantum information

We introduce $\mathrm{QPV}_{\mathrm{BB84}}^{\eta,f}$



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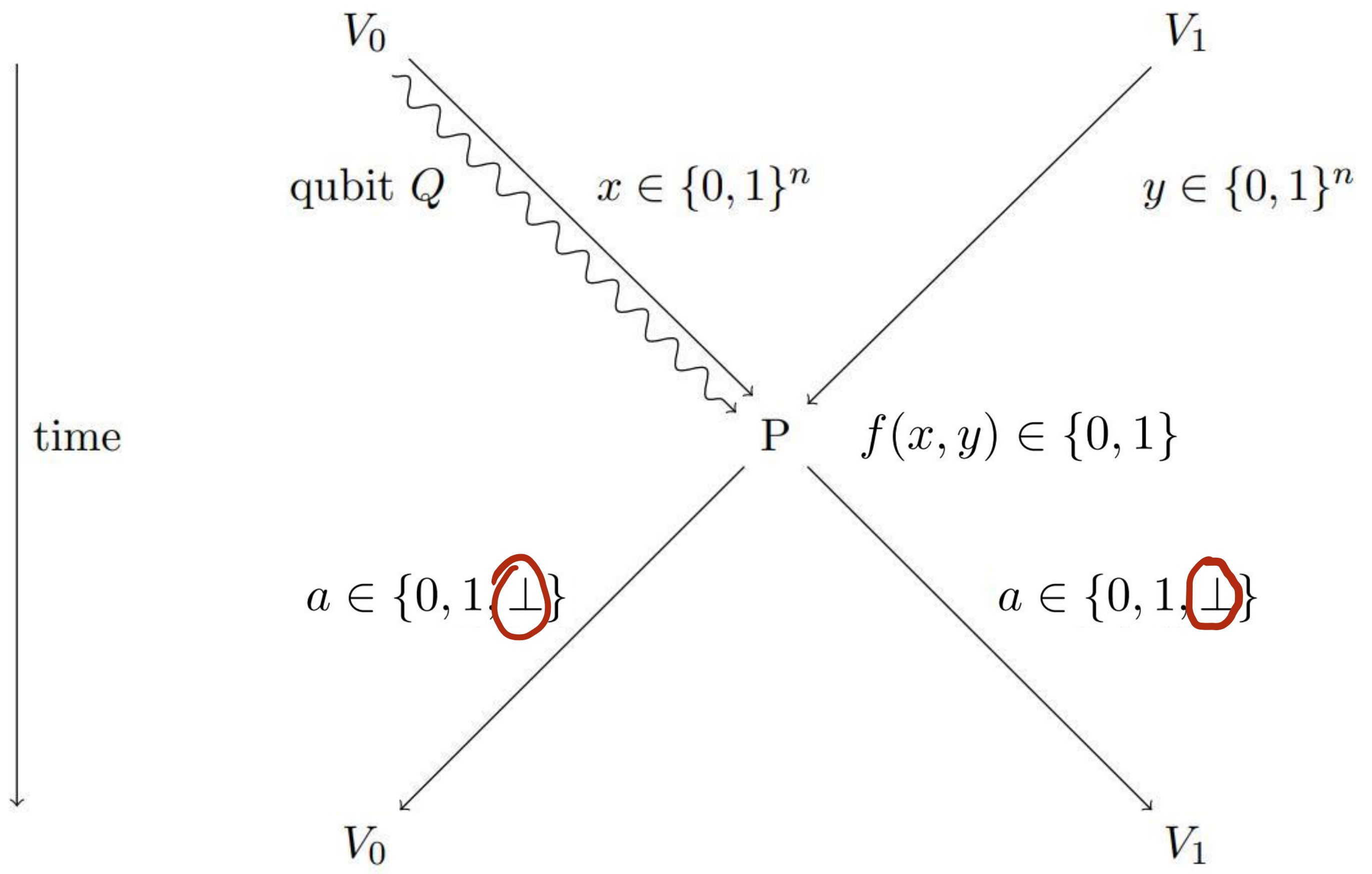
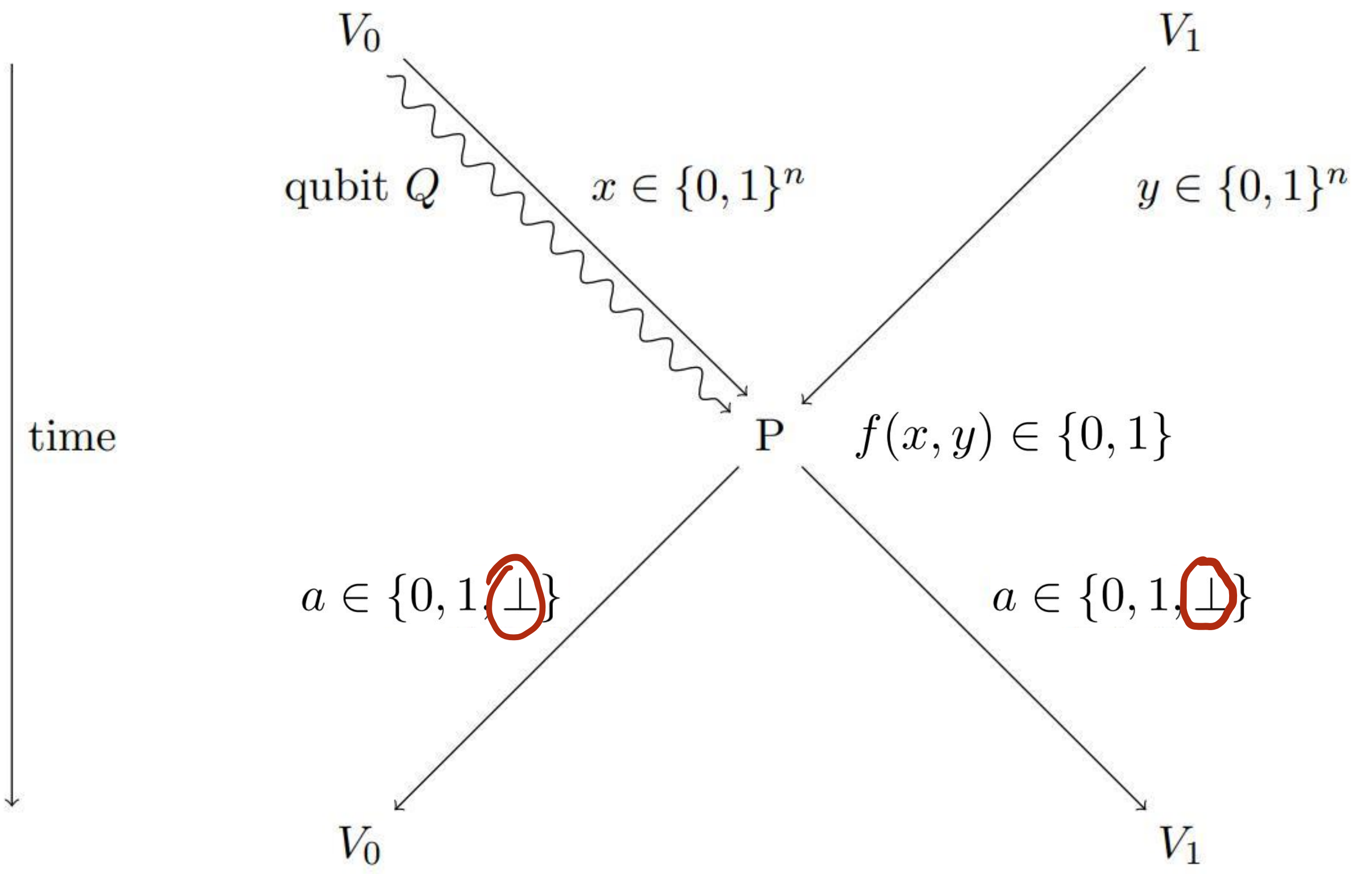


$\mathrm{QPV}^{\eta,f}_{\mathrm{BB84}}$

Previous result with loss



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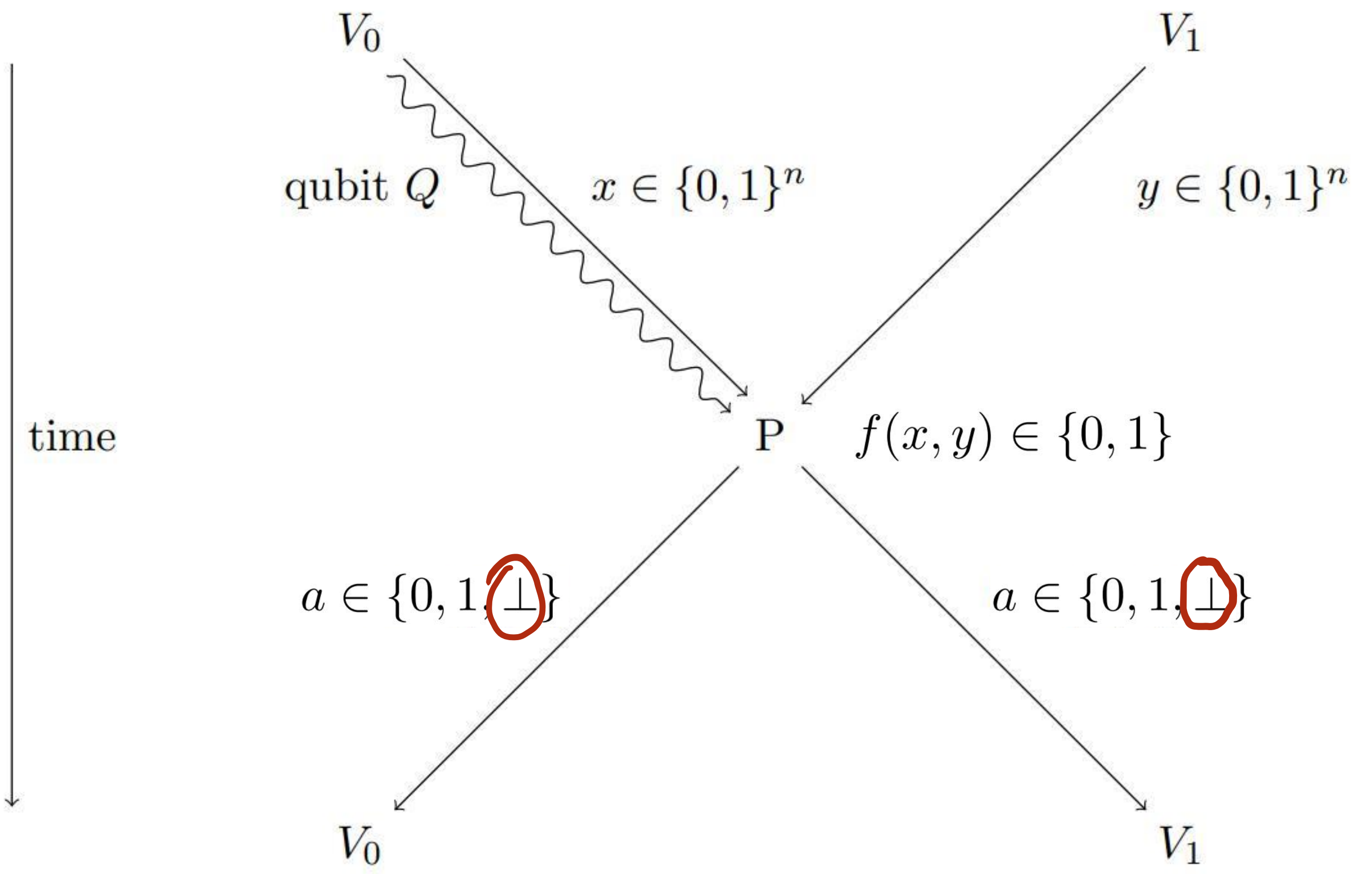

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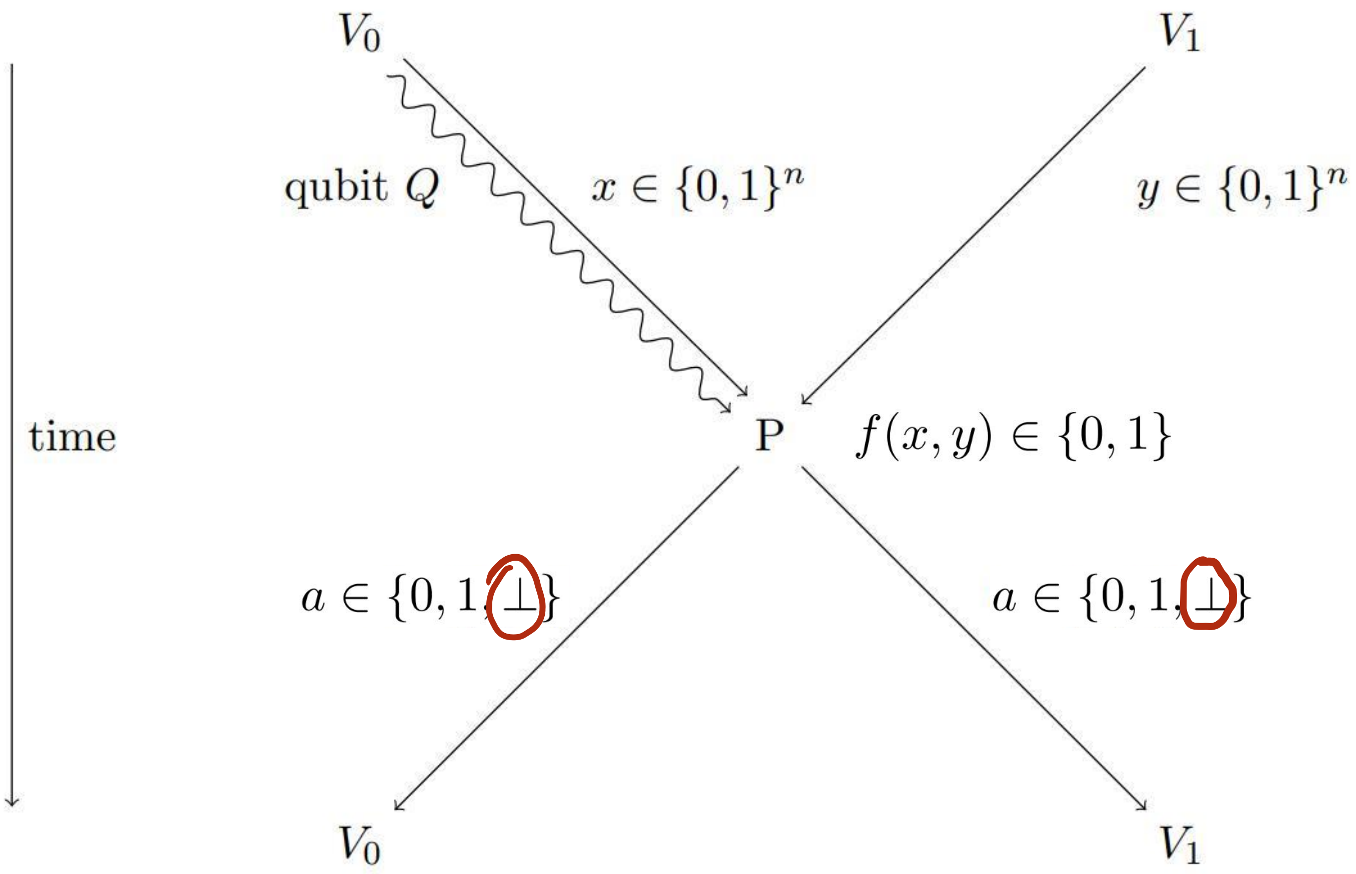
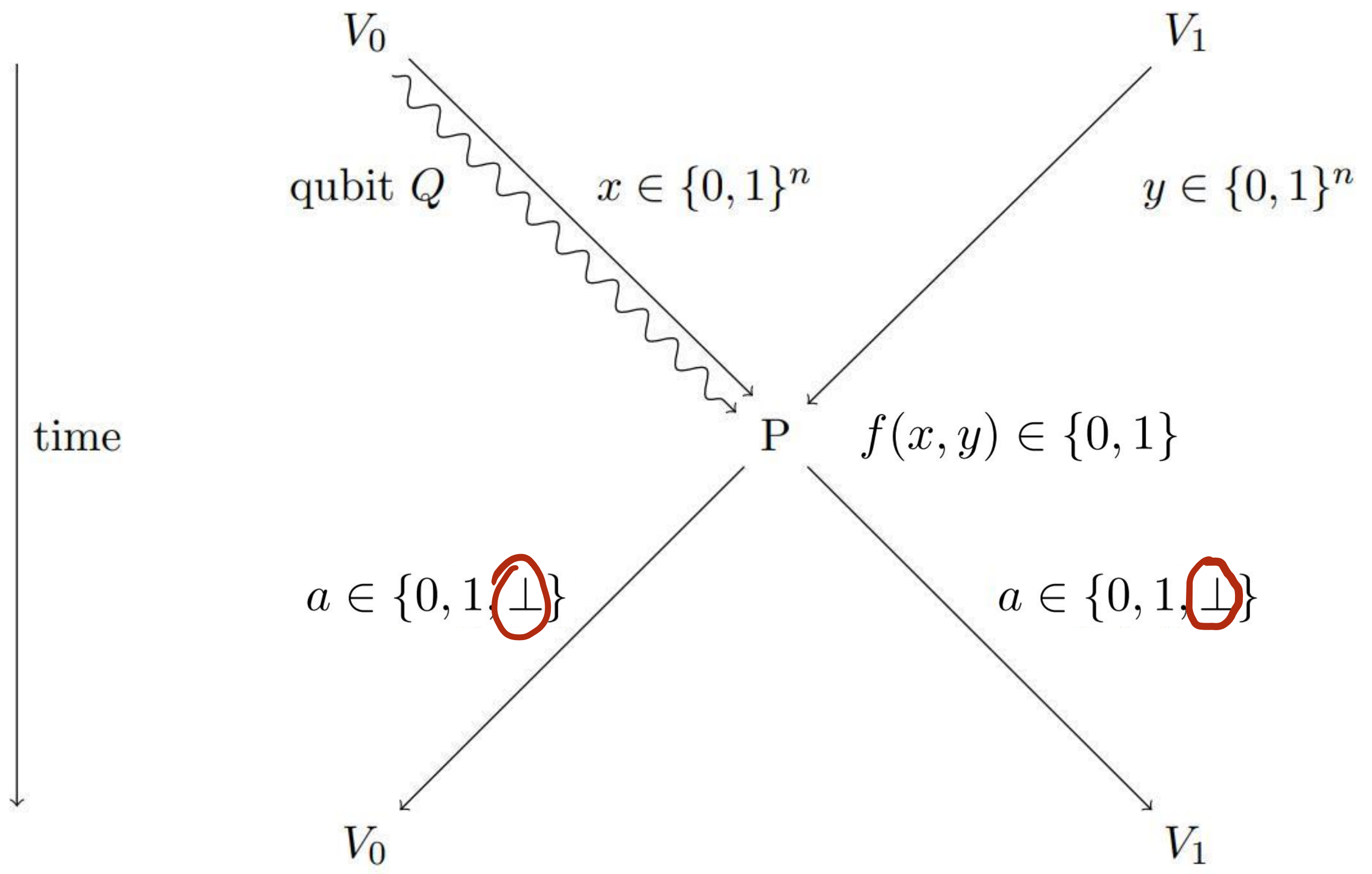


Previous result with loss

Technical lemma

$$\mathrm{QPV}_{\mathrm{BB84}}^{\eta,f}$$



Previous result with loss

↓

Technical lemma

+

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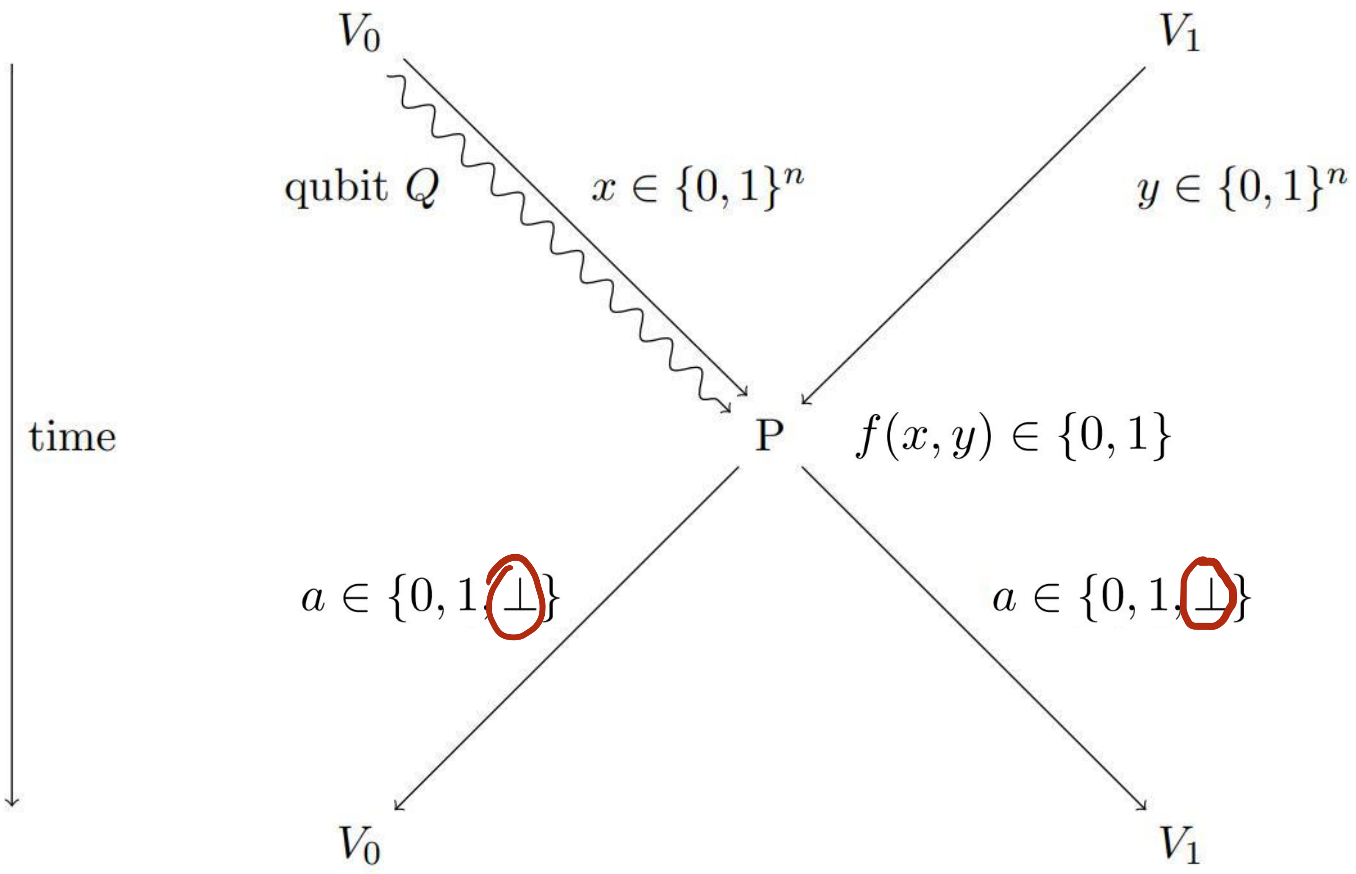

Previous result with loss

Technical lemma

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Techniques: security without loss [BCS22]

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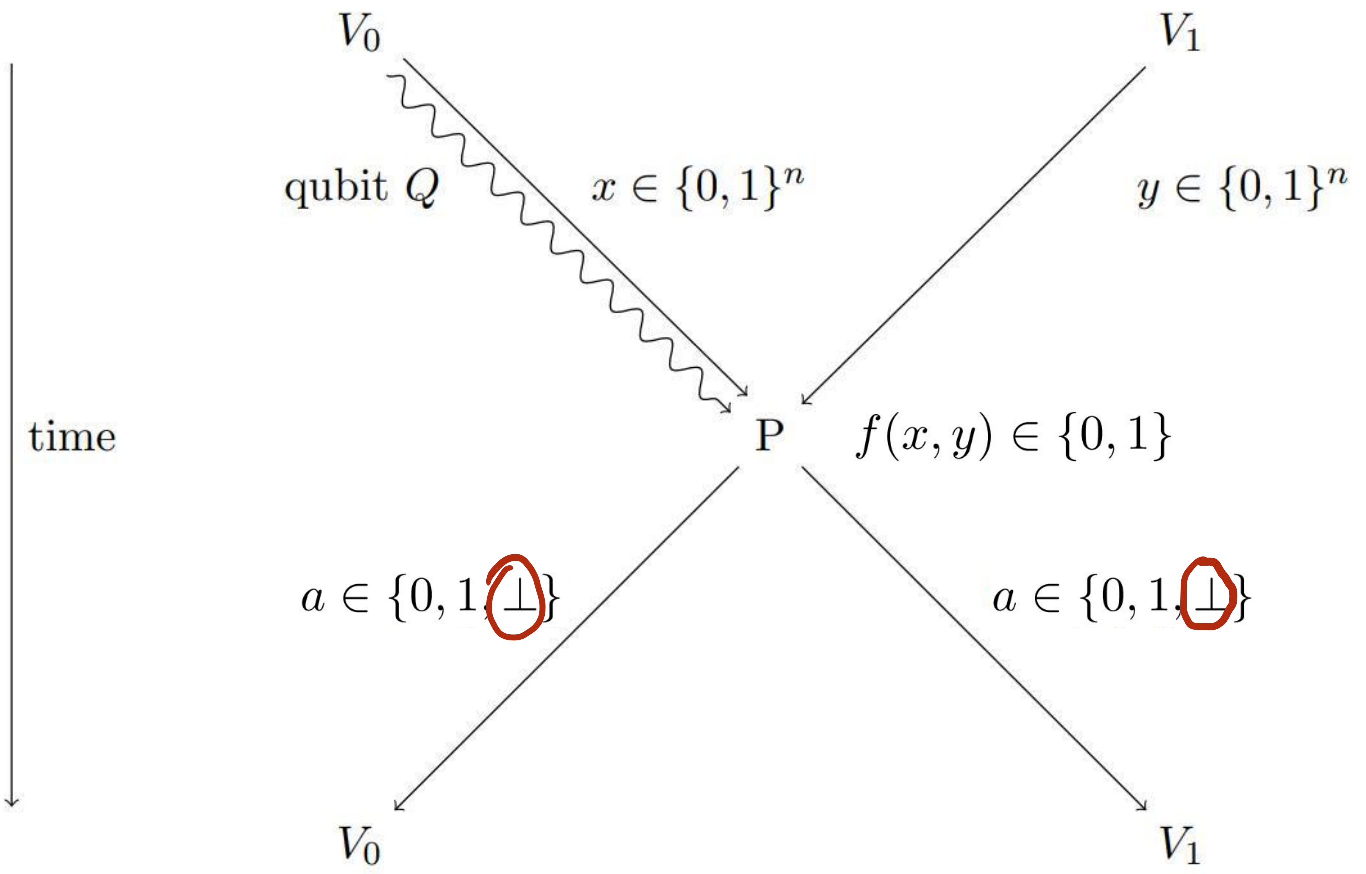

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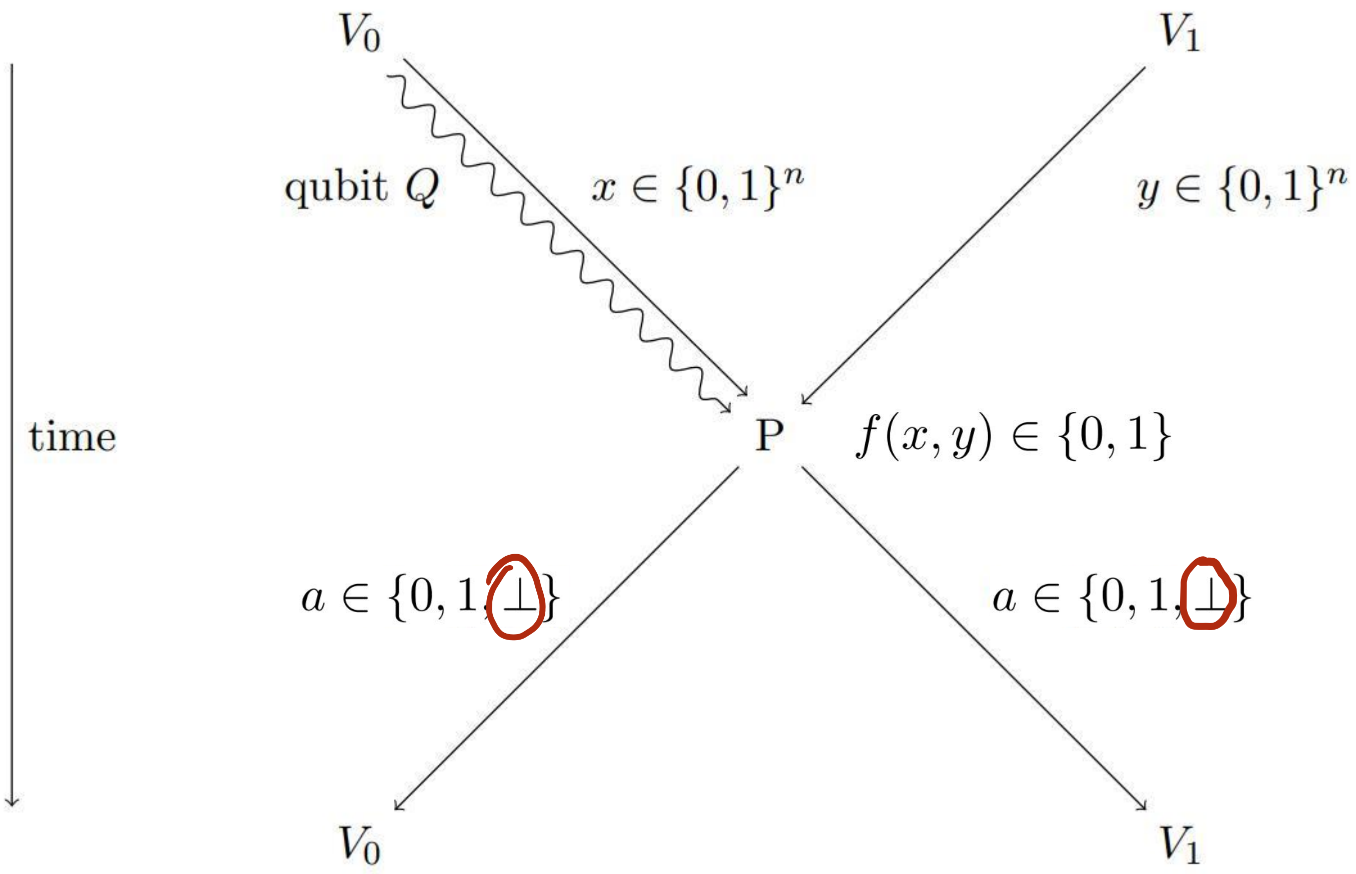

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Techniques: security without loss [BCS22]

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- number of pre-shared qubits $\leq n/2-5$ (ENTANGLED attackers),
- quantum info arbitrarily slow
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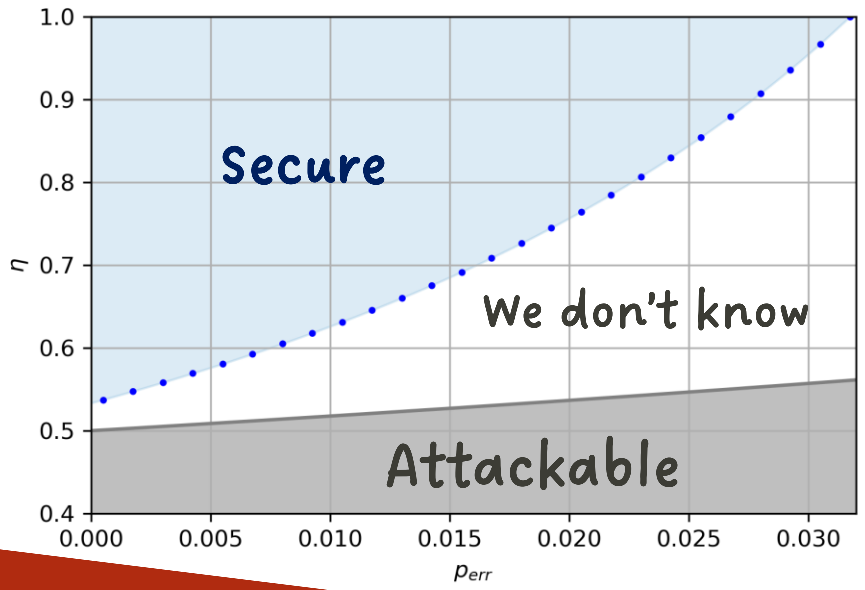
In experimental parameters



This means

This means

👩 🙋 👦

Protocol

This means

Protocol

Attack

This means

Protocol
(With loss)

Attack

This means

Protocol
(With loss)

Attack

Classical info

This means

	 Protocol (With loss)	 Attack
Classical info	2n	2n

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Qubits		

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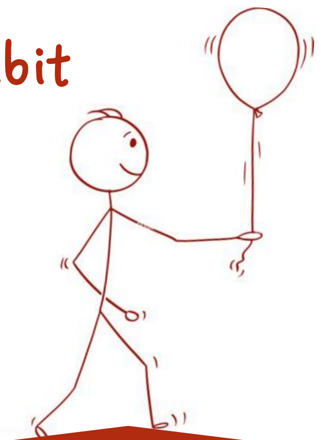

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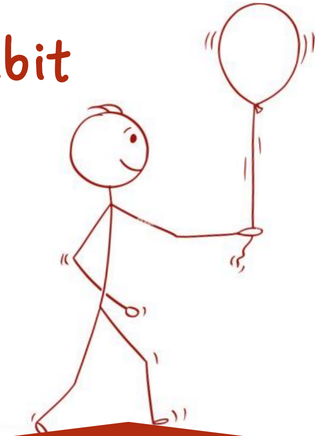


Photon loss
Entanglement
Slow quantum info
SOUTH CAROLINA
ALABAMA
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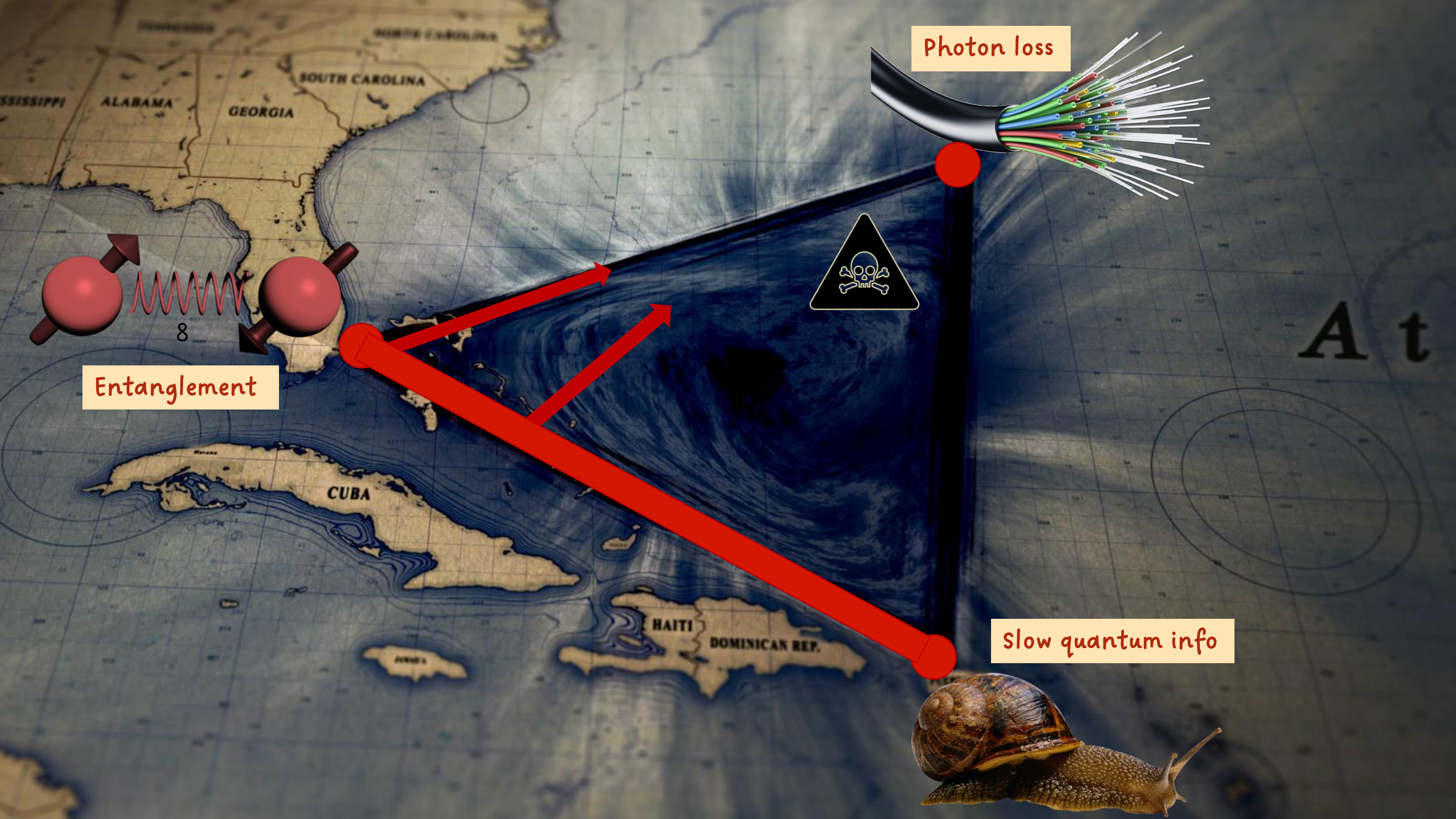
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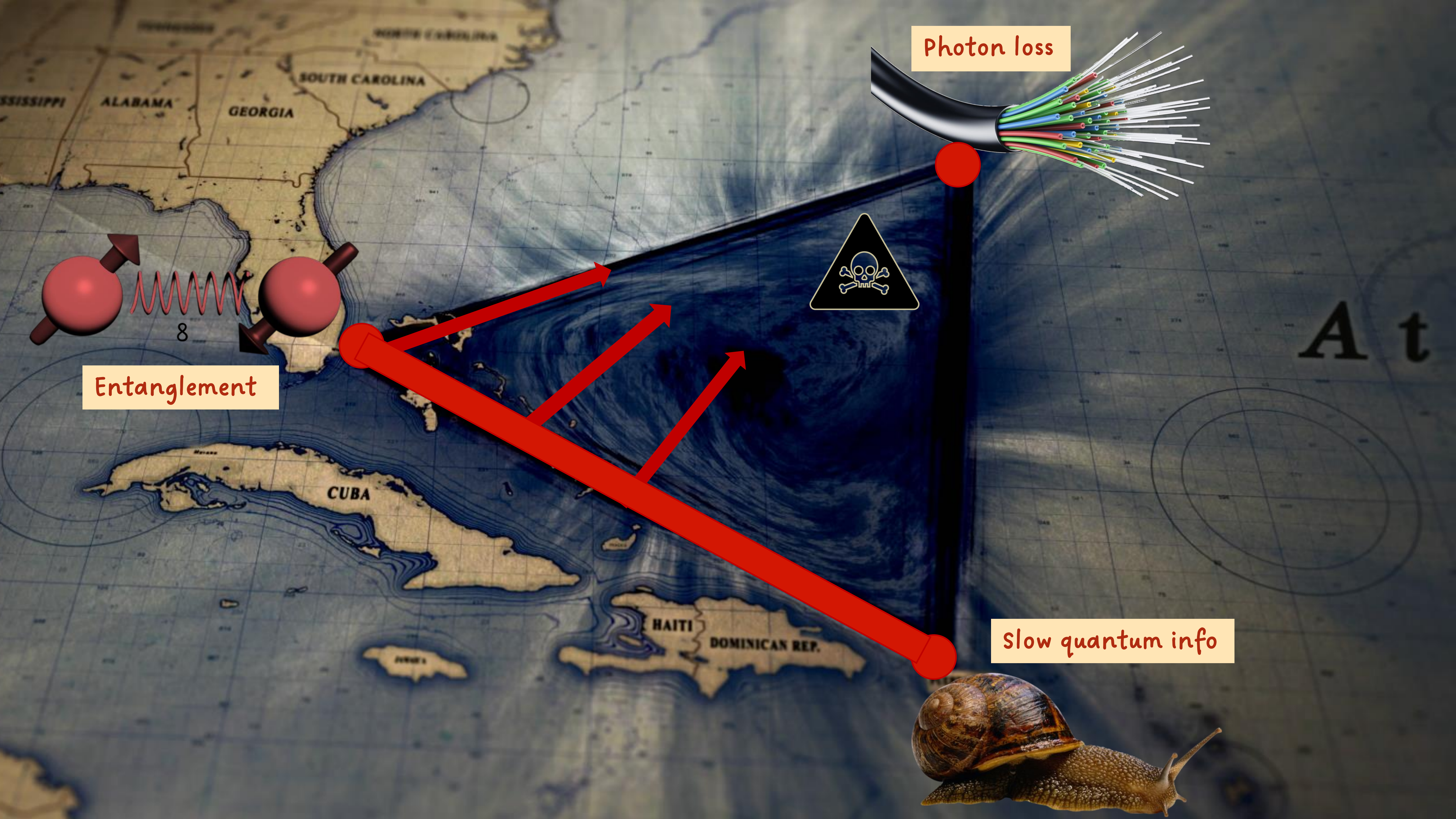
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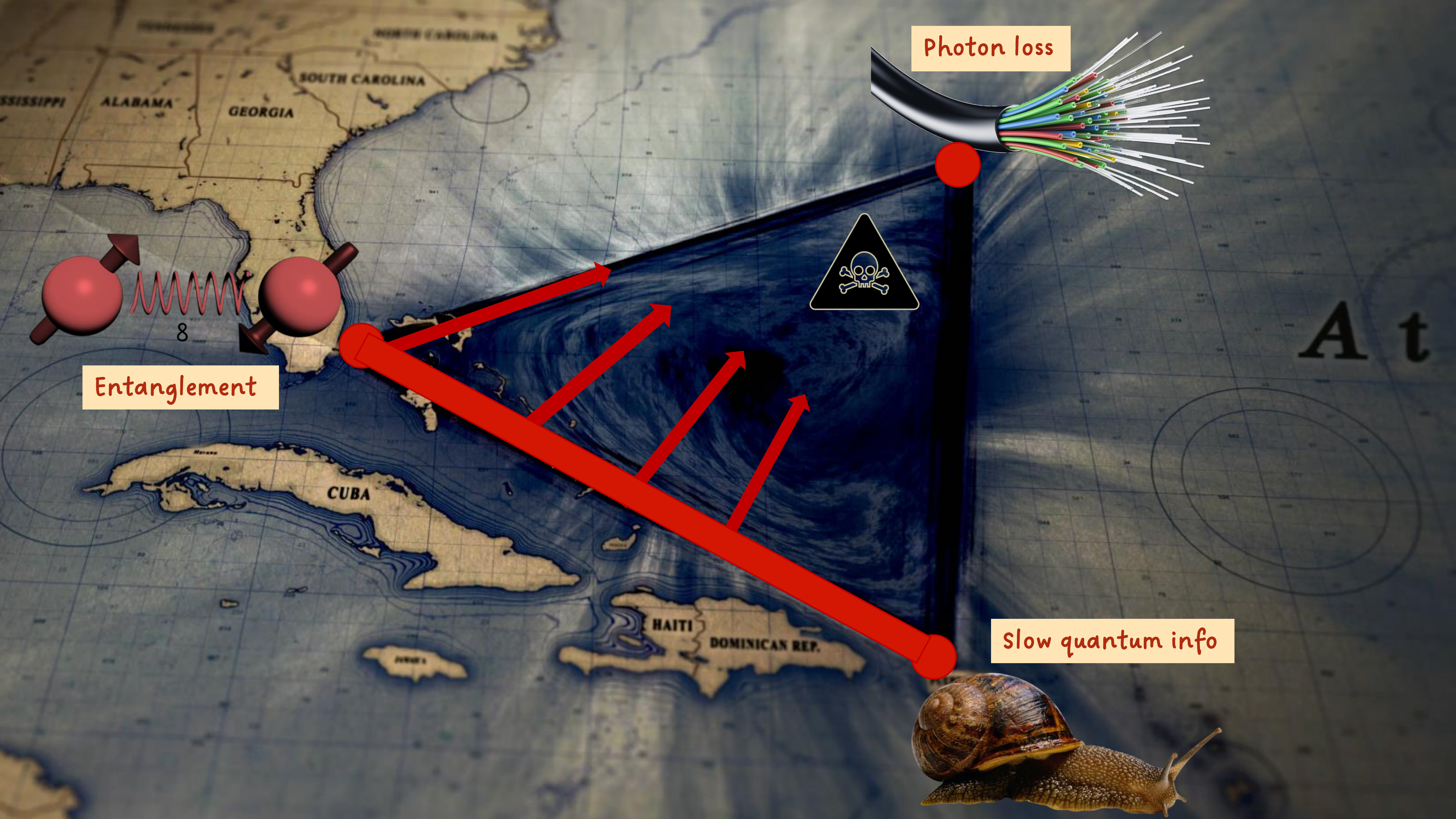

Photon loss
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Slow quantum info
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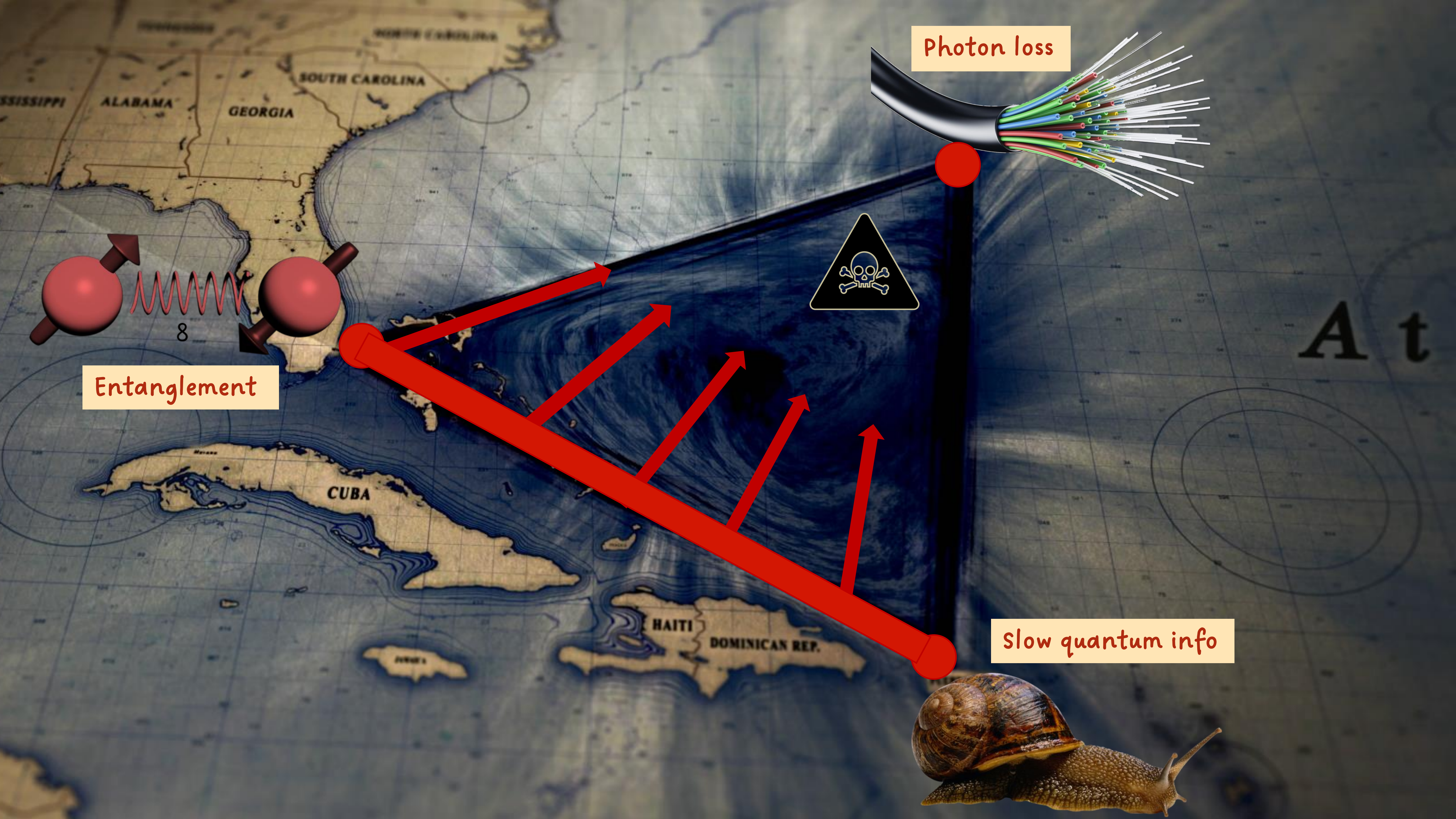

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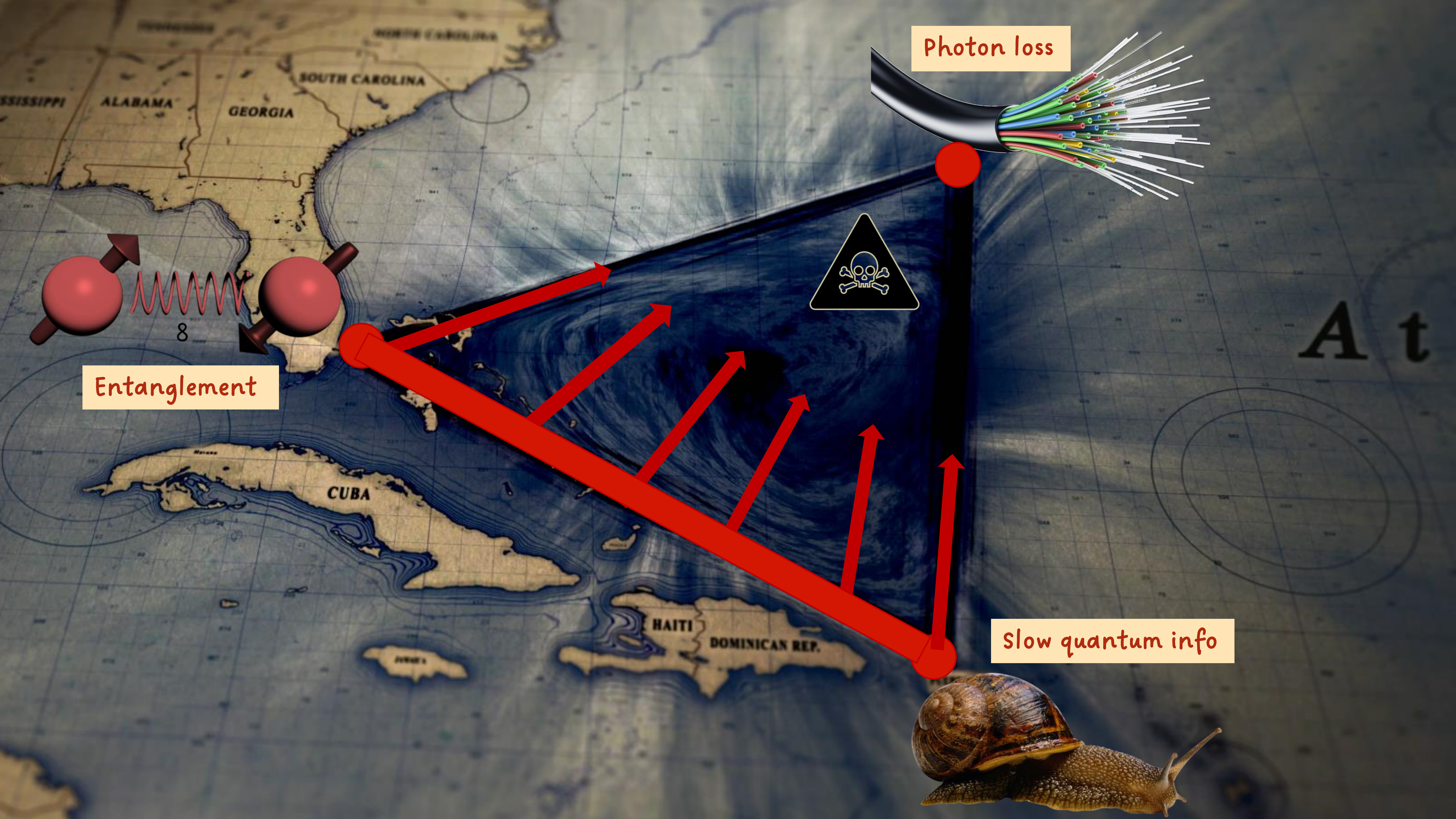
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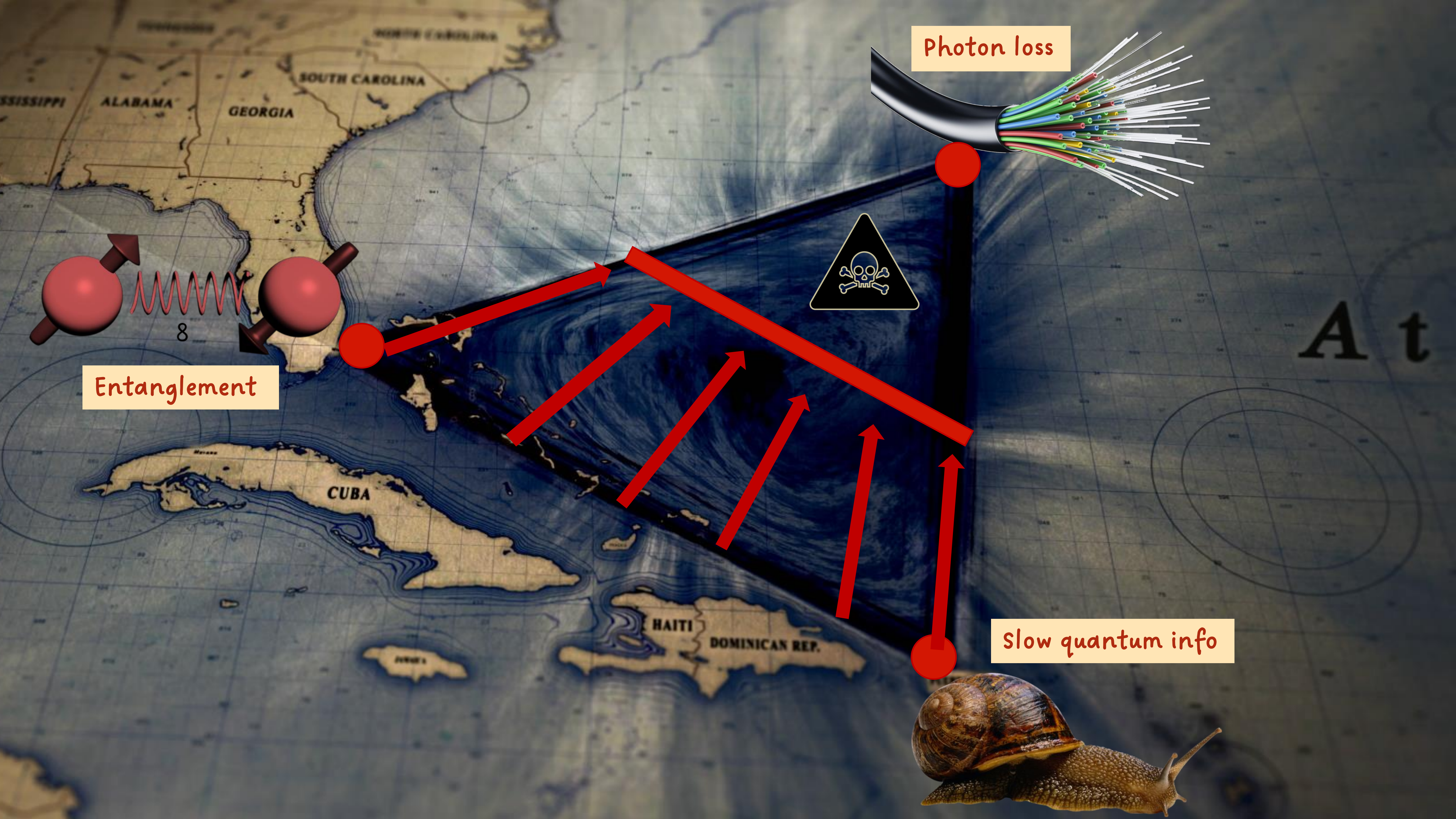

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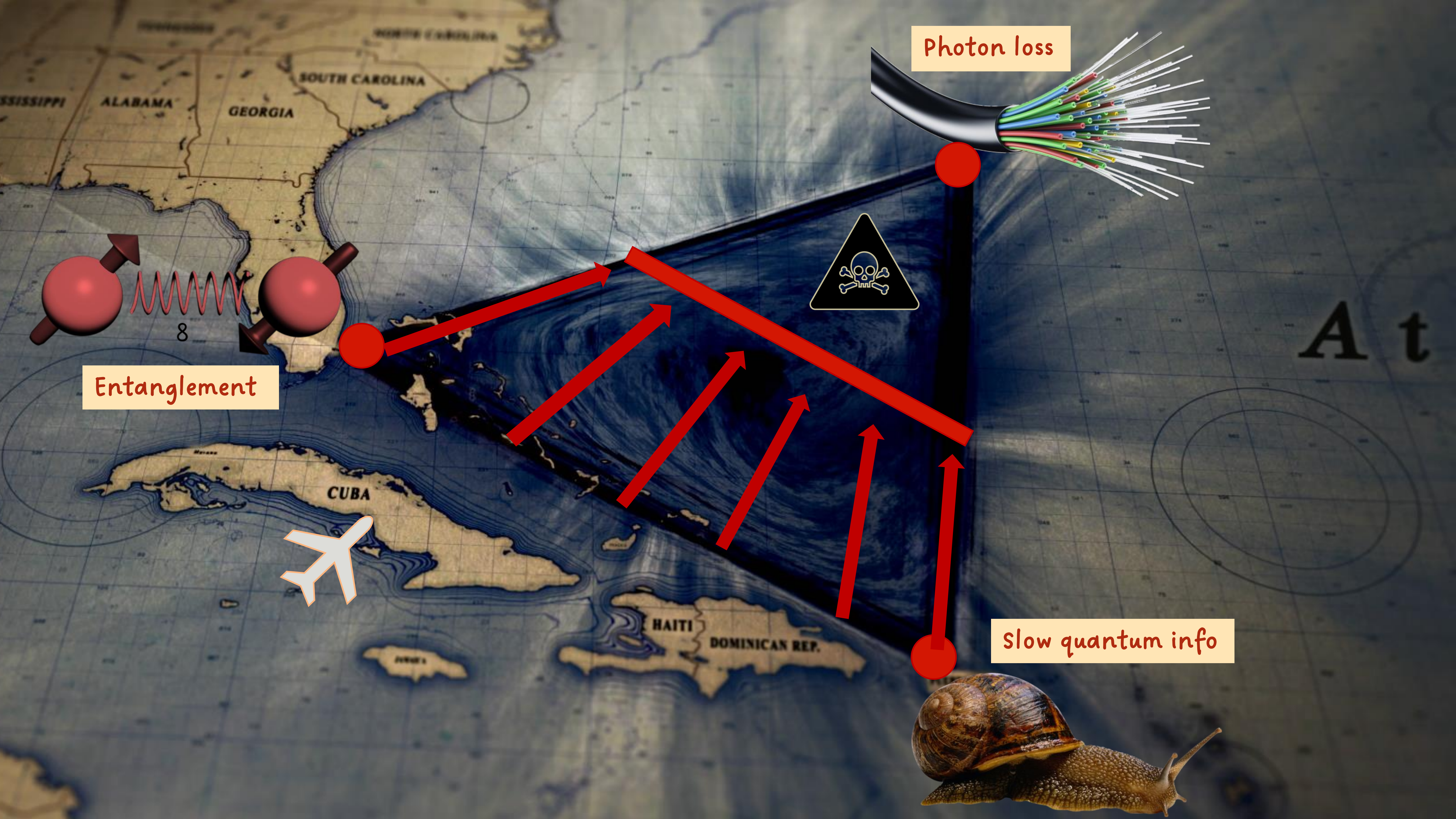

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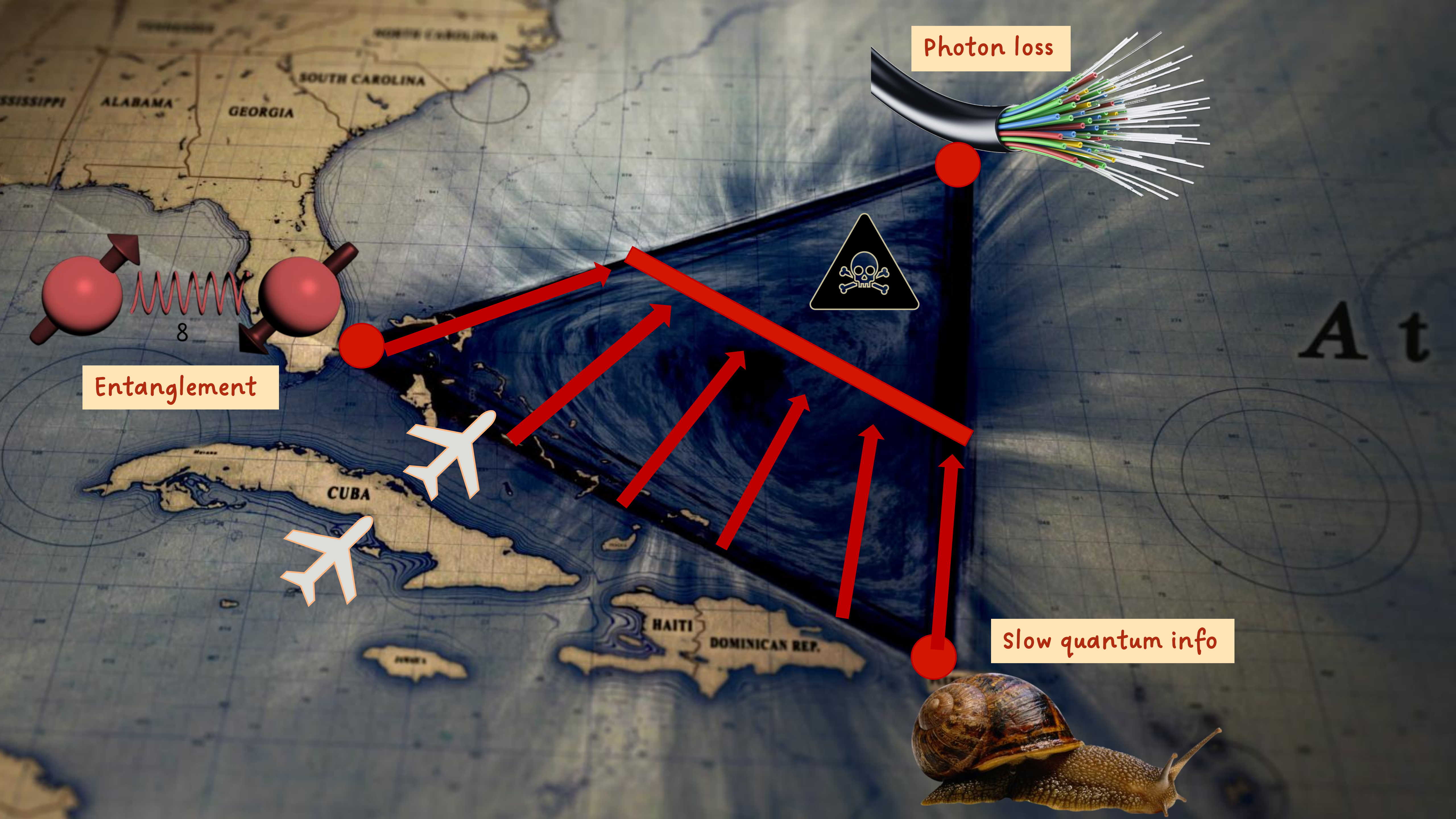

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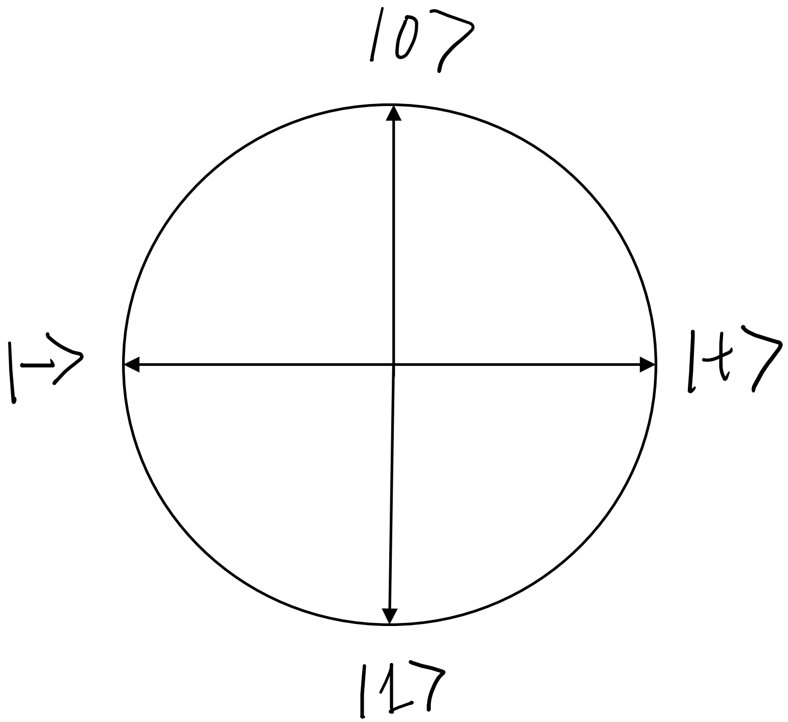

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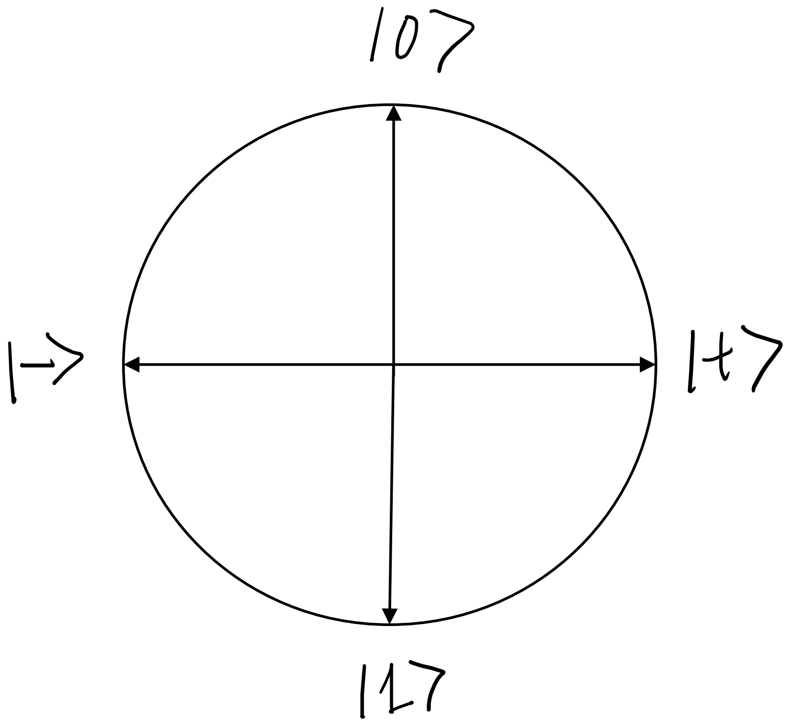

$\eta \geq \frac{1}{2}$

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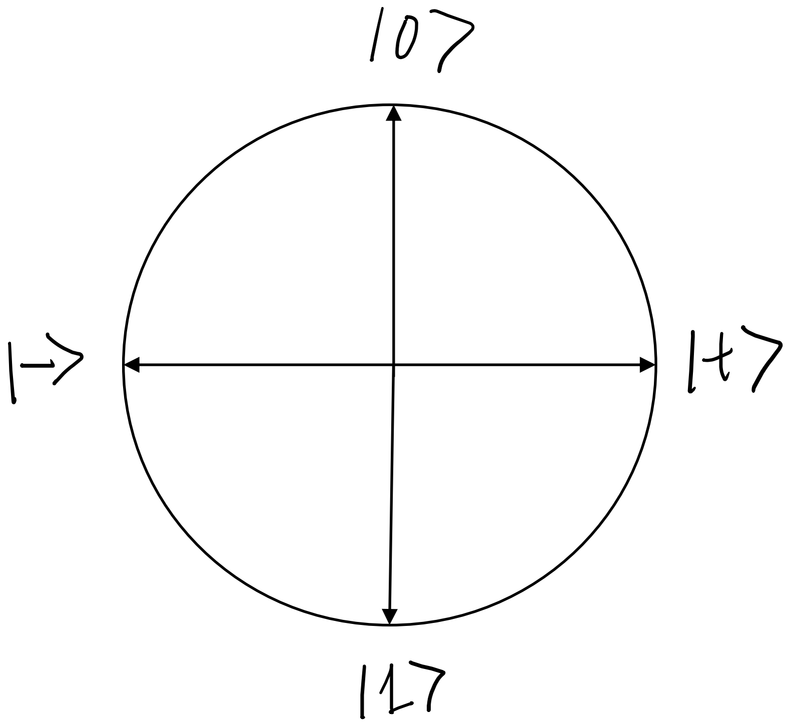

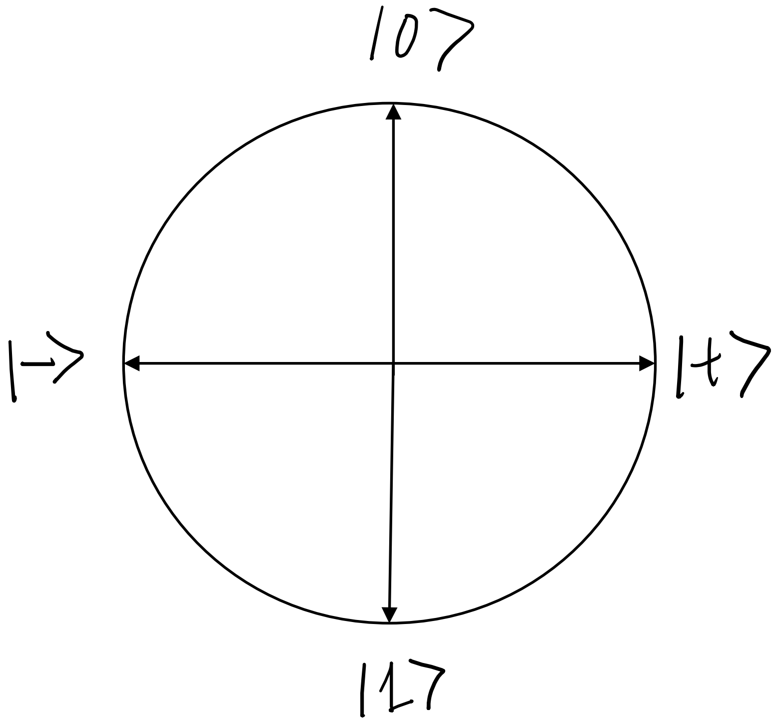

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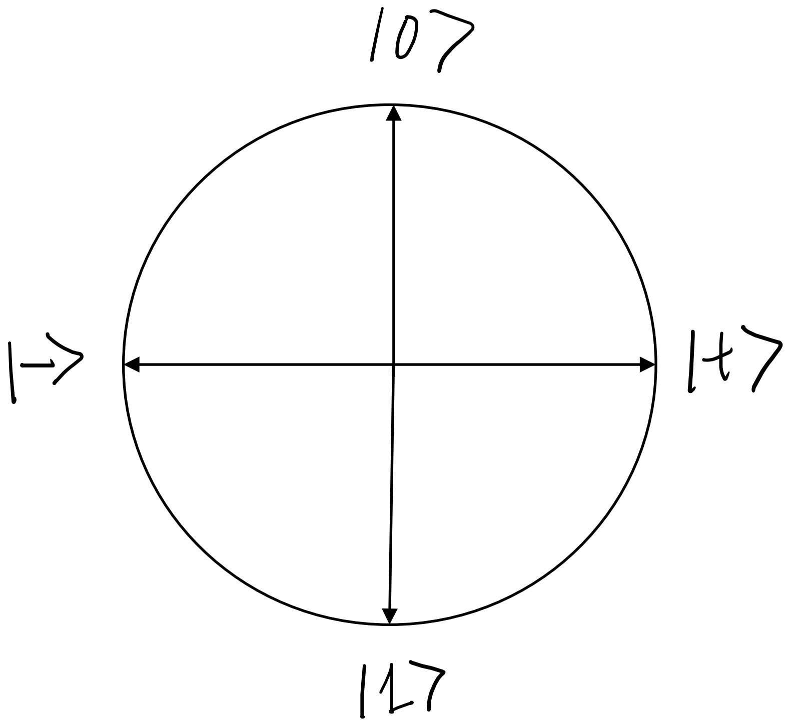

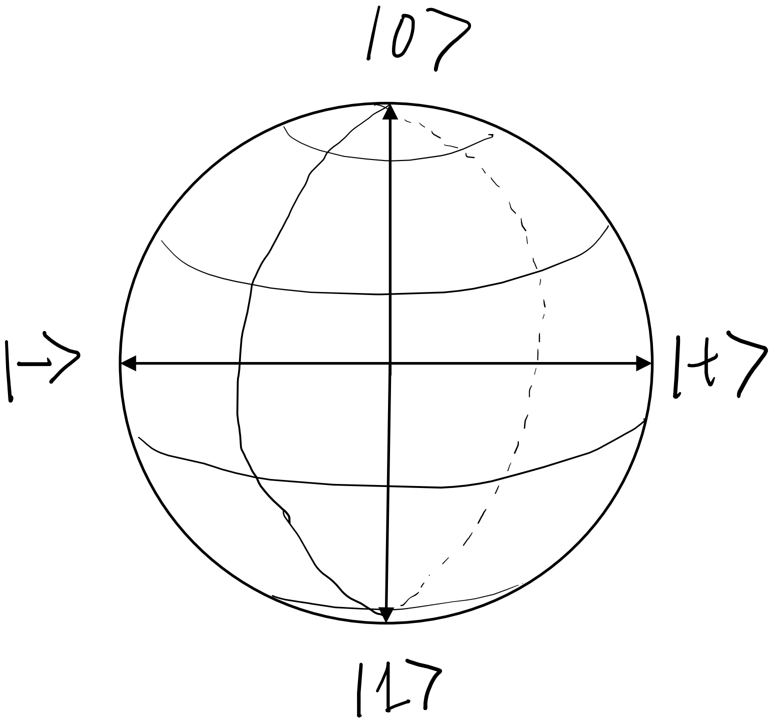

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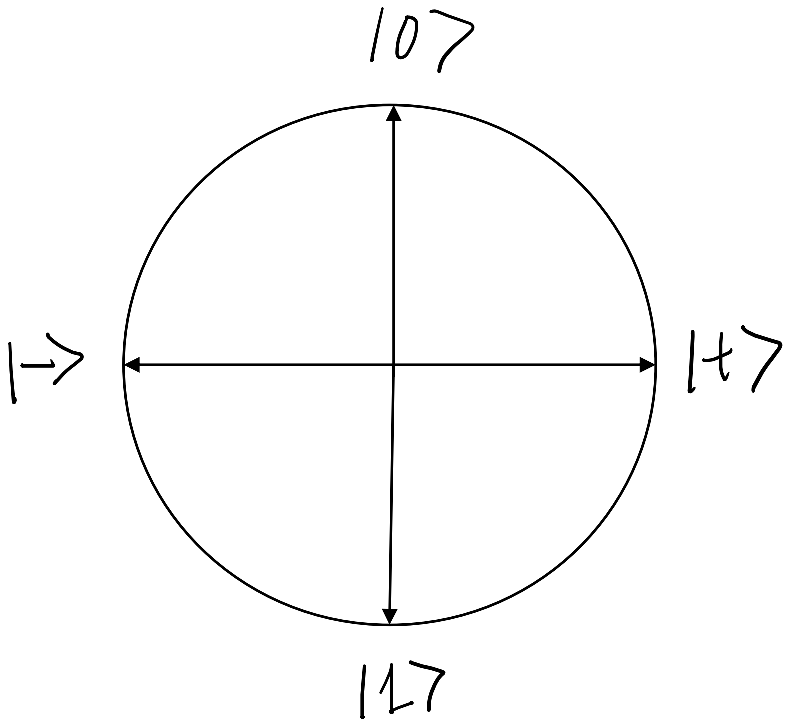

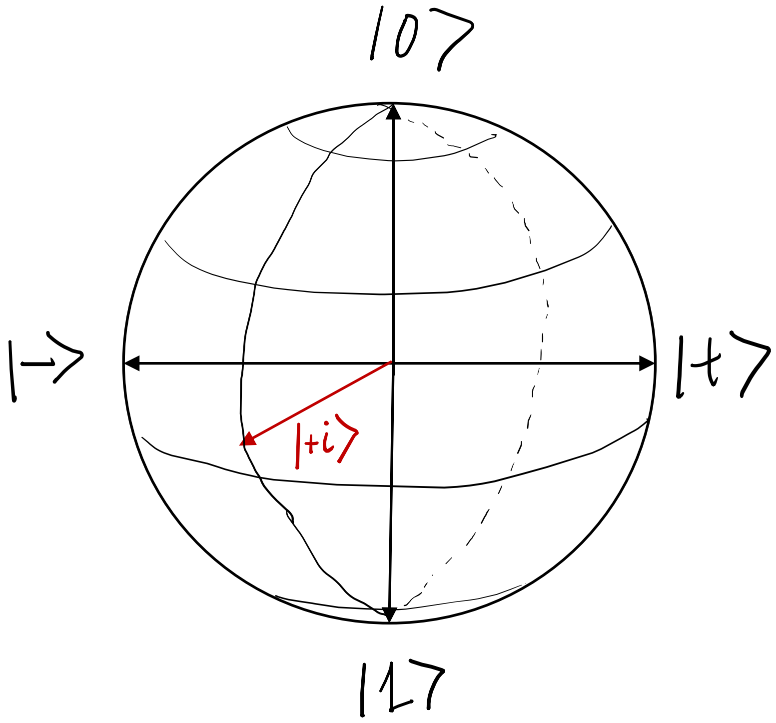

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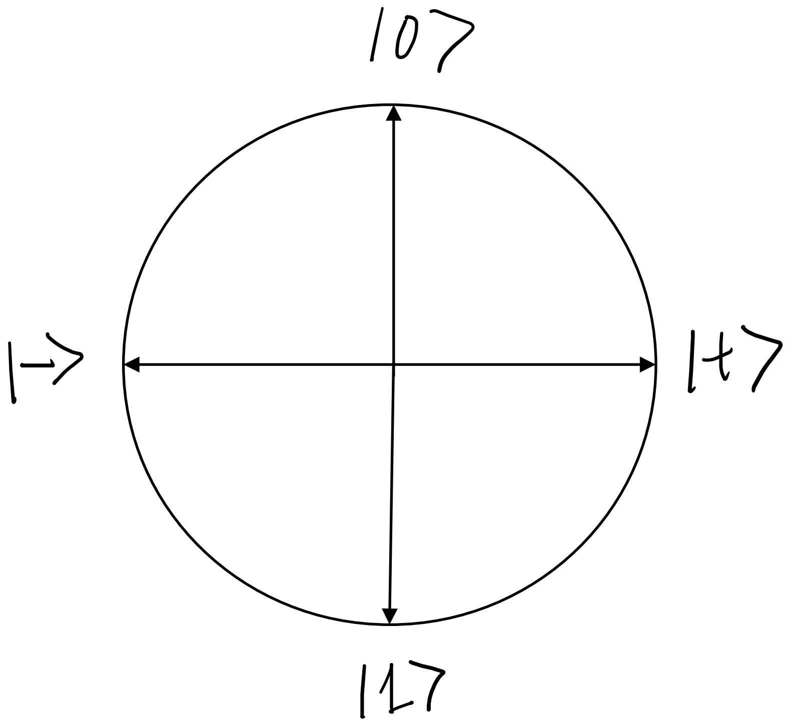

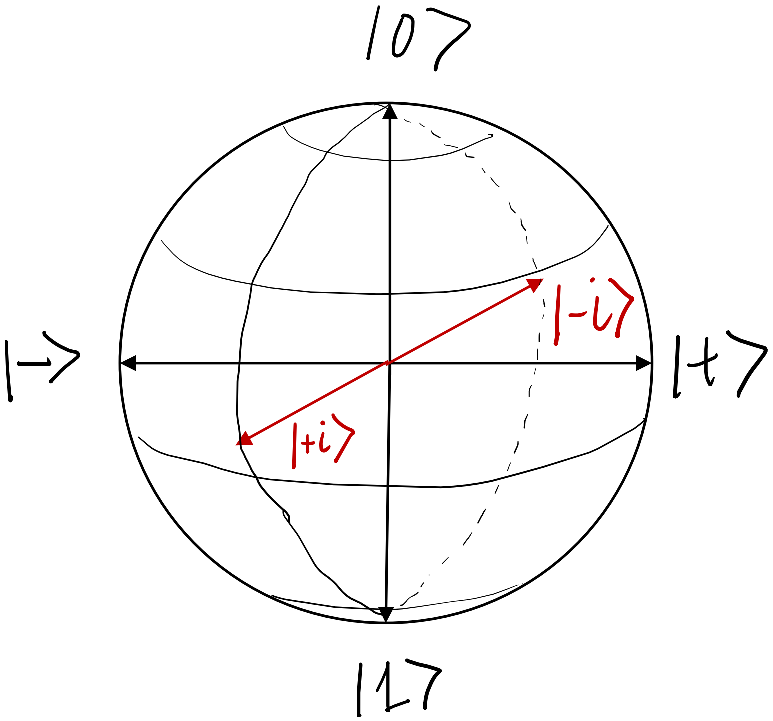

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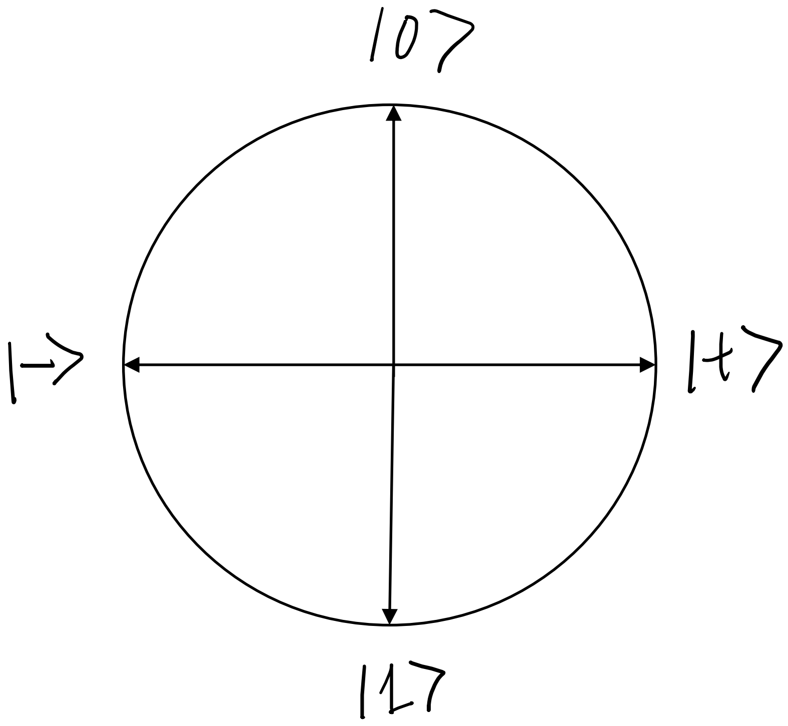

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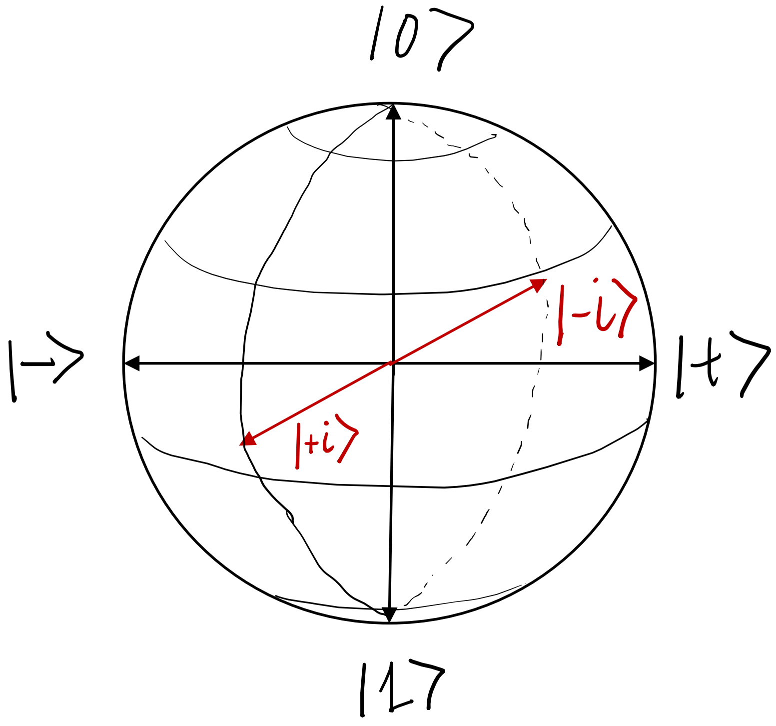
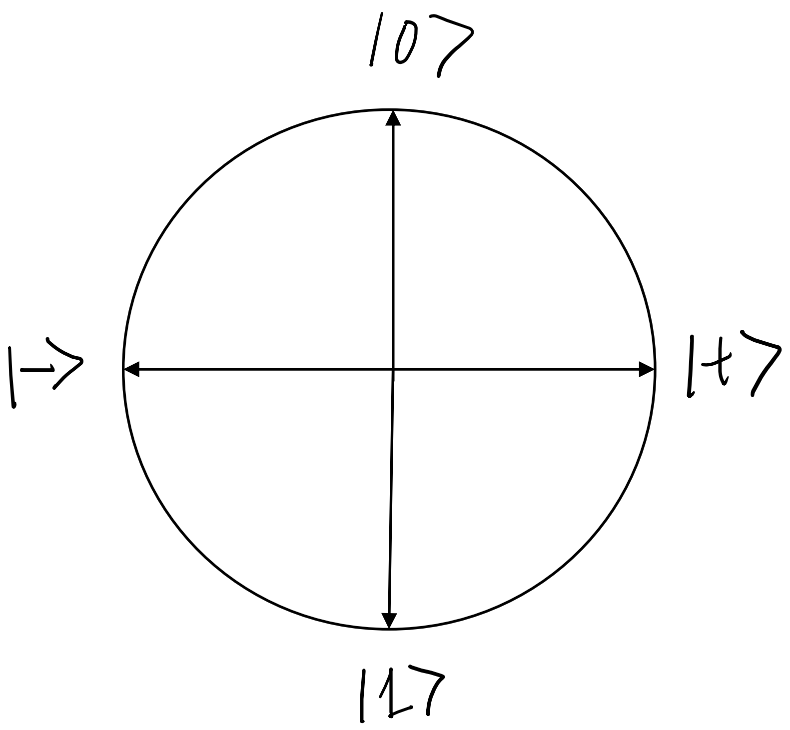
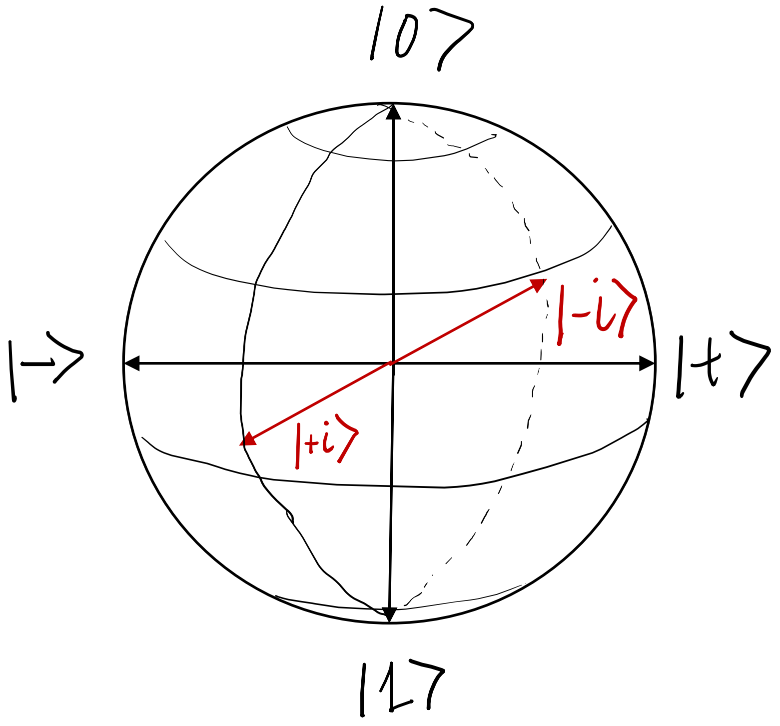


$\eta \geq \frac{1}{3}$

The results can be extended to **multiple bases** and we show that is **more loss-tolerant**





...

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...

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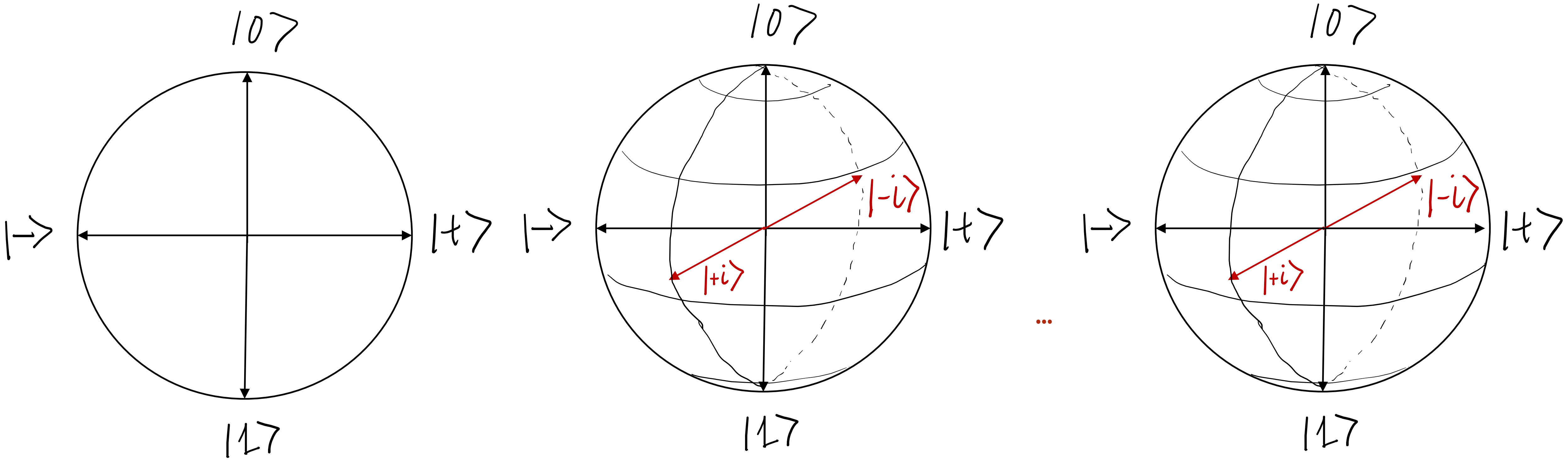

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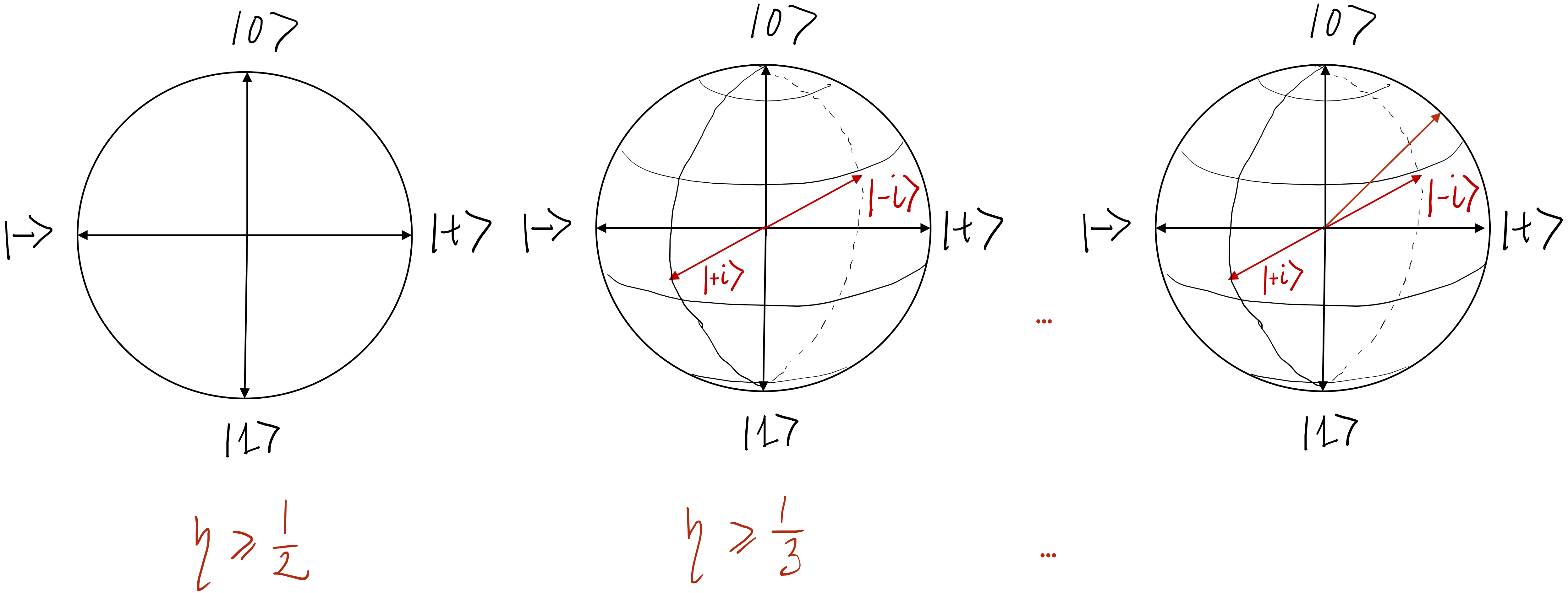

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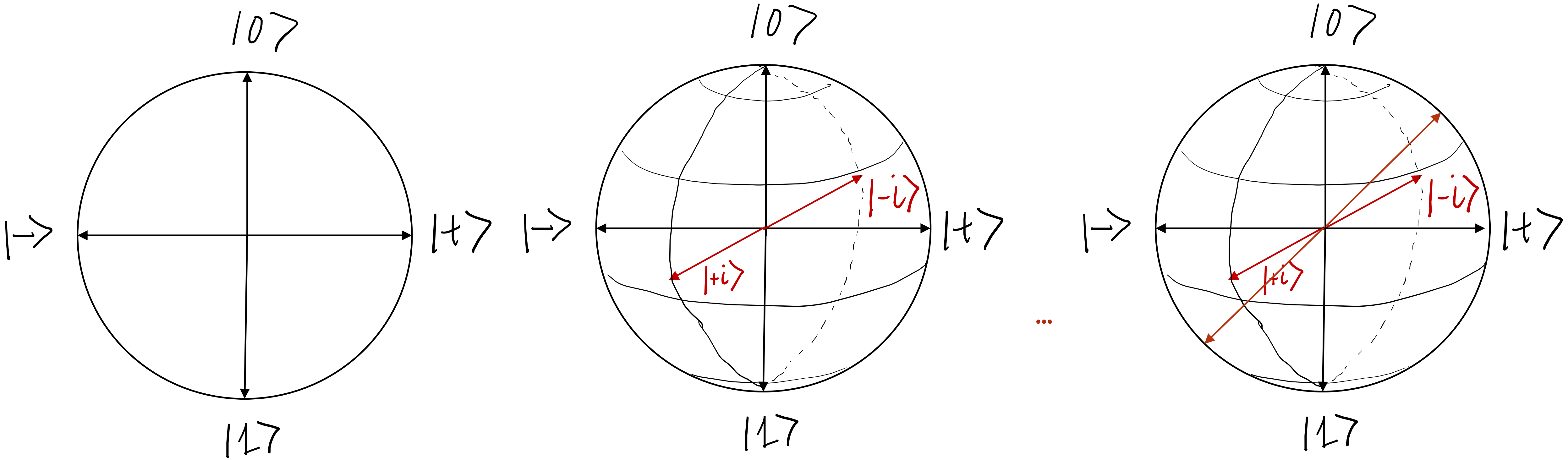

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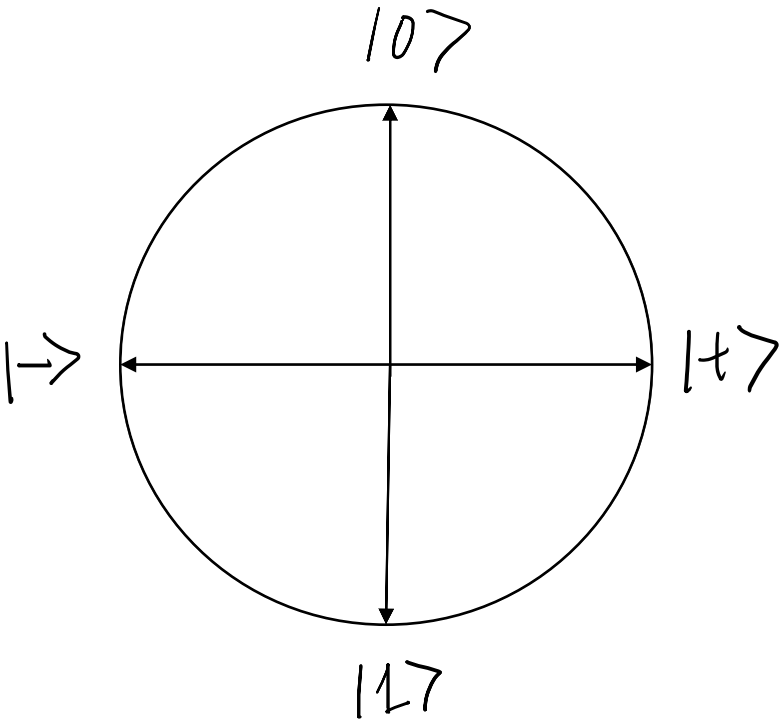

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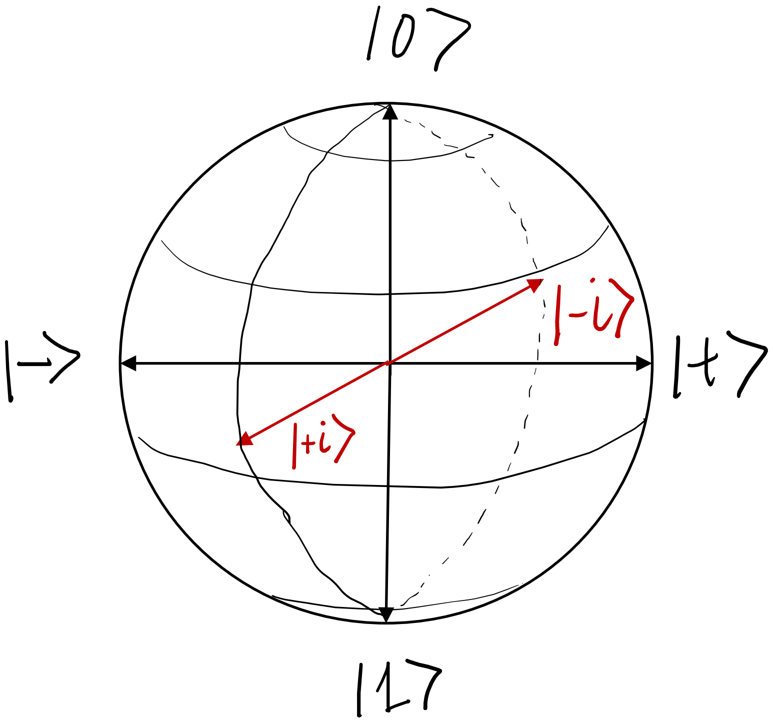

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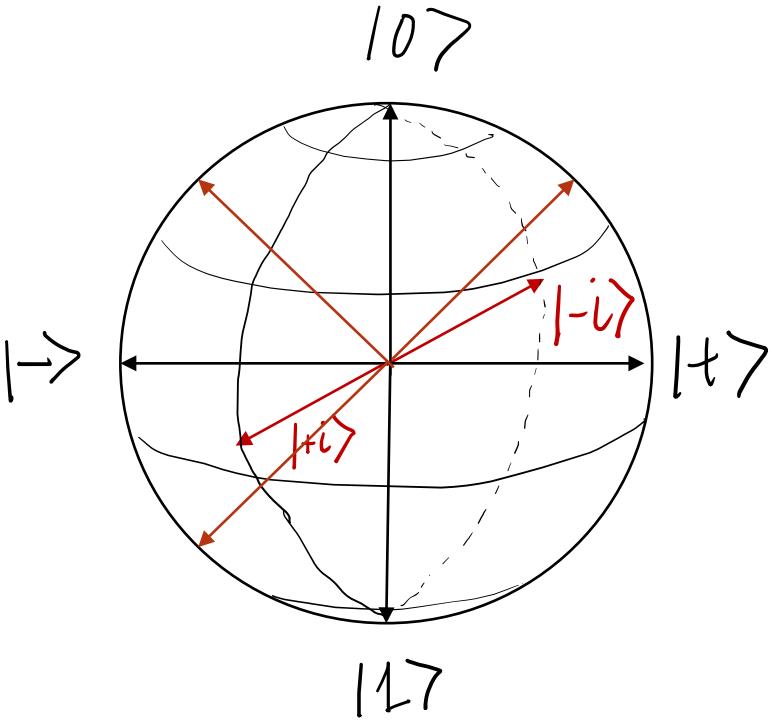

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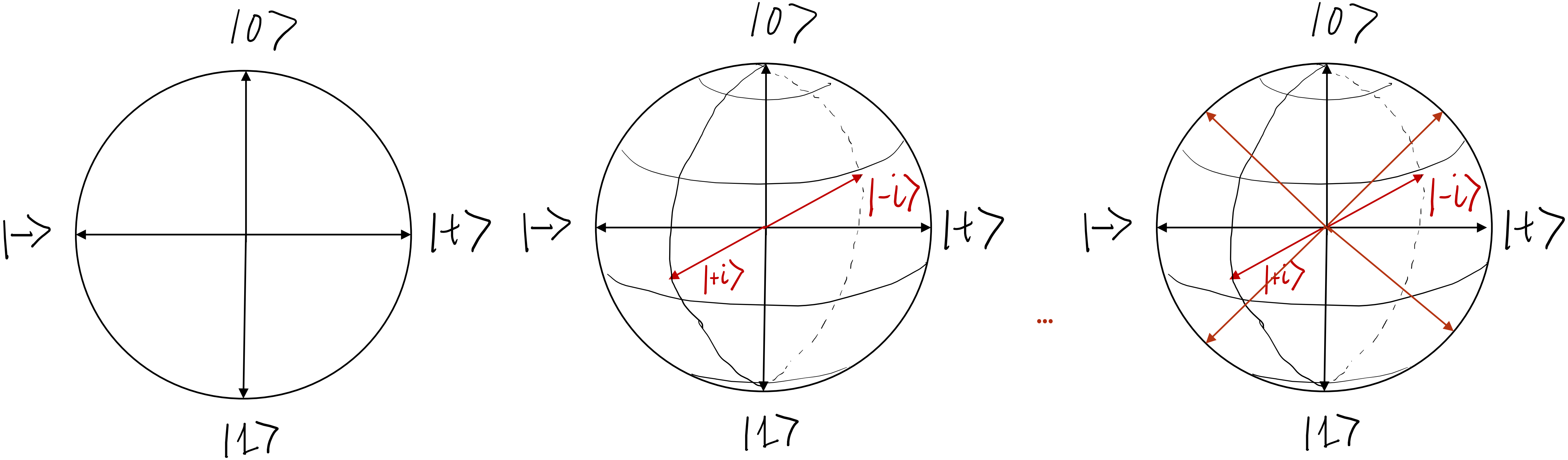

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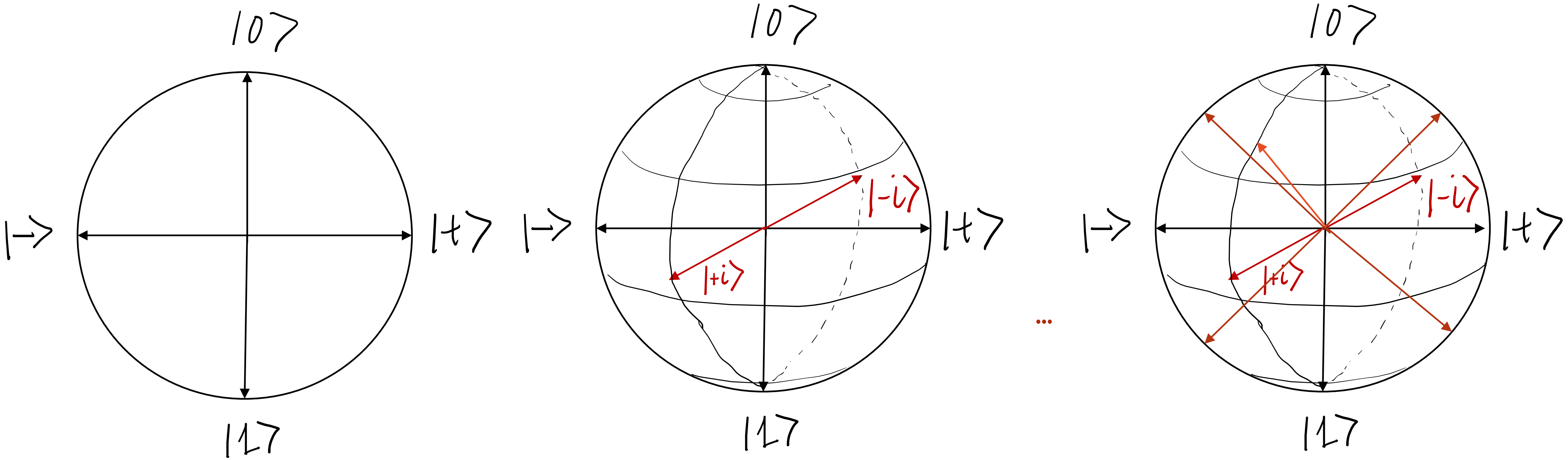

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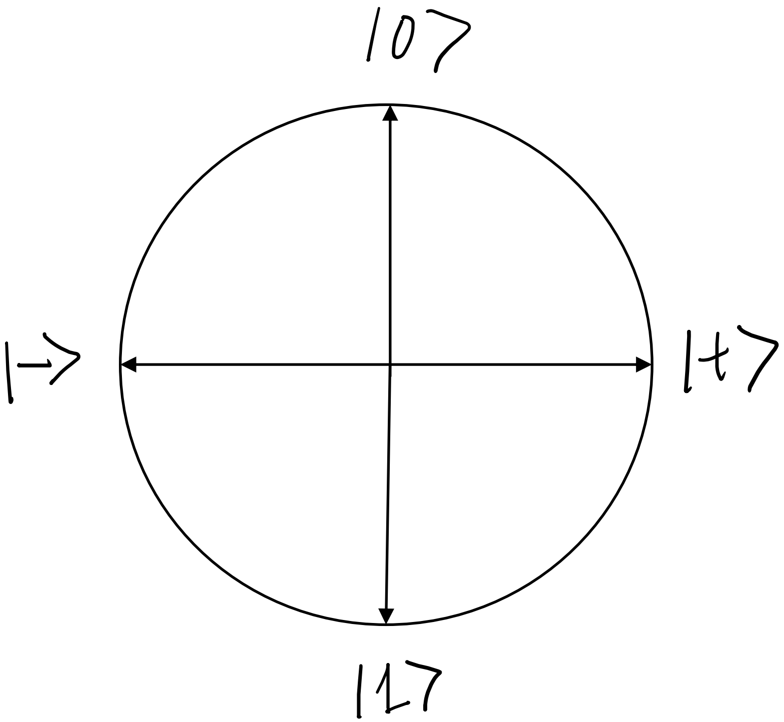

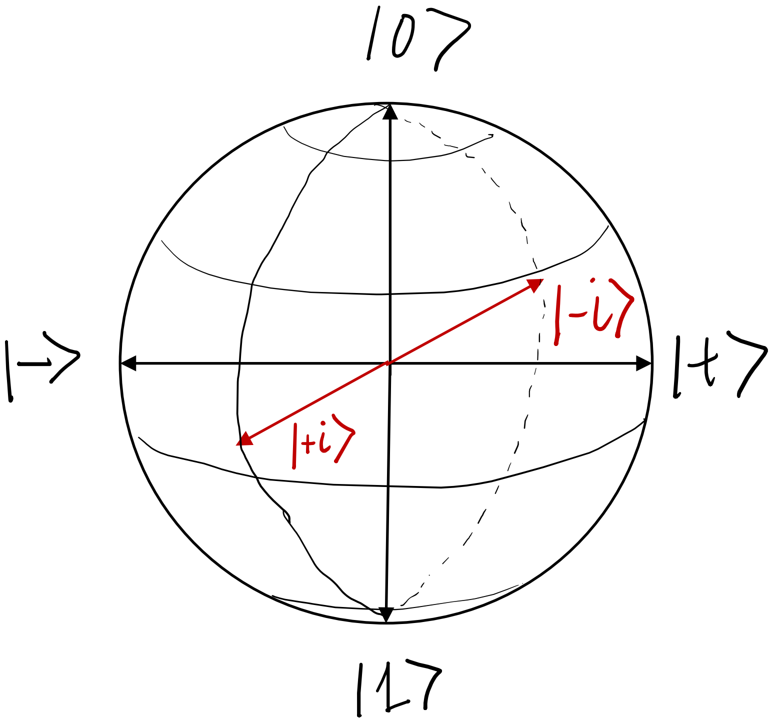

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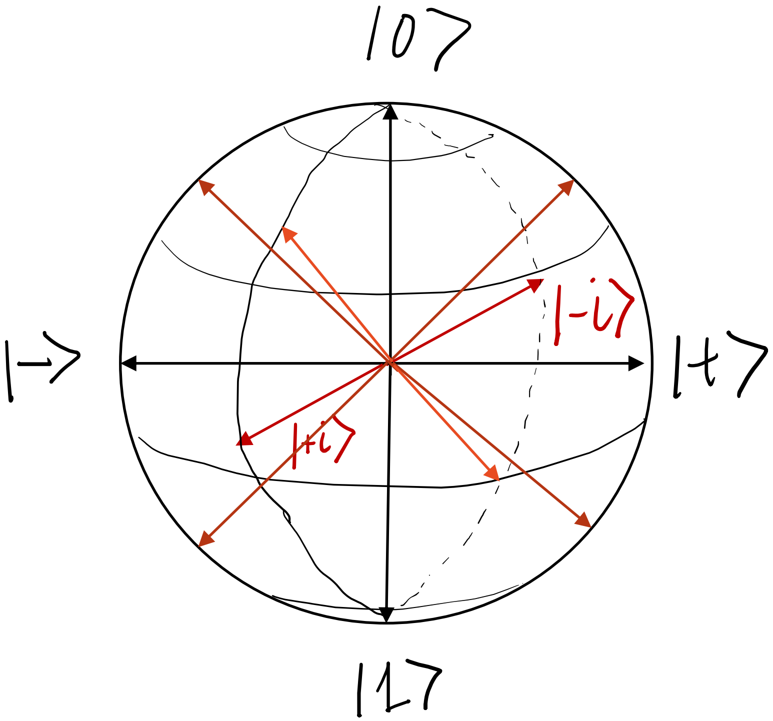

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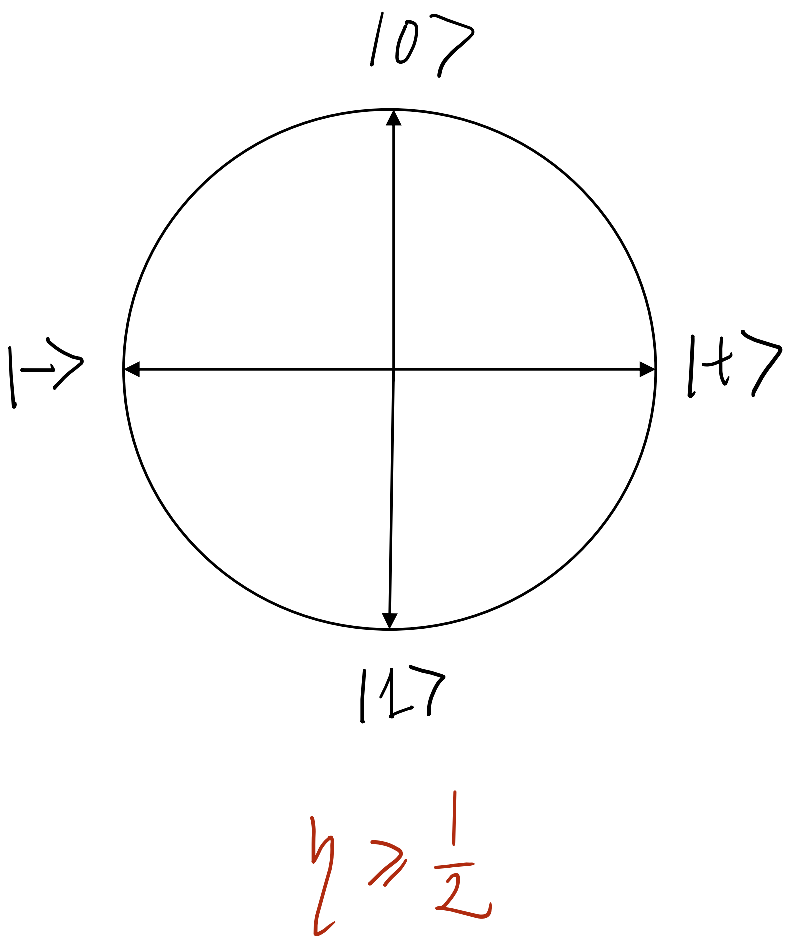

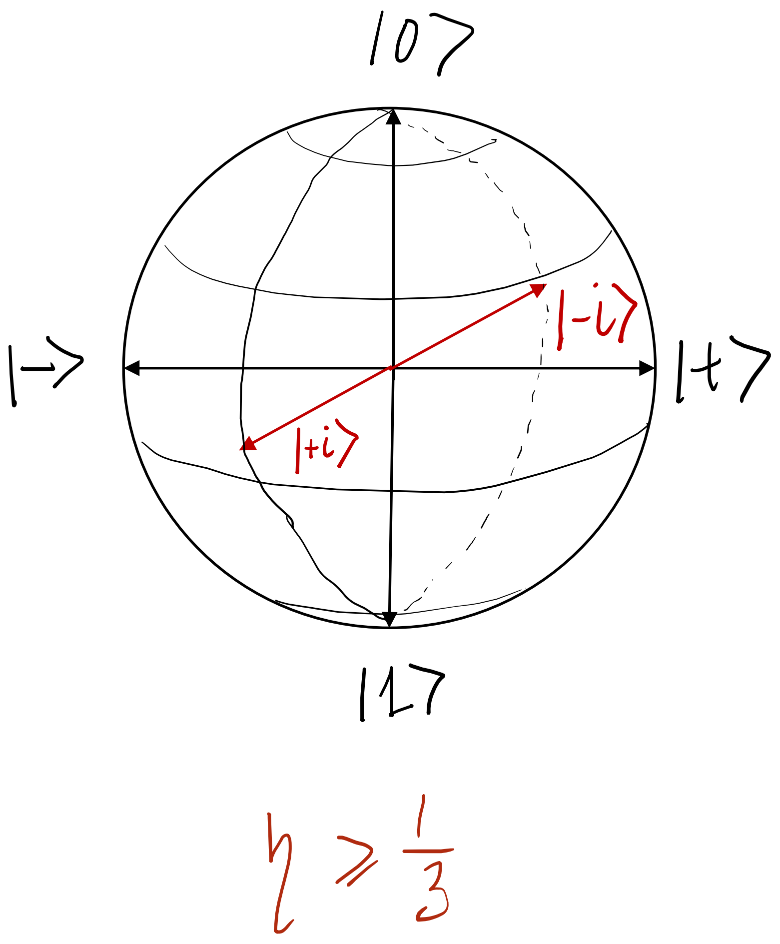

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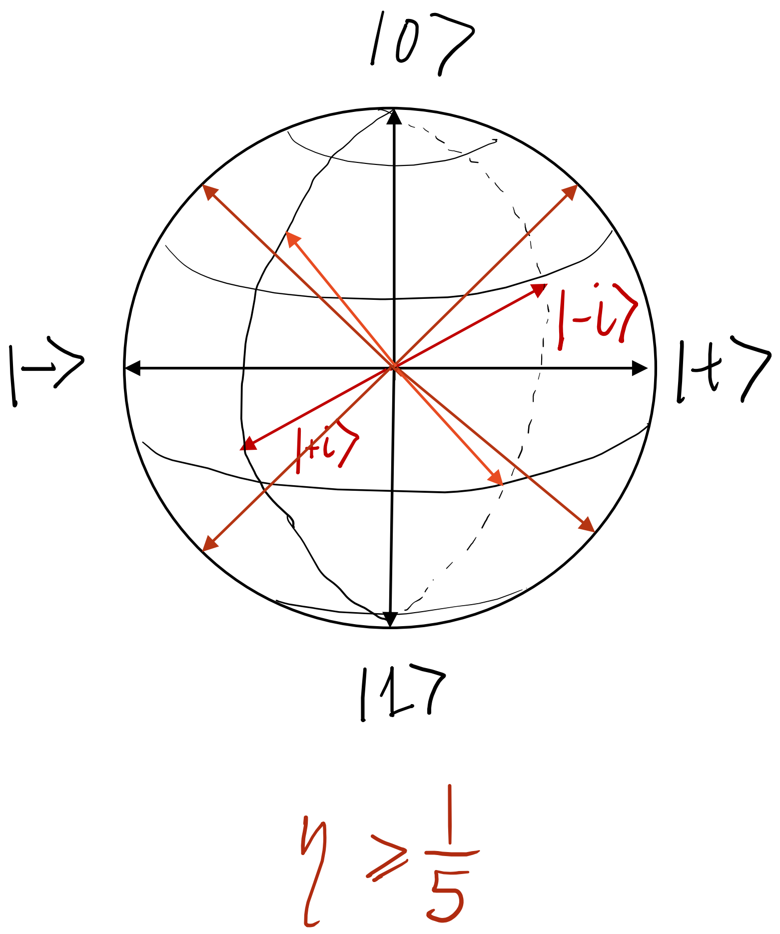


Photon loss
Entanglement
Slow quantum info
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GEORGIA
ALABAMA
CUBA
HAITI
DOMINICAN REP.
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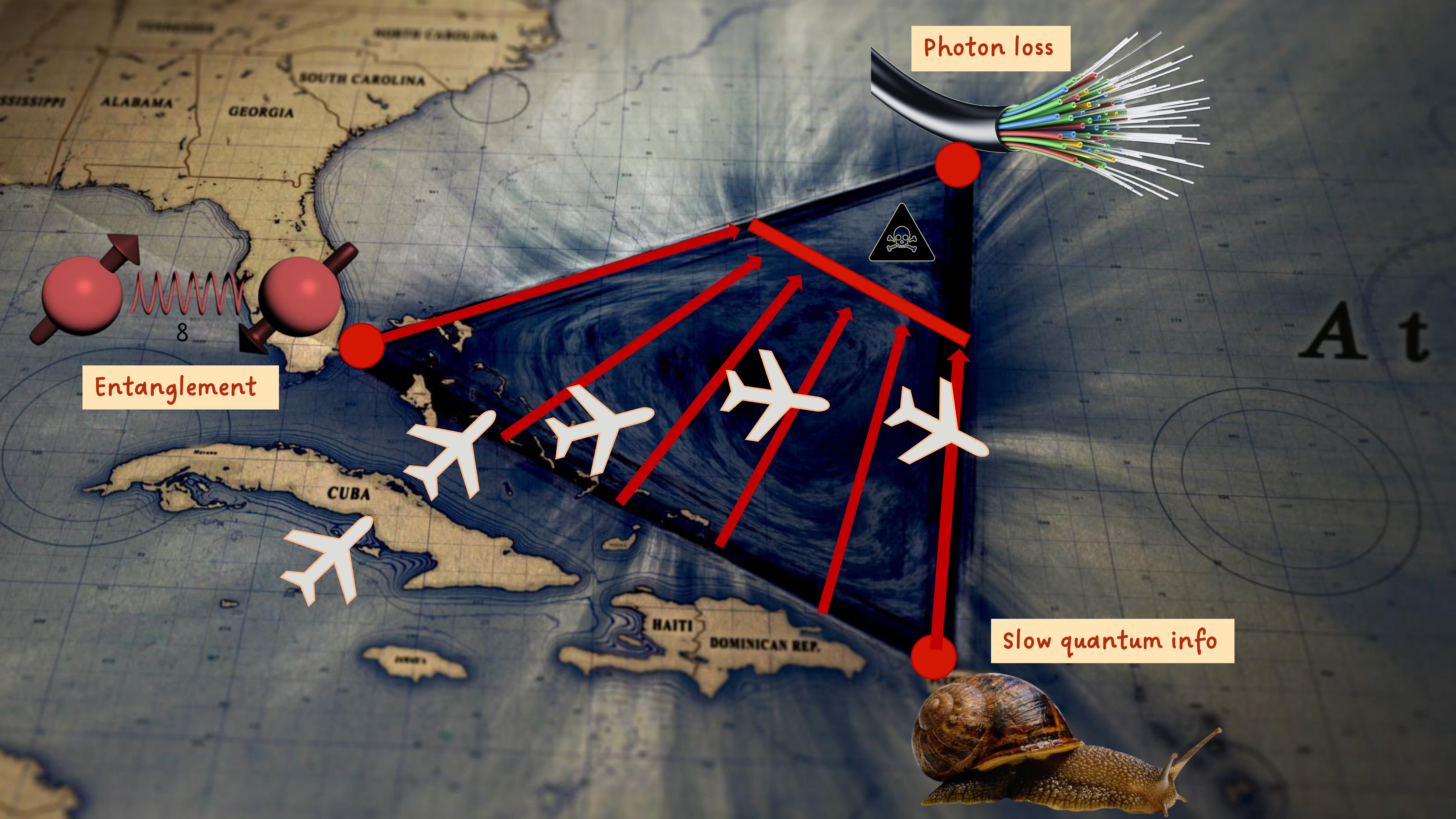

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Thanks for you attention!

Llorenç Escolà Farràs, PhD candidate

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QuSoft

Research Center for Quantum Software

Universiteit van Amsterdam