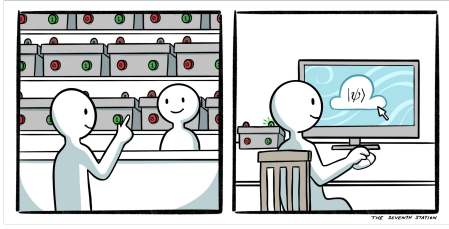
Quantum delegation with an off-the-shelf device

Arthur Mehta

University of Ottawa

Joint work with *Anne Broadbent* and *Yuming Zhao*, based on arXiv:2304.03448



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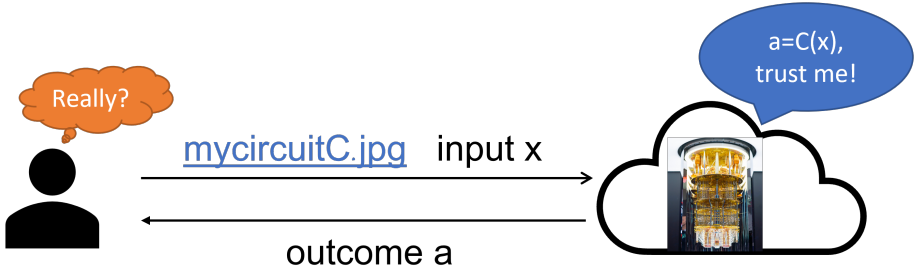
Delegation of quantum computations



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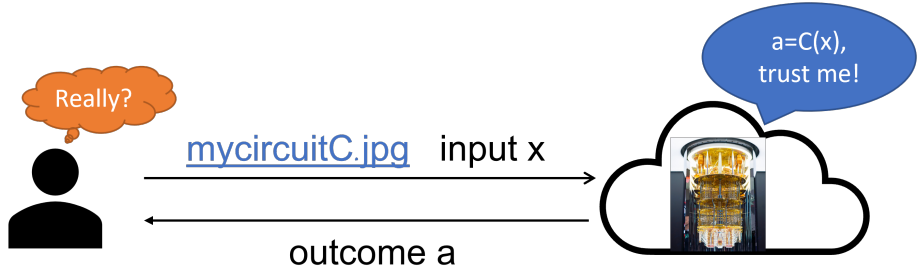

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- I want to be convinced of the correctness,
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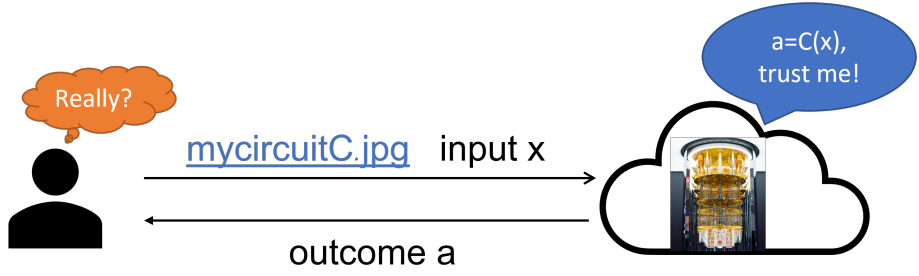

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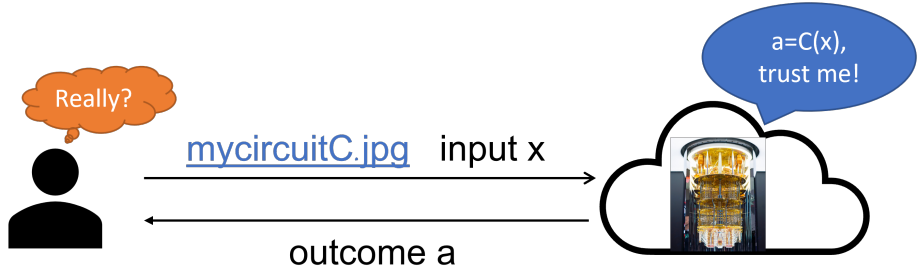

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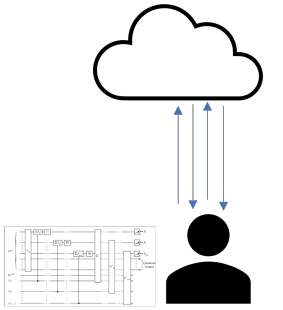
Goal: **zero-knowledge verifiable** delegation of quantum computations

Previous proposed protocols



limited, trusted quantum devices

- Not fully device independent.

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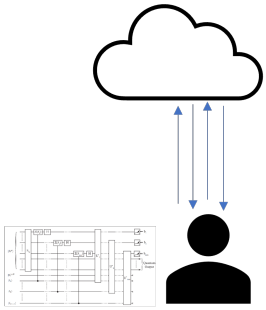

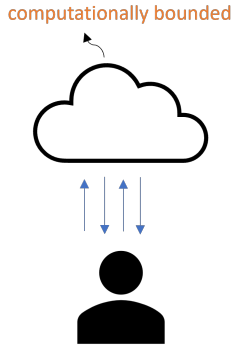

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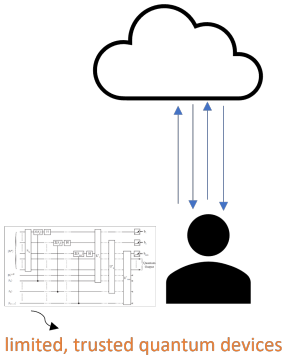

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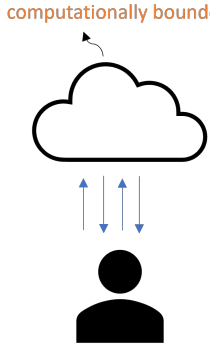

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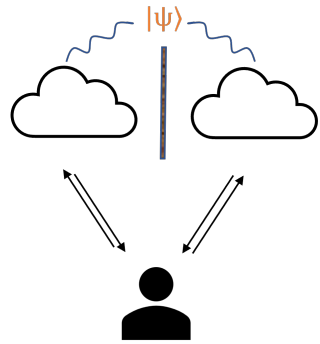

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- Requires 6 or more servers for zero-knowledge.

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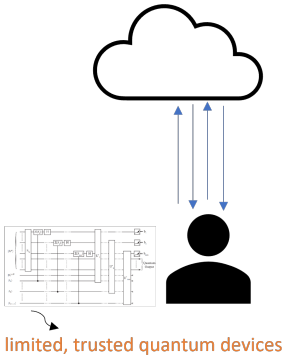

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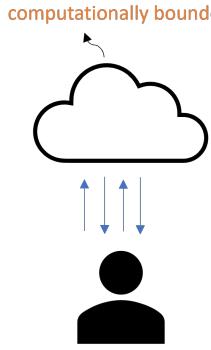

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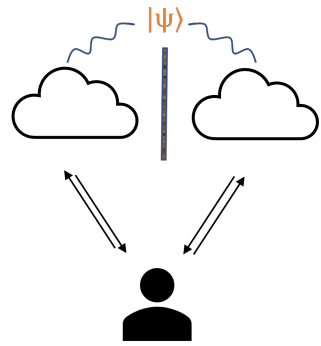

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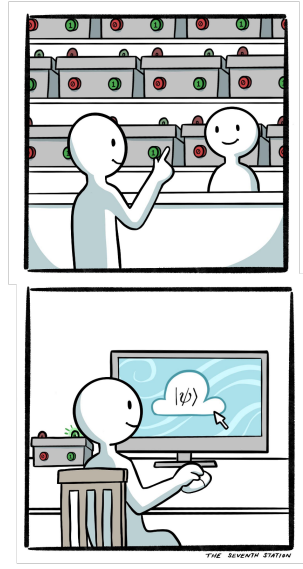
Our model: a **single** quantum server + an **untrusted** device, all in a **single** round

The OTS model

A client wants to delegate a quantum computation $\mathcal{C}(x)$



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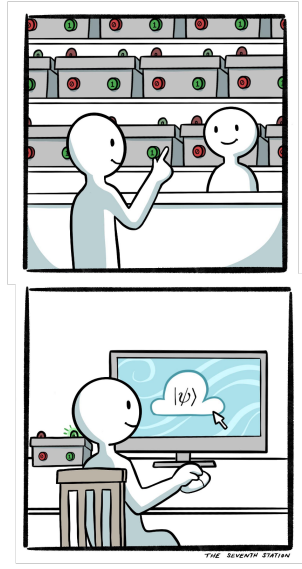

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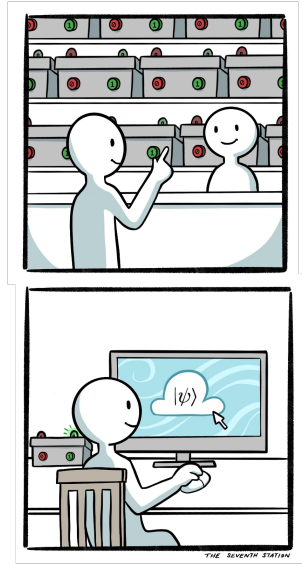

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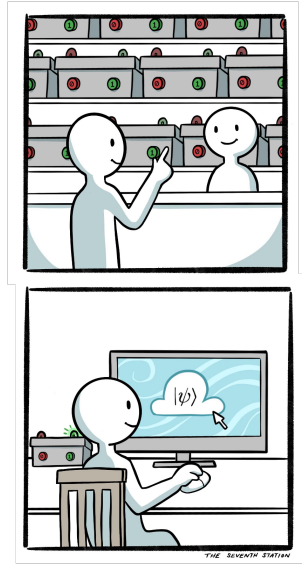
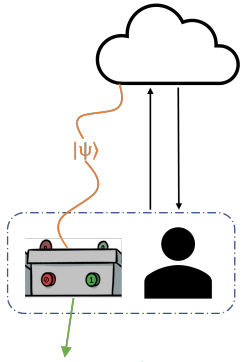
Note The shared state only depends on $|x|$



The OTS model in action



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- Small: measures 6 qubits
- Untrusted: DI techniques
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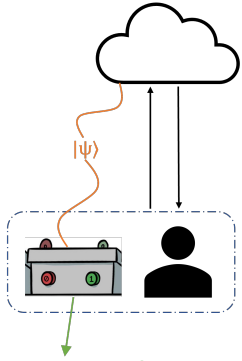

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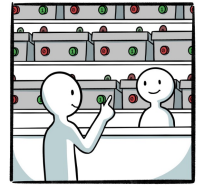

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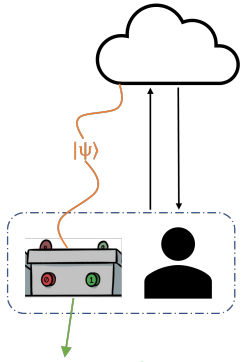

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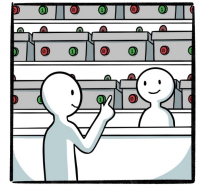

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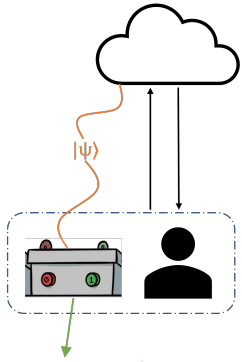

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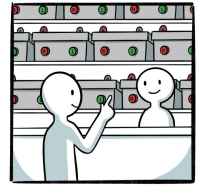

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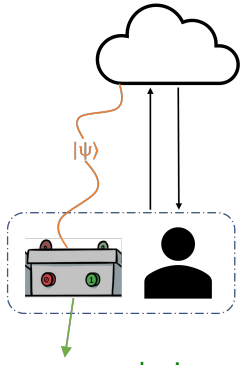

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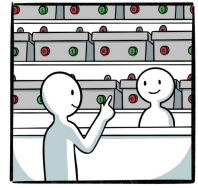

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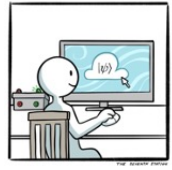

- Accepts if $\lambda_0(H)$ is low
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Main Results

Theorem (Broadbent, M, Zhao 2023)

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Can amplified to constant completeness-soundness gap while preserving ZK.

Obstructions to device independence

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Verifier's Task: Certify many EPR pairs.

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Technical Contributions (Informal)

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- $V_x \mapsto H_x$ *an XZ-Hamiltonian,*
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Very Informal: Prove an enhanced version of Gowers Hatami theorem from approximate representation theory.

Remarks and Open Problems

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2. We know $\mathrm{QMA} \subseteq \mathrm{OTS}$. Is it possible $\mathrm{OTS} = \mathrm{MIP}^*$?
   - What if we lift the constant measurement requirement?
3. Applications to PoQK via self-testing.
   - Can we show our overall protocol self-tests for ground states?