Quantum Secure Non-Malleable Randomness Encoder and its Applications [1] (and) Split-State Non-Malleable Codes and Secret Sharing Schemes for Quantum Messages [2]

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Joint work with Rishabh Batra, Vipul Goyal, Rahul Jain and João Ribeiro

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Outline

Introduction

Non-Malleable Codes (NMCs) [DPW10]

- NMCs encode a message M in a manner such that tampering the codeword results in the decoder either outputting the original message M or a message that is unrelated/independent of M.
- $M \to \mathsf{Enc}(M) \to f(\mathsf{Enc}(M)) \to \mathsf{Dec}(f(\mathsf{Enc}(M))) = M'$.
- $\forall M$, we need $M' \approx_\epsilon p_f M + (1 - p_f)\mathcal{D}_f$, where $p_f, \mathcal{D}_f$ depend only on f (chosen by adversary from family $f \in \mathcal{F}$).
- NMCs can be thought of as a relaxation of error detecting codes.

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Split-state model

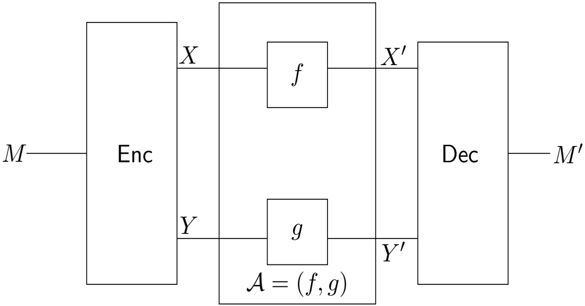

Figure: Split-state model.

Rate of the NMC : $\frac{|M|}{|X|+|Y|}$.

Non-Malleable Randomness Encoder (NMRE) [KOS18]

- “NMRE” can be thought of as a further relaxation of non-malleable codes in the following sense:
  - NMREs output a random message along with its corresponding non-malleable encoding.

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NMRE in the split-state model

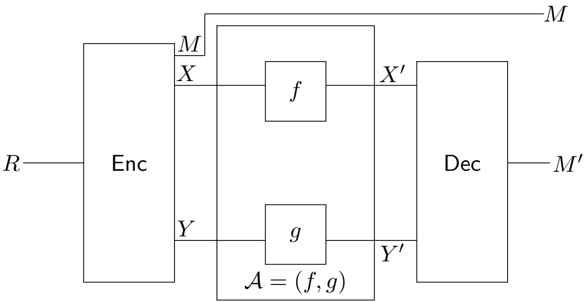


Figure: NMRE in the split-state model.

Rate of the NMRE : $\frac{|M|}{|X|+|Y|}$.

Quantum split-state adversary model [ABJ22]

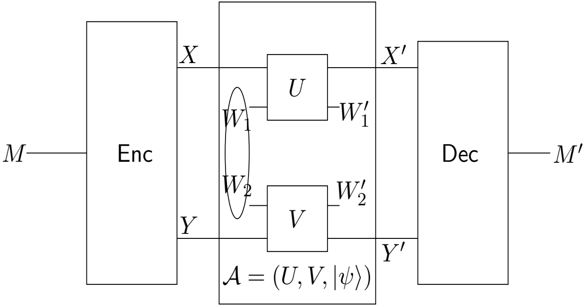


Figure: Quantum split-state adversary model.

Quantum secure NMRE

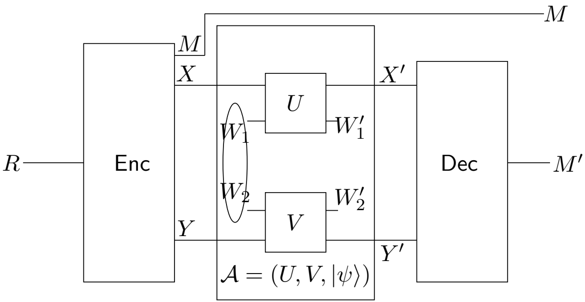


Figure: Quantum secure NMRE.

- NMRE security : $MM' \approx_\varepsilon p_{\mathcal{A}} MM + (1 - p_{\mathcal{A}}) M \otimes M'_{\mathcal{A}}$.
- Analogously, one can consider quantum secure NMC.

Prior work - NMCs in the split-state model

Work by	Rate	Splits	Messages	Adversary
CZ19	$\Omega(1)$	10	classical	classical
KOS18	$1/3$	3	classical	classical
CGL15	$\Omega\left(\frac{1}{\text{poly}(n)}\right)$	2	classical	classical
Li17	$\Omega\left(\frac{1}{\log n}\right)$	2	classical	classical
Li19	$\Omega\left(\frac{\log\log n}{\log n}\right)$	2	classical	classical
AO20	$\Omega(1)$	2	classical	classical
Li23	$\Omega(1)$	2	classical	classical
AKOOS22	$1/3$	2	classical	classical
ABJ22	$\Omega\left(\frac{1}{\text{poly}(n)}\right)$	2	classical	**quantum**

Prior work - NMRE in the split-state model

Work by	Rate	Messages	Adversary	Splits
KOS18	$1/2$	classical	classical	2

- It is not known to be quantum secure to the best of our knowledge.

Applications - NMCs and NMREs

- In construction of non-malleable secret sharing [GK18a, GK18b, ADN+19].
- In construction of non-malleable commitment schemes [GPR16].
- In secure message transmission and non-malleable signatures [SV19].

Results and few technical details

Our results

- We provide a construction of rate $1/2$, 2-split NMRE which is arguably simpler than the construction in [KOS18] and is quantum secure.

Theorem

There exists a rate $1/2$, 2-split quantum secure NMRE.

Prior work - NMRE [KOS18].

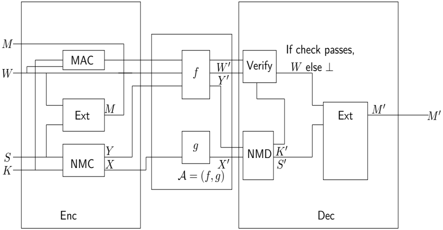


Figure: Rate $1/2$, 2-split NMRE (slightly modified) [KOS18].

- The above construction uses 3 crypto primitives.
  1. MAC - Message authentication code
  2. Ext - Seeded extractor
  3. NMC - Poor rate non-malleable code

Our quantum secure NMRE

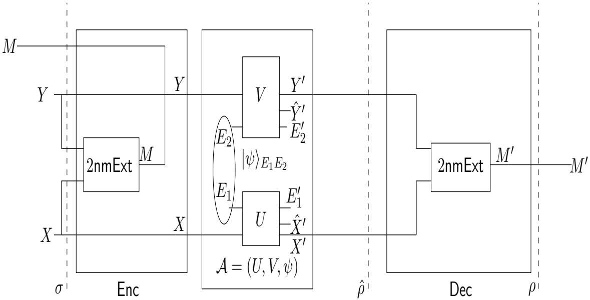
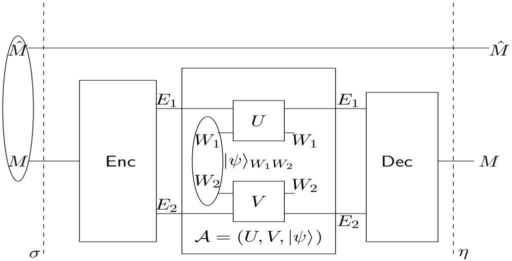


Figure: Rate $1/2$, 2-split quantum secure NMRE.

Our results

- We observe that an NMRE can be constructed using a 2-source non-malleable extractor, 2nmExt.
- Quantum secure 2nmExt construction from earlier work of [BJK21] already gives a rate $1/8$, quantum secure NMRE.
- We modify and optimize parameters of 2nmExt construction from [BJK21] to get a rate $1/2$, quantum secure NMRE.

Definition: Quantum NMCs.



- NMC security: $\forall \sigma_M$, we need $\eta_{M\hat{M}} \approx p_{\mathcal{A}}\sigma_{M\hat{M}} + (1-p_{\mathcal{A}})\gamma_M^{\mathcal{A}} \otimes \sigma_{\hat{M}}$.

Quantum NMC with shared key [AM17]

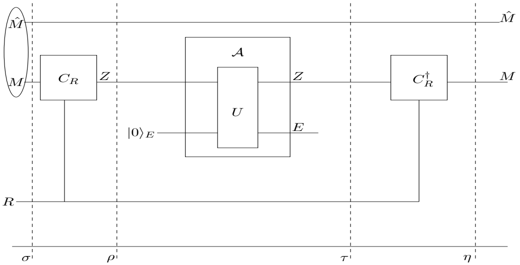


Figure: Quantum NMC with shared key.

- Here, $\{C_r\}_{r \leftarrow R}$ denotes a family of 2-design unitaries.
- Quantum NMC definition from [AM17] is based on mutual information.

3-split quantum NMC

Theorem

There exists a rate $1/11$, 3-split quantum NMC.

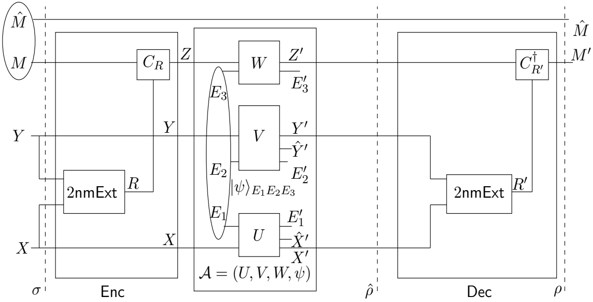


Figure: Rate $1/11$, 3-split quantum NMC.

3-split quantum NMC - High level overview

- Use 2-splits to protect the key R.
- Use the 3rd split to protect the message using 2-design unitaries.
- 1. $R = R'$, security follows from 2-design unitary properties (Pauli mixing and decoupling property).
  2. $RR' = U_R \otimes R'$, security follows from the decoupling property of 2-design unitaries.

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3-split quantum secure NMC

- A similar construction replacing 2-design unitaries by pair-wise independent permutations.
- Rate difference comes from difference in sizes of 2-design unitaries and pair-wise independent permutations.

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There exists a rate $1/5$, 3-split quantum secure NMC.

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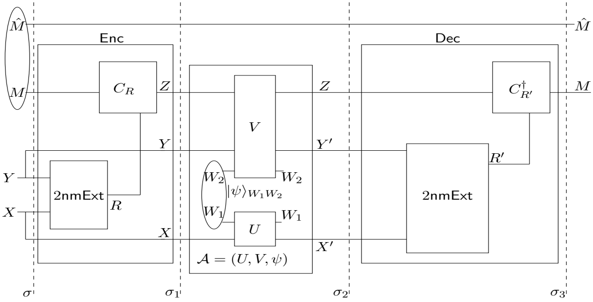
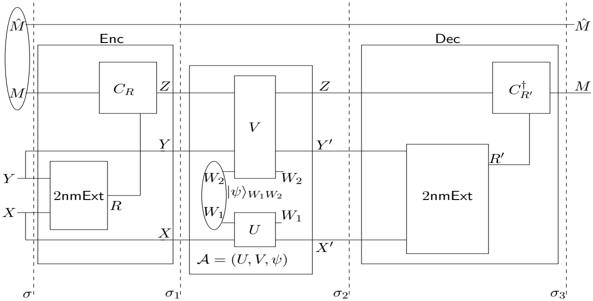
From 3-split to 2-split quantum NMC

- We combine 2-splits as shown below.



From 3-split to 2-split quantum NMC

- Problem: register Z carries information on register R. This implies NMRE security no longer holds.



- Register Z carries no information on R if the input message σ_M is uniform.
- Additionally need - augmented property of 2nmExt.

2-split quantum NMC and quantum secure NMC

Theorem

There exists a rate $1/11$, 2-split quantum NMC for uniform input message.

- Quantum NMC for uniform input message can be thought of as protecting half of maximally entangled state against split-state tamperings.
- Replacing 2-design unitaries by pairwise independent permutations, we get rate $1/5$, 2-split quantum secure NMC for uniform input message.

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Threshold non-malleable secret sharing (NMSS) [GK18a]

- Let M be a classical message and $(\mathrm{Share}, \mathrm{Rec})$ be a t-out-of-p secret sharing scheme.
- Let $\mathrm{Share}(M) = (S_1, \ldots, S_p)$.
- Let adversary Adv tamper $(S_1, \ldots, S_p) \rightarrow (S'_1, \ldots, S'_p)$.
- Let $T = \{1, 2, \ldots, t\}$ be an authorized set to reconstruct the message and $M' = \mathrm{Rec}(S'_1, \ldots, S'_t)$.
- **Non-malleable security:** $MM' \approx p_{\mathsf{Adv}} MM + (1 - p_{\mathsf{Adv}}) M \otimes M'_{\mathsf{Adv}}$.

From 2-split NMC to threshold NMSS [GK18a]

Construction from [GK18a] needs the following:

- a 2-split NMC $(\mathrm{2nmShare}, \mathrm{2nmRec})$.
- additionally:
  - a t-out-of-p secret sharing scheme $(\mathrm{Share}, \mathrm{Rec})$.
  - a 2-out-of-p leakage resilient secret sharing scheme $(\mathrm{lrShare}, \mathrm{lrRec})$.

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From 2-split NMC to threshold NMSS [GK18a]

Candidate threshold NMSS scheme from [GK18a]:

1. Compute the split-state encoding $(L, R) = \mathrm{2nmShare}(M)$;
2. Apply Share to L to obtain p shares stored in $L_1, \ldots, L_p$;
3. Apply $\mathrm{lrShare}$ to R to obtain p shares stored in registers $R_1, \ldots, R_p$;
4. Form the i-th final share $S_i = (L_i, R_i)$.

From 2-split NMC to threshold NMSS [GK18a]

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Reduction from threshold NMSS to 2-split NMC [GK18a]

- Tampering of $R \to R'$ must be performed independent of L.
  - R' depends on $R'_1 R'_2$ which further depend on $L_1 L_2$. But note $L_1 L_2$ information theoretically hides L.
- Tampering of $L \to L'$ must be performed independent of R.
  - L' depends on $L'_1 L'_2 \ldots L'_t$ which further depend on $R_1 R_2 \ldots R_t$. Considering, L'_i as a leakage on R_i, $\mathrm{lrShare}$ property implies now L' is independent of R.
- Overall, they identify random variables $LL'ERR'$ such that
  - $L \otimes E \otimes R$
  - $L'L \leftrightarrow E \leftrightarrow RR'$

Analogous reduction for quantum messages

- Tampering $R \to R'$ is independent of L.
  - Analogous to the classical setting.
- Tampering $L \to L'$ is independent of R.
  - Realizing this argument in the quantum setting requires "**augmented**" leakage-resilient secret sharing scheme.
- We cannot identify registers $LL'ERR'$ such that
  - $L \otimes E \otimes R$
  - $L'L \leftrightarrow E \leftrightarrow RR'$

Theorem

*Using 2-split quantum NMC, quantum secret sharing scheme and **augmented** leakage resilient secret sharing scheme (instead of classical schemes) in the GK18a threshold NMSS scheme gives us the threshold quantum NMSS scheme.*

Difficulty in the quantum setting

- $\{X \otimes E \otimes Y\}$ and adversary modifies $(E, X) \to (E, X, X')$ and $(E, Y) \to (E, Y, Y')$.

  1. When adversary is classical, we have $XX' \leftrightarrow E \leftrightarrow YY'$.
  2. When adversary is quantum, above Markov chain may not be true.

Conclusion and open questions

Improved NMCs

Constant rate 2-split NMCs

- Can we design (worst-case) split-state NMCs for quantum messages with a constant rate? This is open even for classical messages against quantum adversaries with shared entanglement. More ambitiously, can we construct (worst-case) split-state NMSS schemes for quantum messages with a constant rate?

NMSS schemes against joint tamperings

- Can we design NMSS schemes for quantum messages that are secure against joint tampering of shares?

Computationally-bounded adversaries

- What can we achieve if we consider computationally-bounded adversaries instead?

Final slide

That's all from my end! Any questions ?